High-energy OPE for polarized DIS

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Outline

- Review of high-energy OPE
- High-energy evolution equation in the shock-wave background
- High-energy OPE with sub-eikonal corrections
- Evolution equations of sub-eikonal corrections
- Conclusions

Based on:

G.A.C. JHEP 01 (2019) 118 arXiv: 1807.11435 [hep-ph]

G.A.C. arXiv: 2101.12744 [hep-ph] (under review for JHEP publication)

DIS structure functions

DIS differential cross-section: in the laboratory frame for detecting the final lepton in the solid angle $d\Omega$ with final energy within $[E', E' + dE']$

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

- M proton (hadronic target) mass
- $P^\mu = p_2^\mu + \frac{M^2}{s}p_1^\mu$ proton momentum
- $q^\mu = p_1^\mu - x_B p_2^\mu$
- p_1^μ, p_2^μ light-cone vectors
  - $p_1^\mu p_{2\mu} = \frac{s}{2}$ $\quad x_B = \frac{-q^2}{s} \ll 1$

DIS structure functions

DIS differential cross-section

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

Hadronic tensor

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right)F_1(x,Q^2) + \left(P_\mu - q_\mu\frac{q\cdot P}{q^2}\right)\left(P_\nu - q_\nu\frac{q\cdot P}{q^2}\right)\frac{F_2(x,Q^2)}{P\cdot q}$$
$$+i\,\varepsilon_{\mu\nu\lambda\sigma}\,q^\lambda S^\sigma\frac{M}{P\cdot q}\,g_1(x,Q^2) + i\,\varepsilon_{\mu\nu\lambda\sigma}q^\lambda\left(S^\sigma - P^\sigma\frac{q\cdot S}{q\cdot P}\right)\frac{M}{P\cdot q}\,g_2(x,Q^2)$$

To extract the polarized structure functions g_1 and g_2, we need the antisymmetric part of the leptonic tensor. This means that both the incoming lepton and the hadronic target have to be polarized.

- S^μ spin of the target
  - $S^2 = -1$ and $S\cdot P = 0$

DIS structure functions

DIS differential cross-section

$$\frac{d^2\sigma}{d\Omega\, dE'} = \frac{\alpha^2}{Mq^4}\frac{E'}{E}L_{\mu\nu}W^{\mu\nu}$$

Hadronic tensor

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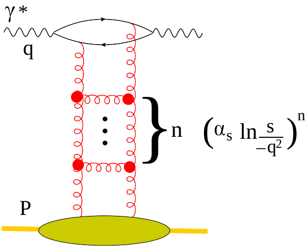
$$+i\,\varepsilon_{\mu\nu\lambda\sigma}\,q^\lambda S^\sigma\frac{M}{P\cdot q}\,g_1(x,Q^2) + i\,\varepsilon_{\mu\nu\lambda\sigma}q^\lambda\left(S^\sigma - P^\sigma\frac{q\cdot S}{q\cdot P}\right)\frac{M}{P\cdot q}\,g_2(x,Q^2)$$

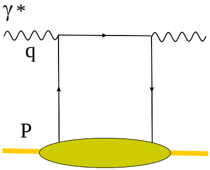
$$W_{\mu\nu} = \frac{1}{\pi}\mathrm{Im}T_{\mu\nu}$$

$$T_{\mu\nu} = i\int d^4x\, e^{iq\cdot x}\langle P,S|\mathrm{T}\{j_\mu(x)j_\nu(0)\}|P,S\rangle$$

electron-proton/nucleus Deep Inelastic Scattering (DIS) $s = (q+P)^2$

$$\langle P|\mathrm{T}\, j^{\mu}(x) j^{\nu}(y)|P\rangle$$

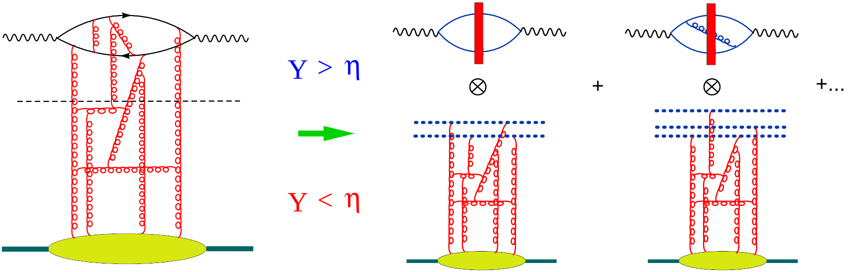




energy suppressed

- BFKL resum $\left(\alpha_s \ln\frac{s}{-q^2}\right)^n$
- Dynamics is linear and it describes proliferation of gluons
  $\Rightarrow$ Violation of Unitarity

remind: High-energy OPE for DIS

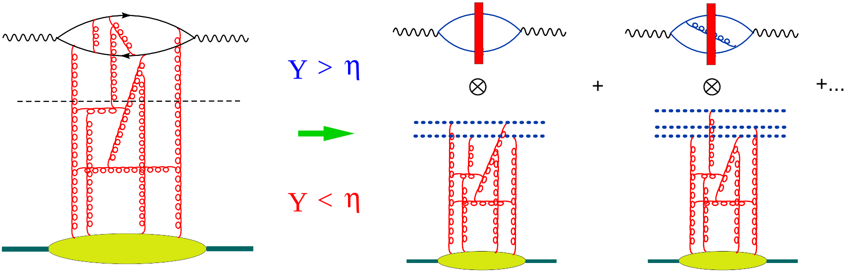


factorization scale: rapidity η

Rapidity Y > η - coefficient function (“impact factor”)

Rapidity Y < η - matrix elements of (light-like) Wilson lines with rapidity divergence cut by η

$$U_x^\eta = \mathrm{Pexp}\Big[ig\int_{-\infty}^{\infty} dx^+ A_+^\eta(x_+, x_\perp)\Big]$$



The high-energy operator product expansion is

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ \mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

$$+\int d^2z_1 d^2z_2 d^2z_3\ \mathcal{I}^{\rm NLO}_{\mu\nu}(z_1,z_2,z_3,x,y)[{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_3}\}{\rm tr}\{\hat{U}^\eta_{z_3}\hat{U}^{\dagger\eta}_{z_2}\} - N_c{\rm tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}]$$

DIS at Leading Log Approximation at high-energy

$$T\{\hat{j}_\mu(x)\hat{j}_\nu(y)\} = \int d^2z_1 d^2z_2\ \mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y){\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$$

- Calculate LO Imapct factor: $\mathcal{I}^{\rm LO}_{\mu\nu}(z_1,z_2,x,y)$
- Calculate evolution of matrix element ${\rm Tr}\{\hat{U}^\eta_{z_1}\hat{U}^{\dagger\eta}_{z_2}\}$: BK/JIMWLK equation
- Solve the evolution equation with initial condition: GBW/MV model
- Convolute the solution of the evolution equation with the impact factor

Electromagnetic gauge invariance

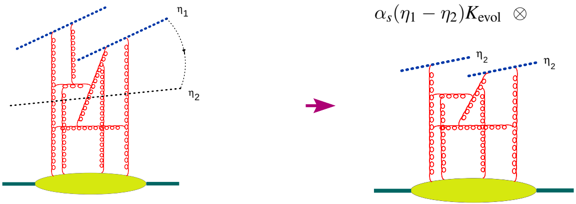
$$\partial^x_\mu \, \mathrm{T}\{j^\mu(x) j^\nu(y)\} = 0 \quad \Rightarrow \quad \partial^x_\mu \, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2) = 0$$

$SL(2,C)$ Möbius invariance (inversion $x^\mu \to \frac{x^\mu}{x^2}$)

$$\int d^2z_1 d^2z_2 \, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2) \stackrel{\text{inv.}}{=} \int d^2z_1 d^2z_2 \, \mathcal{I}^{\mu\nu}(x,y;z_1,z_2)$$

At NLO conformal invariance is restored through the composite conformal Wilson lines (I. Balitsky and G.A.C. 2010)

Evolution Equation



- Separate fields in quantum and classical according to low and large rapidity. Formally we may write:

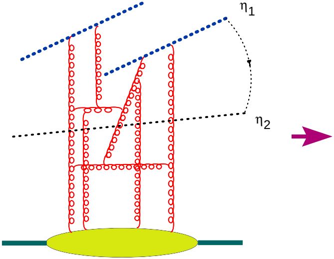
$$\langle B|\mathcal{O}^{\eta_1}|B\rangle \rightarrow \langle \mathcal{O}^{\eta_1}\rangle_A \rightarrow \langle \mathcal{O}'^{\eta_2} \otimes \mathcal{O}'^{\eta_1}\rangle_A$$

- Integrate over the quantum fields and get one-loop rapidity evolution of the operator $\mathcal{O}$

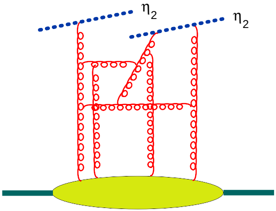
$$\langle \mathcal{O}^{\eta_1}\rangle_A = \alpha_s(\eta_1 - \eta_2)K_{\rm evol} \otimes \langle \mathcal{O}'^{\eta_2}\rangle_A$$

- Where in principle $\mathcal{O}$ and $\mathcal{O}'$ are different operators.

Non-linear evolution equation

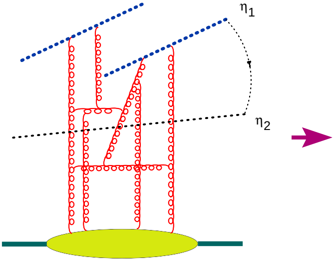


$\alpha_s(\eta_1 - \eta_2)K_{\text{evol}} \;\otimes$

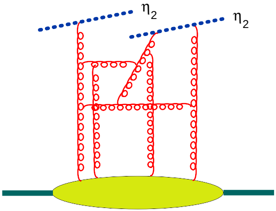


- Linear case $\quad \mathcal{O}^{\eta_1} = \alpha_s \Delta\eta \; K_{\text{evol}} \;\otimes\; \mathcal{O}^{\eta_2}$

Non-linear evolution equation

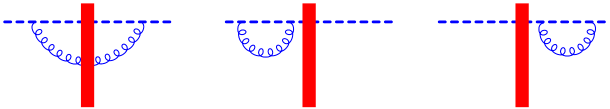


$\alpha_s(\eta_1 - \eta_2)K_{\rm evol} \ \otimes$



- Linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta\, K_{\rm evol} \ \otimes \ \mathcal{O}^{\eta_2}$
- Non-linear case $\mathcal{O}^{\eta_1} = \alpha_s \Delta\eta\, K_{\rm evol} \ \otimes \ \{\mathcal{O}^{\eta_2}\mathcal{O}^{\eta_2}\}$

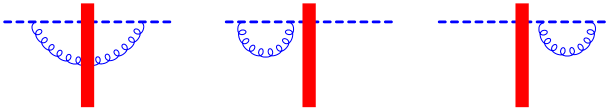
Non-linear evolution equation



$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta\int\frac{d^2z_\perp}{(x-z)_\perp^2}\left[\langle \mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

$\Delta\eta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$



$$\langle\{U_x^{\eta_1}\}_{ij}\rangle_A = \frac{\alpha_s}{2\pi^2}\Delta\eta\int\frac{d^2z_\perp}{(x-z)_\perp^2}\left[\langle\mathrm{tr}\{U_x^{\eta_2}U_z^{\eta_2\dagger}\}\{U_z^{\eta_2}\}_{ij}\rangle_A - \langle\frac{1}{N_c}\{U_x^{\eta_2}\}_{ij}\rangle_A\right]$$

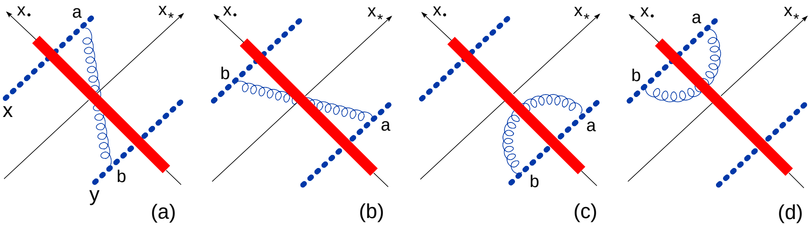
$\Delta\eta = \eta_1 - \eta_2$

$$\{U_x^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\eta_1}\}_{ij},\quad \{U_x^{\eta_1}U_y^{\dagger\eta_1}\}_{ij},\quad \{U_x^{\dagger\eta_1}U_y^{\dagger\eta_1}\}_{ij}$$

- Obtain a set of rules that allow one to get the LO evolution of any trace or product of traces of Wilson lines
- Hierarchy of evolution equation: B-JIMWLK equation.

$$\frac{d}{d\eta}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} = K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\} + \dots \quad \Rightarrow$$

$$\frac{d}{d\eta}\langle\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}} = \langle K_{\mathrm{LO}}\mathrm{tr}\{\hat{U}_x\hat{U}^\dagger_y\}\rangle_{\mathrm{shockwave}}$$



$x_\bullet = \sqrt{\frac{s}{2}}x^-$ $\qquad x_* = \sqrt{\frac{s}{2}}x^+$ $\qquad x^\pm = \frac{x^0 \pm x^3}{\sqrt{2}}$

Non-linear evolution equation

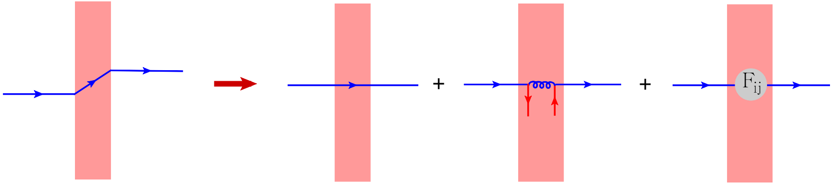
$$\hat{\mathcal{U}}(x,y) \equiv 1 - \frac{1}{N_c}\mathrm{tr}\{\hat{U}(x_\perp)\hat{U}^\dagger(y_\perp)\}$$

$$\frac{d}{d\eta}\hat{\mathcal{U}}(x,y) \;=\; \frac{\alpha_s N_c}{2\pi^2}\int\frac{d^2z\;(x-y)^2}{(x-z)^2(y-z)^2}\Big\{\hat{\mathcal{U}}(x,z)+\hat{\mathcal{U}}(z,y)-\hat{\mathcal{U}}(x,y)-\hat{\mathcal{U}}(x,z)\hat{\mathcal{U}}(z,y)\Big\}$$

- LLA for DIS in pQCD $\Rightarrow$ BFKL
  - (LLA: $\alpha_s \ll 1, \alpha_s\eta \sim 1$): Ladder type of diagrams: proliferation of gluons.

- LLA for DIS in semi-classical-QCD $\Rightarrow$ BK/JIMWLK eqn
  - background field method: describes recombination process.

Quark propagator with sub-eikonal corrections



Quark propagator for g_1 structure function

$$\langle \mathrm{T}\{\psi(x)\bar{\psi}(y)\}\rangle_{A,\psi,\bar{\psi}} = -\frac{1}{2\pi^3 x_*^2 y_*^2}\int \frac{d^2z}{(\mathcal{Z}+i\epsilon)^3}\left(\frac{2}{s}x_*\not{p}_1 + \not{X}_\perp\right)$$
$$\times\left\{i\not{p}_2 U(z_\perp) + \frac{\mathcal{Z}}{8}\left(\frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp) + \gamma_\perp^\mu Q(z_\perp)\gamma_\mu^\perp\right)\right\}\left(\frac{2}{s}y_*\not{p}_1 + \not{Y}_\perp\right)$$

$$\mathcal{Z} \equiv \frac{(x-z)_\perp^2}{x_*} - \frac{(y-z)_\perp^2}{y_*} - \frac{4}{s}(x_\bullet - y_\bullet)$$

$x_* = \sqrt{\frac{s}{2}}x^+ \qquad x_\bullet = \sqrt{\frac{s}{2}}x^-$

Quark propagator with sub-eikonal corrections

$$\langle \mathrm{T}\{\psi(x)\bar{\psi}(y)\}\rangle_{A,\psi,\bar{\psi}} = -\frac{1}{2\pi^3 x_*^2 y_*^2}\int\frac{d^2z}{(\mathcal{Z}+i\epsilon)^3}\Big(\frac{2}{s}x_*\not{p}_1+\not{X}_\perp\Big)$$
$$\times\Big\{i\not{p}_2 U(z_\perp)+\frac{\mathcal{Z}}{8}\Big(\frac{1}{s}\not{p}_2\gamma^5\mathcal{F}(z_\perp)+\gamma_\perp^\mu Q(z_\perp)\gamma_\mu^\perp\Big)\Big\}\Big(\frac{2}{s}y_*\not{p}_1+\not{Y}_\perp\Big)$$

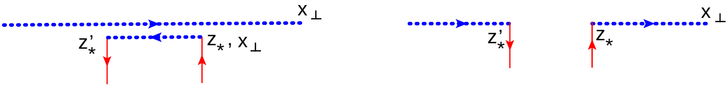
$$Q_{ij}^{\alpha\beta}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big([\infty p_1, z_*]_x t^a\psi^\alpha(z_*,x_\perp)[z_*,z'_*]_x^{ab}\bar{\psi}^\beta(z'_*,x_\perp)t^b[z'_*,-\infty p_1]_x\Big)_{ij}$$

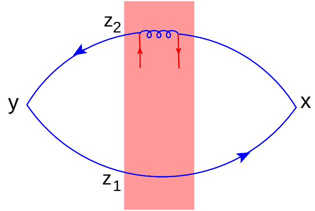
$$\mathcal{F}(z_\perp) \equiv ig\frac{s}{2}\int_{-\infty}^{+\infty}dz_*[\infty p_1, z_*]_z\,\epsilon^{ij}F_{ij}\big(z_*, z_{1\perp}\big)[z_*, -\infty p_1]_z\,.$$

Quark propagator with sub-eikonal corrections

$$Q^{\alpha\beta}_{ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big([\infty p_1, z_*]_x t^a\psi^\alpha(z_*,x_\perp)[z_*,z'_*]^{ab}_x\bar\psi^\beta(z'_*,x_\perp)t^b[z'_*,-\infty p_1]_x\Big)_{ij}$$

$$Q^{\alpha\beta}_{ij}(x_\perp) = -g^2\int_{-\infty}^{+\infty}dz_*\int_{-\infty}^{z_*}dz'_*\Big[\frac{1}{2}\,U^{ij}_x\,\bar\psi^\alpha(z'_*,x_\perp)[z'_*,z_*]_x\psi^\beta(z_*,x_\perp) + \frac{1}{2N_c}\Big([\infty p_1,z_*]_x\psi^\beta(z_*,x_\perp)\bar\psi^\alpha(z'_*,x_\perp)[z'_*,-\infty]\Big)_{ij}\Big]$$





$$
\begin{aligned}
& i\,\bar{\psi}(z'_*, z_{2\perp})\gamma^{\perp}_{\rho}\hat{Y}_2\gamma^{\nu}\hat{Y}_1\not{p}_2\hat{X}_1\gamma^{\mu}\hat{X}_2\gamma^{\rho}_{\perp}\psi(z_*, z_{2\perp}) \\
& = \frac{8}{s}\bar{\psi}(z'_*, z_{2\perp})\, i\not{p}_1\psi(z_*, z_{2\perp})I_1^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) \\
& \quad -\frac{8}{s}\bar{\psi}(z'_*, z_{2\perp})\gamma^5\not{p}_1\psi(z_*, z_{2\perp})I_5^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) + O(\lambda^{-1})
\end{aligned}
$$

$X_i^{\mu} = \frac{2}{s}x_* p_1^{\mu} + X_{i\perp}^{\mu} \qquad X_{i\perp}^{\mu} = x_{\perp}^{\mu} - z_{i\perp}^{\mu} \qquad i = 1, 2$

Impact Factor with sub-eikonal corrections: quark operator

$$
\begin{aligned}
& i\,\bar\psi(z'_*, z_{2\perp})\gamma^\perp_\rho \hat Y_2 \gamma^\nu \hat Y_1 \not{p}_2 \hat X_1 \gamma^\mu \hat X_2 \gamma^\rho_\perp \psi(z_*, z_{2\perp}) \\
&= \frac{8}{s}\bar\psi(z'_*, z_{2\perp})\, i\not{p}_1 \psi(z_*, z_{2\perp}) I_1^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) \\
&\quad - \frac{8}{s}\bar\psi(z'_*, z_{2\perp})\gamma^5 \not{p}_1 \psi(z_*, z_{2\perp}) I_5^{\mu\nu}(x_*, y_*; z_{1\perp}, z_{2\perp}) + O(\lambda^{-1})
\end{aligned}
$$

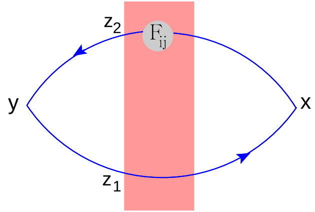
$$
I_1^{\mu\nu}(x, y; z_1, z_2) = \frac{1}{2}x_*^2 y_*^2 \frac{\partial^2}{\partial x_\mu \partial y_\nu}\left(\mathcal{Z}_1 \mathcal{Z}_2 - z_{12\perp}^2 \frac{(x-y)^2}{x_* y_*}\right)
$$

$$
I_5^{\mu\nu}(x, y; z_1, z_2) = \left(x_* \partial_x^\mu - p_2^\mu\right)\left(y_* \partial_y^\nu - p_2^\nu\right)\left[(\vec Y_1 \times \vec Y_2) X_1 \cdot X_2 - (\vec X_1 \times \vec X_2) Y_1 \cdot Y_2\right]
$$

$X_i^\mu = \frac{2}{s} x_* p_1^\mu + X_{i\perp}^\mu \qquad X_{i\perp}^\mu = x_\perp^\mu - z_{i\perp}^\mu \qquad i = 1, 2$

$\vec x \times \vec y = \epsilon^{ij} x_i y_j$

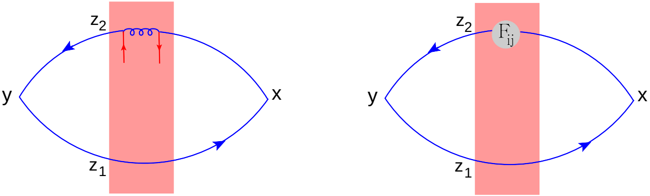
Impact Factor with F_{ij} operator



$$
\begin{aligned}
&\mathrm{tr}\{\not{X}_1\not{p}_2\not{Y}_1\gamma^\nu\not{Y}_2\not{p}_2\sigma_\perp^{\alpha\beta}\not{X}_2\gamma^\mu\} \\
&= 8i\epsilon^{\alpha\beta}\left(x_*\partial_x^\mu - p_2^\mu\right)\left(y_*\partial_y^\nu - p_2^\nu\right)\left[(\vec{X}_1\times\vec{X}_2)Y_1\cdot Y_2 - (\vec{Y}_1\times\vec{Y}_2)X_1\cdot X_2\right] \\
&= 8\,i\,\epsilon^{\alpha\beta}\,I_{\mathcal{F}}^{\mu\nu}
\end{aligned}
$$

$$I_{\mathcal{F}}^{\mu\nu} = -I_5^{\mu\nu}$$

OPE with sub-eikonal corrections



$$
\begin{aligned}
&\mathrm{T}\{\hat{\bar{\psi}}(x)\gamma^\mu\psi(x)\hat{\bar{\psi}}(y)\gamma^\nu\hat{\psi}(y)\} \\
&= \int dz_1 dz_2\, \mathcal{I}^{\mu\nu}_{LO}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\} + \frac{\mathcal{Z}_2}{8\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\mathcal{Q}}_{1z_2}\} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{\mathcal{Q}}^\dagger_{1z_2}\}\Big)\Big] \\
&+\frac{1}{s}\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}_5(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\big(\hat{\mathcal{Q}}_{5z_2} + \hat{\mathcal{F}}_{z_2}\big)\hat{U}^\dagger_{z_1}\} + \mathrm{Tr}\{\big(\hat{\mathcal{Q}}^\dagger_{5z_2} + \hat{\mathcal{F}}^\dagger_{z_2}\big)\hat{U}_{z_1}\}\Big] \\
&+\mathcal{O}(\alpha_s) + \mathcal{O}(\lambda^{-2})
\end{aligned}
$$

$$\mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2) = \frac{\mathcal{Z}_2}{8}\mathcal{I}_{LO}^{\mu\nu} = \frac{1}{4\pi^6 x_*^4 y_*^4}\frac{I_1^{\mu\nu}(x,y;z_1,z_2)}{[\mathcal{Z}_1+i\epsilon]^3[\mathcal{Z}_2+i\epsilon]^2}$$

$$\mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2) = -\frac{1}{4\pi^6 x_*^4 y_*^4}\frac{I_5^{\mu\nu}(x,y;z_1,z_2)}{[\mathcal{Z}_1+i\epsilon]^3[\mathcal{Z}_2+i\epsilon]^2}$$

$$I_1^{\mu\nu}(x,y;z_1,z_2) = \frac{1}{2}x_*^2 y_*^2 \frac{\partial^2}{\partial x_\mu \partial y_\nu}\left(\mathcal{Z}_1\mathcal{Z}_2 - z_{12\perp}^2\frac{(x-y)^2}{x_* y_*}\right)$$

$$I_5^{\mu\nu}(x,y;z_1,z_2) = \left(x_*\partial_x^\mu - p_2^\mu\right)\left(y_*\partial_y^\nu - p_2^\nu\right)\left[(\vec{Y}_1\times\vec{Y}_2)X_1\cdot X_2 - (\vec{X}_1\times\vec{X}_2)Y_1\cdot Y_2\right]$$

$X_i^\mu = \frac{2}{s}x_* p_1^\mu + X_{i\perp}^\mu \qquad X_{i\perp}^\mu = x_\perp^\mu - z_{i\perp}^\mu \qquad i = 1,2$

$\vec{x}\times\vec{y} = \epsilon^{ij}x_i y_j$

Symmetry of the sub-eikonal Impact Factors

Sub-eikonal Impact Factors are electromagnetic gauge invariant

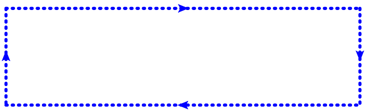
$$\partial_\mu \mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2) = 0$$
$$\partial_\mu \mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2) = 0$$

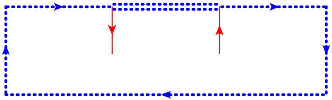
and $SL(2,C)$ Möbius invariant (inv. $x^\mu \to \frac{x^\mu}{x^2}$)

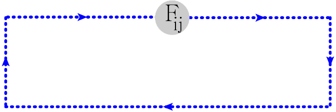
$$\int d^2z_2 d^2z_2\, \mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2) \stackrel{\text{inv.}}{=} \int d^2z_2 d^2z_2\, \mathcal{I}_1^{\mu\nu}(x,y;z_1,z_2)$$
$$\int d^2z_2 d^2z_2\, \mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2) \stackrel{\text{inv.}}{=} \int d^2z_2 d^2z_2\, \mathcal{I}_5^{\mu\nu}(x,y;z_1,z_2)$$

High-Energy OPE with sub-eikonal corrections: flavor singlet

$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^\mu\psi(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\\
&=\int dz_1 dz_2\,\mathcal{I}^{\mu\nu}_{LO}(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}+\frac{\mathcal{Z}_2}{8\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\mathcal{Q}}_{1z_2}\}+\mathrm{Tr}\{\hat{U}_{z_1}\hat{\mathcal{Q}}^\dagger_{1z_2}\}\Big)\Big]\\
&+\frac{1}{s}\int d^2z_1 d^2z_2\,\mathcal{I}^{\mu\nu}_5(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\big(\hat{\mathcal{Q}}_{5z_2}+\hat{\mathcal{F}}_{z_2}\big)\hat{U}^\dagger_{z_1}\}+\mathrm{Tr}\{\big(\hat{\mathcal{Q}}^\dagger_{5z_2}+\hat{\mathcal{F}}^\dagger_{z_2}\big)\hat{U}_{z_1}\}\Big]\\
&+\mathcal{O}(\alpha_s)+\mathcal{O}(\lambda^{-2})
\end{aligned}
$$

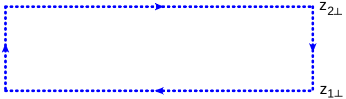


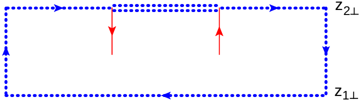




High-Energy OPE with sub-eikonal corrections: flavor non-singlet

$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^{\mu}\psi(x)\bar{\hat{\psi}}(y)\gamma^{\nu}\hat{\psi}(y)\}\\
&=\int dz_1 dz_2\, \mathcal{I}^{\mu\nu}_{LO}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^{\dagger}_{z_2}\} + \frac{\mathcal{Z}_2}{8\,s}\Big(\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}\hat{\mathcal{Q}}_{1z_2}\} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{\mathcal{Q}}^{\dagger}_{1z_2}\}\Big)\Big]\\
&+\frac{1}{s}\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}_{5}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{\mathcal{Q}}_{5z_2}\hat{U}^{\dagger}_{z_1}\} + \mathrm{Tr}\{\hat{\mathcal{Q}}^{\dagger}_{5z_2}\hat{U}_{z_1}\}\Big]\\
&+\mathcal{O}(\alpha_s) + \mathcal{O}(\lambda^{-2})
\end{aligned}
$$





Fierz identity

$\not{p}_1 = \sqrt{\frac{s}{2}}\gamma_+ = \sqrt{\frac{s}{2}}\gamma^-$

$$Q_{1x} = -g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_*\, \bar\psi(z'_*, x_\perp)\, i\not{p}_1 [z'_*, z_*]_x \psi(z_*, x_\perp)$$

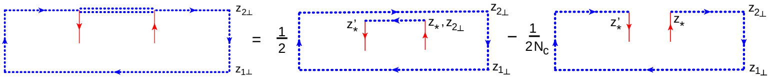
$$Q_{5x} \equiv -g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_*\, \bar\psi(z'_*, x_\perp)\, \gamma^5\not{p}_1 [z'_*, z_*]_x \psi(z_*, x_\perp)$$

$$\tilde{Q}_{1ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* \left([\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, i\not{p}_1\}[z'_*, -\infty p_1]\right)_{ij}$$

$$\tilde{Q}_{5ij}(x_\perp) \equiv g^2\int_{-\infty}^{+\infty} dz_* \int_{-\infty}^{z_*} dz'_* \left([\infty p_1, z_*]_x \mathrm{tr}\{\psi(z_*, x_\perp)\bar\psi(z'_*, x_\perp)\, \gamma^5\not{p}_1\}[z'_*, -\infty p_1]\right)_{ij}$$

$$\mathrm{Tr}\{\hat{Q}_{1x}\hat{U}_y^\dagger\} = \frac{1}{2}\,\mathrm{Tr}\{\hat{U}_y^\dagger\hat{U}_x\}\hat{Q}_{1\,x} - \frac{1}{2N_c}\,\mathrm{Tr}\{\hat{U}_y^\dagger\hat{\tilde{Q}}_{1x}\}$$

$$\mathrm{Tr}\{\hat{Q}_{5x}\hat{U}_y^\dagger\} = \frac{1}{2}\,\mathrm{Tr}\{\hat{U}_y^\dagger\hat{U}_x\}\hat{Q}_{5\,x} - \frac{1}{2N_c}\,\mathrm{Tr}\{\hat{U}_y^\dagger\hat{\tilde{Q}}_{5x}\}$$



High-Energy OPE with sub-eikonal corrections: flavor singlet

$$
\begin{aligned}
&\mathrm{T}\{\hat{\bar{\psi}}(x)\gamma^{\mu}\psi(x)\hat{\bar{\psi}}(y)\gamma^{\nu}\hat{\psi}(y)\} \\
&= \int dz_1 dz_2\, \mathcal{I}^{\mu\nu}_{LO}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}\hat{U}^{\dagger}_{z_2}\} \\
&\quad + \frac{\mathcal{Z}_2}{16\,s}\Big(\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}\hat{U}_{z_2}\}\hat{Q}_{1z_2} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^{\dagger}_{z_2}\}\hat{Q}^{\dagger}_{1z_2} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}\hat{\hat{Q}}_{1z_2}\} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}\hat{\hat{Q}}^{\dagger}_{1z_2}\}\Big)\Big] \\
&\quad + \frac{1}{s}\int d^2z_1 d^2z_2\, \mathcal{I}^{\mu\nu}_{5}(z_{1\perp}, z_{2\perp}; x, y)\Big[\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}\hat{U}_{z_2}\}\hat{Q}_{5z_2} + \mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^{\dagger}_{z_2}\}\hat{Q}^{\dagger}_{5z_2} \\
&\quad - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}^{\dagger}_{z_1}(\hat{\hat{Q}}_{5z_2} - 2N_c\hat{\mathcal{F}}_{z_2})\} - \frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}(\hat{\hat{Q}}^{\dagger}_{5z_2} - 2N_c\hat{\mathcal{F}}^{\dagger}_{z_2})\}\Big] \\
&\quad + \mathcal{O}(\alpha_s) + \mathcal{O}(\lambda^{-2})
\end{aligned}
$$

High-Energy OPE with sub-eikonal corrections: flavor non-singlet

$$
\begin{aligned}
&\mathrm{T}\{\bar{\hat{\psi}}(x)\gamma^\mu\psi(x)\bar{\hat{\psi}}(y)\gamma^\nu\hat{\psi}(y)\}\\
&=\int dz_1 dz_2\,\mathcal{I}^{\mu\nu}_{LO}(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}^\dagger_{z_2}\}\\
&\quad+\frac{\mathcal{Z}_2}{16\,s}\Big(\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}_{z_2}\}\hat{Q}_{1z_2}+\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\hat{Q}^\dagger_{1z_2}-\frac{1}{N_c}\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\tilde{Q}}_{1z_2}\}-\frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}\hat{\tilde{Q}}^\dagger_{1z_2}\}\Big)\Big]\\
&\quad+\frac{1}{s}\int d^2z_1 d^2z_2\,\mathcal{I}^{\mu\nu}_5(z_{1\perp},z_{2\perp};x,y)\Big[\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{U}_{z_2}\}\hat{Q}_{5z_2}+\mathrm{Tr}\{\hat{U}_{z_1}\hat{U}^\dagger_{z_2}\}\hat{Q}^\dagger_{5z_2}\\
&\quad-\frac{1}{N_c}\mathrm{Tr}\{\hat{U}^\dagger_{z_1}\hat{\tilde{Q}}_{5z_2}\}-\frac{1}{N_c}\mathrm{Tr}\{\hat{U}_{z_1}\hat{\tilde{Q}}^\dagger_{5z_2}\}\Big]\\
&\quad+\mathcal{O}(\alpha_s)+\mathcal{O}(\lambda^{-2})
\end{aligned}
$$

Parametrization of the forward matrix elements

Quark distributions

$S^\mu \simeq \frac{\lambda}{M}P^\mu + S_\perp^\mu \qquad \Delta_\perp^\mu = (x-y)_\perp^\mu$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[Q_1(x_\perp)\,\mathrm{Tr}\{U_xU_y^\dagger\} + \text{a.c.}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(q_1(k_\perp^2,x) + \frac{\vec{S}\times\vec{k}}{M}q_{1T}(k_\perp^2,x)\Big)$$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[\,\mathrm{Tr}\{\tilde{Q}_1(x_\perp)U_y^\dagger\} + \text{a.c.}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\tilde{q}_1(k_\perp^2,x) + \frac{\vec{S}\times\vec{k}}{M}\tilde{q}_{1T}(k_\perp^2,x)\Big)$$

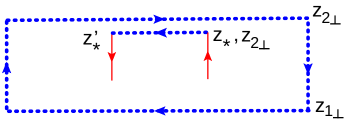
$$\int d^2\Delta e^{i(\Delta,k)}\langle\langle P,S|\Big[Q_5(x_\perp)\mathrm{Tr}\{U_xU_y^\dagger\} + \text{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\lambda q_{5L}(k_\perp^2,x) - \frac{(S,k)_\perp}{M}q_{5T}(k_\perp^2,x)\Big)$$

$$\int d^2\Delta\, e^{i(\Delta,k)}\langle\langle P,S|\Big[\,\mathrm{Tr}\{\tilde{Q}_5(x_\perp)U_y^\dagger\} + \text{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big(\lambda\tilde{q}_{5L}(k_\perp^2,x) - \frac{(S,k)_\perp}{M}\tilde{q}_{5T}(k_\perp^2,x)\Big)$$

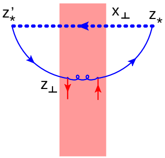
Gluon distributions

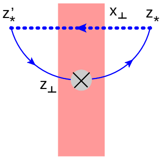
$S^\mu \simeq \frac{\lambda}{M}P^\mu + S^\mu_\perp$ $\qquad\Delta^\mu_\perp = (x-y)^\mu_\perp$

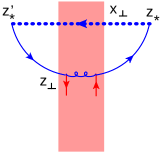
$$\int d^2\Delta e^{i(\Delta,k)_\perp}\langle\langle P,S|\Big[\mathrm{Tr}\{\mathcal{F}(x_\perp)U^\dagger_y\} + \mathrm{a.c}\Big]|P,S\rangle\rangle = \frac{s}{2}\Big[\lambda\frac{k^2_\perp}{M^2}G_L(k^2_\perp,x) + \frac{(S,k)_\perp}{M}G_T(k^2_\perp,x)\Big]$$

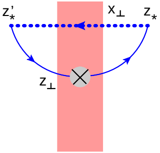


Diagrams at one loop: quantum quark field









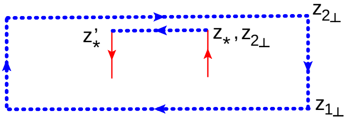
$$\langle \mathrm{Tr}\{U_y^\dagger U_x\} Q_{1x}\rangle = \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \frac{\mathrm{Tr}\{U_y^\dagger U_x\}}{(x-z)_\perp^2}\Big[\mathrm{Tr}\{U_x^\dagger U_z\}\, Q_{1z} - \frac{1}{N_c}\mathrm{Tr}\{U_x^\dagger \tilde{Q}_{1z}\}\Big]$$

and

$$\langle \mathrm{Tr}\{U_y^\dagger U_x\} Q_{5x}\rangle = \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \frac{\mathrm{Tr}\{U_y^\dagger U_x\}}{(x-z)_\perp^2} \times\Big[\mathrm{Tr}\{U_x^\dagger U_z\}\, Q_{5z} - \frac{1}{N_c}\mathrm{Tr}\{U_x^\dagger\big(\tilde{Q}_{5z} - 2N_c\mathcal{F}_z\big)\}\Big]$$

Sanity check: operators of different parity do not mix

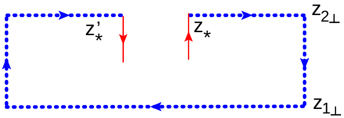
Evolution equation of sub-eikonal corrections



Diagrams at one loop: classical quark field (BK diagrams)

$$Q_{1x}\langle \mathrm{Tr}\{U_x U_y^\dagger\}\rangle = \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \frac{(x-y)_\perp^2}{(x-z)_\perp^2(y-z)_\perp^2} \times\Big[\mathrm{Tr}\{U_x U_z^\dagger\}\mathrm{Tr}\{U_z U_y^\dagger\} - N_c \mathrm{Tr}\{U_x U_y^\dagger\}\Big] Q_{1x}$$

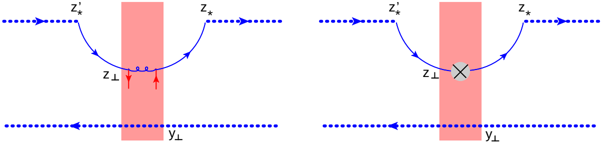
Evolution equation of sub-eikonal corrections



Diagrams at one loop: quantum quark field

Figure : Diagrams with $\tilde{Q}_{1x}$ and $\tilde{Q}_{5x}$ quantum.

Evolution equation of sub-eikonal corrections



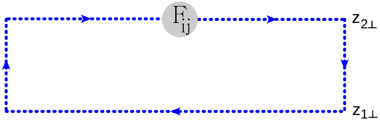
$$\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1x}\}\rangle = \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\frac{d^2z}{(x-z)_\perp^2}\Big[\mathrm{Tr}\{U_y^\dagger U_z\}Q_{1z} - \frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1z}\}\Big]$$

and

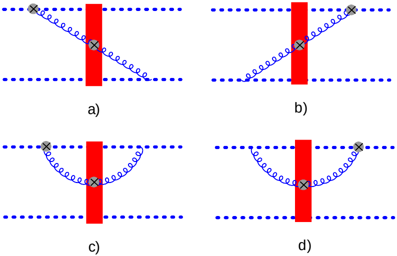
$$\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{5x}\}\rangle \;=\; \frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\frac{d^2z}{(x-z)_\perp^2} \times\Big[\mathrm{Tr}\{U_y^\dagger U_z\}Q_{5z} - \frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\big(\tilde{Q}_{5z} - 2N_c\mathcal{F}_z\big)\}\Big]$$

Sanity check: operators of different parity do not mix

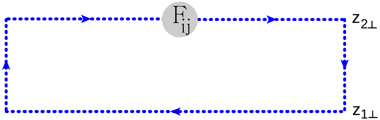
Evolution equation of sub-eikonal corrections



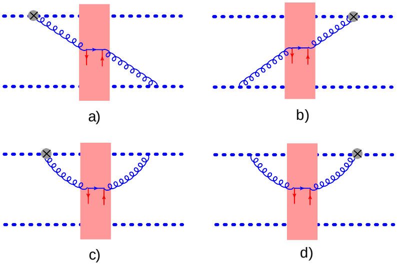
Diagrams with F_{ij} quantum



Evolution equation of sub-eikonal corrections



Diagrams with F_{ij} quantum



Evolution equation of sub-eikonal corrections

Diagrams with F_{ij} quantum

$$\langle \mathrm{Tr}\{\mathcal{F}_x\, U^\dagger_y\}\rangle \;=\; -\frac{\alpha_s}{\pi^2}\,\mathrm{Tr}\{U_x t^a U^\dagger_y t^b\}\,\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2 z$$
$$\times\Bigg[\frac{(\vec{x}-\vec{z})\times(\vec{z}-\vec{y})}{(x-z)^2_\perp\,(y-z)^2_\perp}\Big(\mathcal{Q}^{ba}_{1z}-\mathcal{Q}^{ba\,\dagger}_{1z}\Big)$$
$$-\Big(\frac{(x-z,z-y)}{(x-z)^2_\perp\,(y-z)^2_\perp}+\frac{1}{(x-z)^2_\perp}\Big)\Big(\mathcal{Q}^{ba}_{5z}+\mathcal{Q}^{ba\,\dagger}_{5z}+\mathcal{F}^{ba}_z\Big)\Bigg]$$

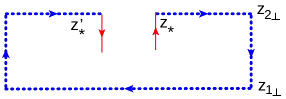
$$\mathcal{Q}^{ab}_5(z_\perp)\equiv g^2\int_{-\infty}^{+\infty}dz_{1*}\int_{-\infty}^{z_{1*}}dz_{2*}\bar{\psi}(z_{1*},z_\perp)\,\gamma^5\not{p}_1\,[z_{1*},\infty p_1]_z\,t^a U_z\,t^b\,[-\infty p_1,z_{2*}]_z\,\psi(z_{2*},z_\perp)$$

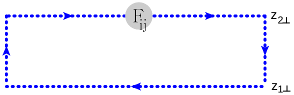
$$\mathcal{Q}^{ab}_1(z_\perp)\equiv g^2\int_{-\infty}^{+\infty}dz_{1*}\int_{-\infty}^{z_{1*}}dz_{2*}\bar{\psi}(z_{1*},z_\perp)\,i\not{p}_1\,[z_{1*},\infty p_1]_z\,t^a U_z\,t^b\,[-\infty p_1,z_{2*}]_z\,\psi(z_{2*},z_\perp)$$

Evolution equation of sub-eikonal corrections

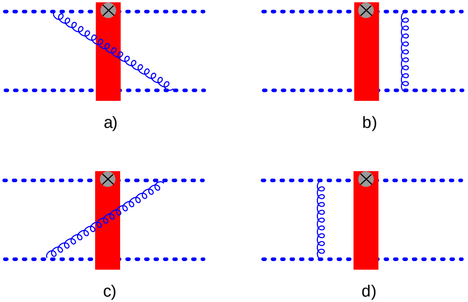
Diagrams with F_{ij} quantum

$$
\begin{aligned}
\langle \mathrm{Tr}\{\mathcal{F}_x\, U^\dagger_y\}\rangle =\ & \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
& \times\Bigg\{\frac{1}{2}\frac{(\vec{x}-\vec{z})\times(\vec{z}-\vec{y})}{(x-z)^2_\perp\,(y-z)^2_\perp}\Big[\mathrm{Tr}\{U^\dagger_y\tilde{Q}_{1\,z}\}\mathrm{Tr}\{U^\dagger_z U_x\} - \mathrm{Tr}\{U_x\tilde{Q}^\dagger_{1z}\}\mathrm{Tr}\{U^\dagger_y U_z\} \\
& + \frac{1}{N_c}\Big(\mathrm{Tr}\{U_x U^\dagger_y\tilde{Q}_{1z}U^\dagger_z\} + \mathrm{Tr}\{U^\dagger_y U_x U^\dagger_z\tilde{Q}_{1z}\} - \mathrm{Tr}\{U_x U^\dagger_y U_z\tilde{Q}^\dagger_{1z}\} - \mathrm{Tr}\{U^\dagger_y U_x\tilde{Q}^\dagger_{1z}U_z\}\Big) \\
& + \frac{1}{N_c^2}\mathrm{Tr}\{U^\dagger_y U_x\}\Big(Q^\dagger_{1z} - Q_{1z}\Big)\Big] - \frac{1}{2}\Big[\frac{(x-z,z-y)}{(x-z)^2_\perp\,(y-z)^2_\perp} + \frac{1}{(x-z)^2_\perp}\Big] \\
& \times\Big[\mathrm{Tr}\{U^\dagger_y\big(\tilde{Q}_{5z} - 2\mathcal{F}_z\big)\}\mathrm{Tr}\{U^\dagger_z U_x\} + \mathrm{Tr}\{U_x\big(\tilde{Q}^\dagger_{5z} - 2\mathcal{F}^\dagger_z\big)\}\mathrm{Tr}\{U^\dagger_y U_z\} \\
& - \frac{1}{N_c}\Big(\mathrm{Tr}\{U_x U^\dagger_y U_z\tilde{Q}^\dagger_{5z}\} + \mathrm{Tr}\{U^\dagger_y U_x\tilde{Q}^\dagger_{5z}U_z\} + \mathrm{Tr}\{U_x U^\dagger_y\tilde{Q}_{5z}U^\dagger_z\} + \mathrm{Tr}\{U^\dagger_y U_x U^\dagger_z\tilde{Q}_{5z}\}\Big) \\
& + \frac{1}{N_c^2}\mathrm{Tr}\{U^\dagger_y U_x\}\Big(Q_{5z} + Q^\dagger_{5z}\Big)\Big]\Bigg\}
\end{aligned}
$$





Diagrams with F_{ij} or $\tilde{Q}_1$ (and $\tilde{Q}_5$) classical: BK-type diagrams



+ self-energy diagrams

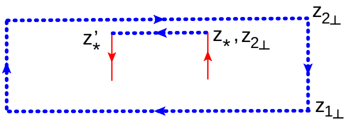
Evolution equation of sub-eikonal corrections

Diagrams with F_{ij} or $\tilde{Q}_1$ (and $\tilde{Q}_5$) classical: BK-type diagrams

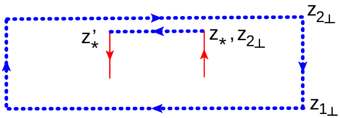
$$\langle \mathrm{Tr}\{\tilde{Q}_{1x}\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\! d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp} \times\Big[\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1x}\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\tilde{Q}_{1x}\}\Big]$$

$$\langle \mathrm{Tr}\{\tilde{Q}_{5x}\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\! d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp} \times\Big[\mathrm{Tr}\{U^\dagger_z\tilde{Q}_{1x}\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\tilde{Q}_{1x}\}\Big]$$

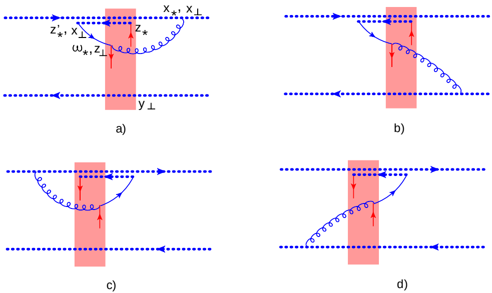
$$\langle \mathrm{Tr}\{\mathcal{F}_x\, U^\dagger_y\}\rangle \;=\; \frac{\alpha_s}{2\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int\! d^2z\,\frac{(x-y)^2_\perp}{(x-z)^2_\perp\,(y-z)^2_\perp} \times\Big[\mathrm{Tr}\{U^\dagger_z\mathcal{F}_x\}\,\mathrm{Tr}\{U^\dagger_y U_z\} - N_c\,\mathrm{Tr}\{U^\dagger_y\mathcal{F}_x\}\Big]$$



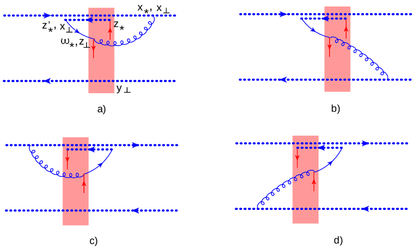
quark-to-gluon diagrams



quark-to-gluon diagrams



Evolution equation of sub-eikonal corrections

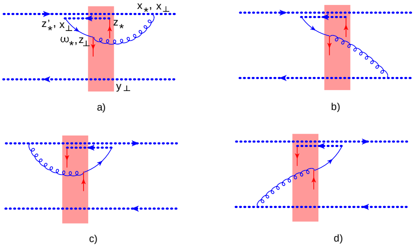


$$\langle \mathrm{Tr}\{U_x U_y^\dagger\} Q_{1x} \rangle = \frac{\alpha_s}{4\pi^2} \int_0^{+\infty} \frac{d\alpha}{\alpha} \int d^2 z \Bigg\{ \frac{1}{(x-z)_\perp^2} \Big[\mathrm{Tr}\left\{ U_x U_y^\dagger U_z \mathcal{X}_{1zx}^\dagger \right\}$$

$$+ \mathrm{Tr}\{U_z U_y^\dagger U_x \mathcal{X}_{1xz}^\dagger\} + \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{1xz}^- + \mathcal{H}_{1zx}^+ \Big) \Big] + \frac{(x-z, z-y)_\perp}{(y-z)_\perp^2 (x-z)_\perp^2}$$

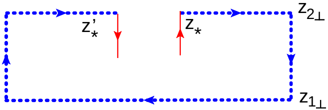
$$\times \Big[\mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}_{1zx}^\dagger\} + \mathrm{Tr}\left\{ U_z U_y^\dagger U_x \mathcal{X}_{1xz}^\dagger \right\} + \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{1xz}^- + \mathcal{H}_{1zx}^+ \Big) \Big]$$

$$+ \frac{(\vec{x}-\vec{z}) \times (\vec{y}-\vec{z})}{(y-z)_\perp^2 (x-z)_\perp^2} \Big[\mathrm{Tr}\{U_x U_y^\dagger U_z \mathcal{X}_{5zx}^\dagger\} - \mathrm{Tr}\left\{ U_z U_y^\dagger U_x \mathcal{X}_{5xz}^\dagger \right\} - \frac{1}{N_c} \mathrm{Tr}\{U_x U_y^\dagger\} \Big(\mathcal{H}_{5xz}^- - \mathcal{H}_{5zx}^+ \Big) \Big] \Bigg\}.$$

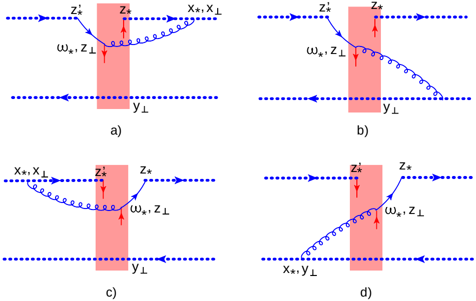


$$
\begin{aligned}
\langle \mathrm{Tr}\{U_xU_y^\dagger\}Q_{5x}\rangle = &-\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z\,\Bigg\{\frac{1}{(x-z)_\perp^2}\Big[\mathrm{Tr}\left\{U_xU_y^\dagger U_z\mathcal{X}_{5zx}^\dagger\right\}\\
&+\mathrm{Tr}\left\{U_zU_y^\dagger U_x\mathcal{X}_{5xz}^\dagger\right\}-\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger U_x\}\Big(\mathcal{H}_{5xz}^- + \mathcal{H}_{5zx}^+\Big)\Big] + \frac{(x-z,z-y)_\perp}{(y-z)_\perp^2(x-z)_\perp^2}\\
&\times\Big[\mathrm{Tr}\{U_xU_y^\dagger U_z\mathcal{X}_{5zx}^\dagger\}+\mathrm{Tr}\left\{U_zU_y^\dagger U_x\mathcal{X}_{5xz}^\dagger\right\}-\frac{1}{N_c}\mathrm{Tr}\{U_xU_y^\dagger\}\Big(\mathcal{H}_{5xz}^- + \mathcal{H}_{5zx}^+\Big)\Big]\\
&+\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(x-z)_\perp^2}\Big[\mathrm{Tr}\{U_zU_y^\dagger U_x\mathcal{X}_{1xz}^\dagger\}-\mathrm{Tr}\{U_xU_y^\dagger U_z\mathcal{X}_{1zx}^\dagger\}+\frac{1}{N_c}\mathrm{Tr}\{U_xU_y^\dagger\}\Big(\mathcal{H}_{1zx}^+ - \mathcal{H}_{1xz}^-\Big)\Big]\Bigg\}.
\end{aligned}
$$

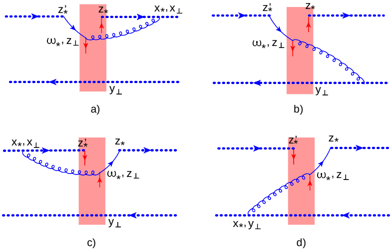
Evolution equation of sub-eikonal corrections



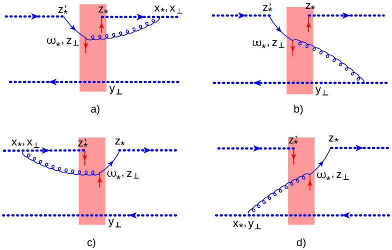
quark-to-gluon diagrams



Evolution equation of sub-eikonal corrections



$$
\begin{aligned}
\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{1x}\}\rangle = & -\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
& \times\Bigg\{\frac{1}{(x-z)_\perp^2}\left[\mathrm{Tr}\left\{U_zU_y^\dagger\right\}\left(\mathcal{H}^+_{1xz}+\mathcal{H}^-_{1zx}\right)-\frac{1}{N_c}\mathrm{Tr}\left\{U_y^\dagger\left(\mathcal{X}_{1xz}+\mathcal{X}_{1zx}\right)\right\}\right] \\
& +\frac{(x-z,z-y)}{(y-z)_\perp^2(z-x)_\perp^2}\left[\mathrm{Tr}\{U_zU_y^\dagger\}\left(\mathcal{H}^+_{1xz}+\mathcal{H}^-_{1zx}\right)-\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{1xz}+\mathcal{X}_{1zx}\right)\}\right] \\
& +\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(z-x)_\perp^2}\left[\mathrm{Tr}\{U_zU_y^\dagger\}\left(\mathcal{H}^-_{5zx}-\mathcal{H}^+_{5xz}\right)+\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{5xz}-\mathcal{X}_{5zx}\right)\}\right]\Bigg\}
\end{aligned}
$$



$$
\begin{aligned}
\langle \mathrm{Tr}\{U_y^\dagger \tilde{Q}_{5x}\}\rangle = & -\frac{\alpha_s}{4\pi^2}\int_0^{+\infty}\frac{d\alpha}{\alpha}\int d^2z \\
& \times\Bigg\{\frac{1}{(x-z)_\perp^2}\Big[\mathrm{Tr}\left\{U_z U_y^\dagger\right\}\Big(\mathcal{H}^+_{5xz}+\mathcal{H}^-_{5zx}\Big)-\frac{1}{N_c}\mathrm{Tr}\left\{U_y^\dagger\left(\mathcal{X}_{5xz}+\mathcal{X}_{5zx}\right)\right\}\Big] \\
& +\frac{(x-z,z-y)}{(y-z)_\perp^2(z-x)_\perp^2}\Big[\mathrm{Tr}\{U_z U_y^\dagger\}\Big(\mathcal{H}^+_{5xz}+\mathcal{H}^-_{5zx}\Big)-\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\Big(\mathcal{X}_{5xz}+\mathcal{X}_{5zx}\Big)\}\Big)\Big] \\
& +\frac{(\vec{x}-\vec{z})\times(\vec{y}-\vec{z})}{(y-z)_\perp^2(z-x)_\perp^2}\Big[\mathrm{Tr}\{U_z U_y^\dagger\}\Big(\mathcal{H}^+_{1xz}-\mathcal{H}^-_{1zx}\Big)+\frac{1}{N_c}\mathrm{Tr}\{U_y^\dagger\left(\mathcal{X}_{1zx}-\mathcal{X}_{1xz}\right)\}\Big]\Bigg\}
\end{aligned}
$$

$$\mathcal{X}_1(x_\perp, y_\perp) = -g^2 \int_{-\infty}^{+\infty} dz_* d\omega_* \bar{\psi}(z_*, y_\perp)[z_*, -\infty p_1]_y \, i\not{p}_1 \, [\infty p_1, \omega_*]_x \psi(\omega_*, x_\perp)$$

$$\mathcal{X}_1^\dagger(x_\perp, y_\perp) = g^2 \int_{-\infty}^{+\infty} dz_* d\omega_* \, \bar{\psi}(\omega_*, x_\perp)[\omega_*, \infty p_1]_x \, i\not{p}_1 \, [-\infty p_1, z_*]_y \psi(z_*, y_\perp)$$

$$\mathcal{X}_5(x_\perp, y_\perp) = -g^2 \int_{-\infty}^{+\infty} dz_* d\omega_* \bar{\psi}(z_*, y_\perp)[z_*, -\infty p_1]_y \, \gamma^5 \not{p}_1 \, [\infty p_1, \omega_*]_x \psi(\omega_*, x_\perp)$$

$$\mathcal{X}_5^\dagger(x_\perp, y_\perp) = -g^2 \int_{-\infty}^{+\infty} dz_* d\omega_* \bar{\psi}(\omega_*, x_\perp)[\omega_*, \infty p_1]_x \, \gamma^5 \not{p}_1 \, [-\infty p_1, z_*]_y \psi(z_*, y_\perp)$$

$\mathcal{H}$-operators

$$\mathcal{H}_1^+(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*,y_\perp)[\omega_*,\infty p_1]_y\, i\not{p}_1\, [\infty p_1, z_*]_x \psi(z_*,x_\perp)$$

$$\mathcal{H}_5^+(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*,y_\perp)[\omega_*,\infty p_1]_y\, \gamma^5\not{p}_1\, [\infty p_1, z_*]_x \psi(z_*,x_\perp)$$

$$\mathcal{H}_1^-(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*,y_\perp)[\omega_*,-\infty p_1]_y\, i\not{p}_1\, [-\infty p_1, z_*]_x \psi(z_*,x_\perp)$$

$$\mathcal{H}_5^-(x_\perp,y_\perp) = -g^2\int_{-\infty}^{+\infty} dz_* d\omega_*\, \bar\psi(\omega_*,y_\perp)[\omega_*,-\infty p_1]_y\, \gamma^5\not{p}_1\, [-\infty p_1, z_*]_x \psi(z_*,x_\perp)$$

Conclusions

- New Impact factors have been derived
- New evolution equations which describe the high-energy spin dynamics
- New quark and gluon distributions
- Outlook
  - Extract double log of energy and compare with Bartels-Ermolaev-Ryskin
  - Compare with results obtained by Kovchegov's group (20015-2021)
  - Extend formalism to other observables