Resummation of kinematic power corrections in DVCS: Part 1: Operator Product Expansion

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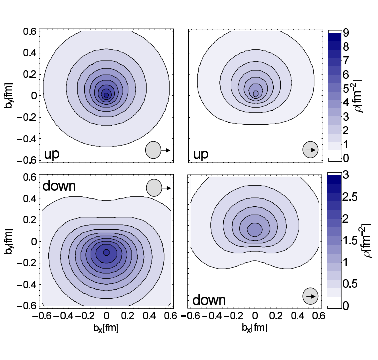
based on

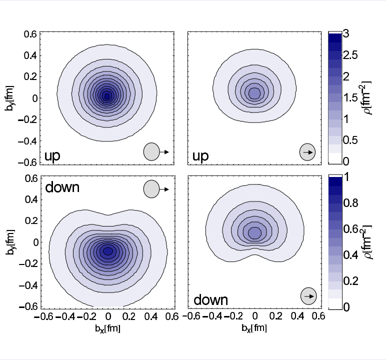
V. Braun, Yao Ji, A. Manashov 2011.04533

Regensburg, 19.02.2021

Nucleon Tomography

access to three-dimensional picture of the nucleon (M. Burkardt)





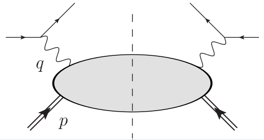
↪ **first two moments of transverse spin parton density**

computer simulations: M. Göckeler *et al.*, Phys.Rev.Lett. 98 (2007) 222001

- **Momentum transfer t defines the resolution of spacial imaging**

Planar vs. non-planar kinematics

"Natural" separation of longitudinal and transverse d.o.f. in DIS

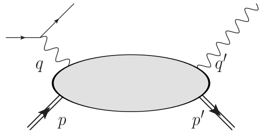


$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ parton fraction = Bjorken x_B

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"Laboratory frame"

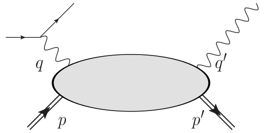
$$p = (p_0, \vec{0}_\perp, p_z)$$
$$q = (q_0, \vec{0}_\perp, q_z)$$

$\Rightarrow$ asymmetry parameter $\xi \simeq x_B/(2 - x_B)$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ (almost) transverse

Planar vs. non-planar kinematics (2)

Many possible choices in DVCS



"Photon frame"

$$\begin{aligned} q' &= (q'_0, \vec{0}_\perp, q'_z) \\ q &= (q_0, \vec{0}_\perp, q_z) \end{aligned}$$

$\Rightarrow$ skewedness parameter $\xi = \frac{x_B(1+t/Q^2)}{2-x_B(1-t/Q^2)}$

$\Rightarrow$ momentum transfer $\Delta = p' - p$ longitudinal

Relating the laboratory and photon reference frames

- **Compton form factors are different, related by Lorentz trafo**

$$\mathcal{F}^{\text{lab}}_{++} = \mathcal{F}^{\text{phot}}_{++} + \frac{\varkappa}{2}\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big] - \varkappa_0\, \mathcal{F}^{\text{phot}}_{0+},$$
$$\mathcal{F}^{\text{lab}}_{0+} = -(1+\varkappa)\,\mathcal{F}^{\text{phot}}_{0+} + \varkappa_0\Big[\mathcal{F}^{\text{phot}}_{++} + \mathcal{F}^{\text{phot}}_{-+}\Big]$$

$$\boxed{\mathcal{F} \in \{\mathcal{H}, \mathcal{E}, \widetilde{\mathcal{H}}, \widetilde{\mathcal{E}}\}}$$

where

$$\varkappa_0 \sim \sqrt{(t_{\min} - t)/Q^2}\,, \qquad \varkappa \sim (t_{\min} - t)/Q^2$$

- **different definition of the skewedness parameter**

$$\xi^{\text{lab}} \simeq \frac{x_B}{2 - x_B} \quad \text{vs.} \quad \xi^{\text{phot}} = \frac{x_B(1 + t/Q^2)}{2 - x_B(1 - t/Q^2)}$$

Defining the Leading Twist approximation

Kumerički-Müller convention (KM)

$$\mathrm{LT}_{\mathrm{KM}}:\begin{cases}\mathcal{F}^{\text{lab}}_{++}=T_0\otimes F, & \mathcal{F}^{\text{lab}}_{0+}=0,\\ \mathcal{F}^{\text{lab}}_{-+}=0, & \xi_{\mathrm{KM}}=\xi^{\text{lab}}\end{cases}$$

Braun-Manashov-Pirnay convention (BMP)

$$\mathrm{LT}_{\mathrm{BMP}}:\begin{cases}\mathcal{F}^{\text{phot}}_{++}=T_0\otimes F, & \mathcal{F}^{\text{phot}}_{0+}=0,\\ \mathcal{F}^{\text{phot}}_{-+}=0, & \xi_{\mathrm{BMP}}=\xi^{\text{phot}}\end{cases}$$

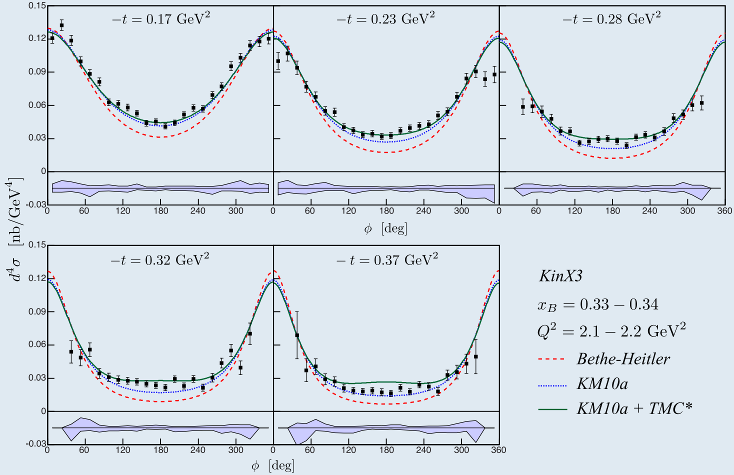
$$\Downarrow$$

$$\mathrm{LT}_{\mathrm{BMP}}:\begin{cases}\mathcal{F}^{\text{lab}}_{++}=\left(1+\frac{\varkappa}{2}\right)T_0\otimes F, & \mathcal{F}_{0+}=\varkappa_0\,T_0\otimes F\\ \mathcal{F}^{\text{lab}}_{-+}=\frac{\varkappa}{2}T_0\otimes F, & \xi=\xi_{\mathrm{BMP}},\end{cases}$$

- **Changing frame of reference results in**
  - — **Different skewedness parameter for a given x_B**
  - — **Numerically significant excitation of helicity-flip CFFs**
- **Different results for experimental observables**

Large effects for the total cross section

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453

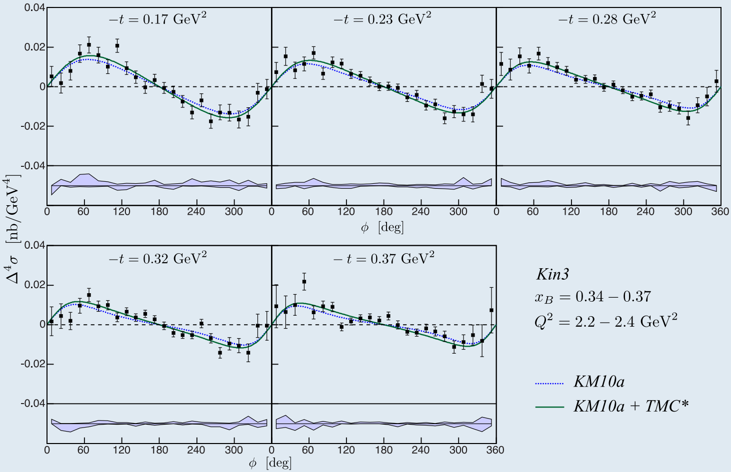


- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

Small/moderate effects for asymmetries

M. Defurne *et al.* [Hall A Collaboration] arXiv:1504.05453



- **TMC* curves very close to BMP LT**

GPD model: KM10a (Kumericki, Mueller, Nucl. Phys. B **841** (2010) 1)

To summarize:

- **noncomplanarity makes separation of collinear directions ambiguous**

  — hence “leading twist approximation” ambiguous

  — related to violation of translation invariance and EM Ward identities

- **have to be repaired by adding power corrections of special type, “kinematic” PC**

Operator Product Expansion

schematically

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\dots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\dots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\dots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\dots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\dots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\dots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\dots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\dots\mu_N}}_{\text{descendants}} + \dots \Big\}$$

$$+ \text{ quark-gluon operators}$$

"kinematic" corrections that repair the frame dependence and Ward identities come from

(1) corrections m/Q and $\sqrt{-t}/Q$ to the ME of twist-two operators (Nachtmann)
(2) higher-twist operators that are obtained from twist-two by adding total derivatives

Operator Product Expansion (2)

Problem: matrix elements of some descendant operators over free quarks vanish

Ferrara, Grillo, Parisi, Gatto, '71-'73

Example

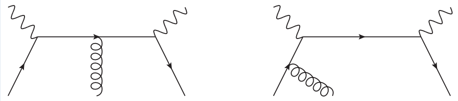
$$\partial^\mu O_{\mu\nu} = 2i\bar{q}gF_{\nu\mu}\gamma^\mu q\,, \qquad\qquad O_{\mu\nu} = (1/2)[\bar{q}\gamma_\mu \overleftrightarrow{D}_\nu q + (\mu \leftrightarrow \nu)]$$

- Usual procedure to calculate the coefficient functions does not work
- How to separate "kinematic" and "genuine" (quark-gluon) contributions?

Twist-four: t/Q^2 and m^2/Q^2

VB, A. Manashov, D. Müller, B. Pirnay '11-'14

- Consider quark-gluon matrix elements



- Use hermiticity of evolution equations for twist-4 operators to separate "kinematic" terms

- **Complete results available to t/Q^2, m^2/Q^2 accuracy**

  — translation and gauge invariance restored

  — factorization valid

  — correct threshold behavior $t \to t_{\min}$, $\xi \to 1$

  — for many observables, complete results close to LT in "photon frame"

. . . and beyond

All orders in $(\sqrt{-t}/Q)^k$, $(m/Q)^k$?

apart from theoretical completeness

- Factor-two effects in some kinematic regions, need resummation to all twists
- Problems with some newer data ?
- Mass corrections in coherent DVCS on ^{4}He ?

Here the first step:

VB, Yao Ji, A. Manashov 2011.04533 (to appear in JHEP)

$$\mathsf{T}\{j(x)j(0)\} = \sum_N \Big\{ A_N^{\mu_1\cdots\mu_N} \underbrace{\mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{twist-2 operators}} + B_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu \mathcal{O}^N_{\mu,\mu_1\ldots\mu_N}}_{\text{descendants of twist 2}}$$

$$+ C_N^{\mu_1\cdots\mu_N} \underbrace{\partial^2 \mathcal{O}^N_{\mu_1\ldots\mu_N}}_{\text{descendants}} + D_N^{\mu_1\cdots\mu_N} \underbrace{\partial^\mu\partial^\nu \mathcal{O}^N_{\mu,\nu,\mu_1\ldots\mu_N}}_{\text{descendants}} + \ldots \Big\} + \ldots$$

$$\equiv \sum_N C_N^{\mu_1\cdots\mu_N}(x,\partial)\, \mathcal{O}^N_{\mu_1\ldots\mu_N} + \text{ quark-gluon operators}$$

S. Ferrara, A. F. Grillo and R. Gatto, 1971-1973: "Conformally covariant OPE"

In conformal field theories, the CFs of descendants are related to the CFs of twist-2 operators by symmetry and do not need to be calculated directly

$$A_N^{\mu_1\cdots\mu_N} \overset{O(4,2)}{\mapsto} C_N^{\mu_1\cdots\mu_N}(x,\partial)$$

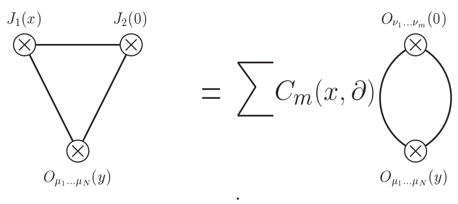
Conformal triangles

A.M. Polyakov, 1970:

$$\langle O_1(x_1)\, O_2(x_2)\rangle = \frac{\text{const}}{|x_1-x_2|^{2\Delta_1}}\,\delta_{\Delta_1\Delta_2}$$

$$\langle O_1(x_1)\, O_2(x_2)\, O_3(x_3)\rangle = \frac{\text{const}}{|x_1-x_2|^{\Delta_1+\Delta_2-\Delta_3}|x_1-x_3|^{\Delta_1+\Delta_3-\Delta_2}|x_2-x_3|^{\Delta_3+\Delta_2-\Delta_1}}$$

- $\leftarrow$ Δ_k is a scaling dimension (canonical + anomalous)



- $\leftarrow$ exact to all orders of perturbation theory and beyond

QCD?

QCD is not a conformal theory, but

$$\mathcal{A}_{\rm QCD} = \mathcal{A}^{\rm conf}_{\rm QCD} + O(\beta(\alpha_s))$$

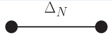
"Conformal QCD": QCD in $d-2\epsilon$ at Wilson-Fischer critical point $\beta(\alpha_S)=0$

V.B., A. Manashov, Eur.Phys.J.C 73 (2013) 2544

Tensor operators

$$\vec{\mu}_N = (\mu_1, \dots, \mu_N)$$

- two-point function

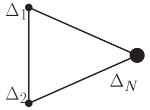


$$= \left\langle \mathcal{O}^{\vec{\mu}_N}_{\Delta_N}(x_1)\mathcal{O}^{\vec{\nu}_N}_{\Delta_N}(x_2)\right\rangle = c_N \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x_{12}),$$

$$x_{12} = x_1 - x_2$$

$$\mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x) = \frac{1}{|x|^{2\Delta_N}}\Big(\eta^{\mu_1\nu_1}(x)\dots\eta^{\mu_N\nu_N}(x) + \text{permutations} - \text{traces}\Big)$$

- three-point function with two scalar operators



$$= \left\langle \mathcal{O}(x_1)\mathcal{O}(x_2)\mathcal{O}^{\vec{\mu}_N}_{\Delta_N}(x_3)\right\rangle = c'_N T^{\vec{\mu}_N}_{\Delta_N}(x_1,x_2,x_3),$$

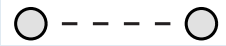
$$T^{\vec{\mu}_N}_{\Delta_N}(x_1,x_2,x_3) = \frac{\Lambda^{\mu_1}(x_1,x_2,x_3)\dots\Lambda^{\mu_N}(x_1,x_2,x_3)\; - \;\text{traces}}{|x_{12}|^{\Delta_1+\Delta_2-\Delta_N+N}|x_{13}|^{\Delta_1+\Delta_N-N-\Delta_2}|x_{23}|^{\Delta_2+\Delta_N-N-\Delta_1}},$$

with

$$\eta^{\mu\nu}(x) = g^{\mu\nu} - \frac{2x^\mu x^\nu}{x^2}, \qquad \Lambda^\mu(x_1,x_2,x_3) = \frac{x^\mu_{13}}{x^2_{13}} - \frac{x^\mu_{23}}{x^2_{23}}$$

Shadow operator formalism   Ferrara et al., 1972

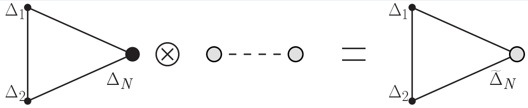
$$\widetilde{\Delta}_N = d - \Delta_N$$



$$= \left\langle \mathcal{O}^{\vec{\mu}_N}_{\tilde{\Delta}_N}(x_1)\mathcal{O}^{\vec{\nu}_N}_{\tilde{\Delta}_N}(x_2)\right\rangle = c_N \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\tilde{\Delta}_N}(x_{12})$$

$$= \int d^d y\, \mathcal{D}^{\vec{\mu}_N\,\vec{\nu}_N}_{\Delta_N}(x_1 - y)\mathcal{D}^{\vec{\nu}_N\,\vec{\rho}_N}_{\tilde{\Delta}_N}(y - x_2)$$

$$= c_N\, \delta^{(d)}(x_{12}) \left(g^{\mu_1\rho_1}\dots g^{\mu_N\rho_N} + \dots\right)$$



Shadow operator formalism (2)

- The coefficient function of the operator $\mathcal{O}_N^{\vec{\mu}_N}$ (including the descendants) is given by the Fourier transform of the "shadow triangle"

$$C_{\Delta_N}^{\vec{\mu}_N}(x_{12}, ip) = \mathcal{N}\int d^d y\, e^{ipy}\, T_{\tilde{\Delta}_N}^{\vec{\mu}_N}(x_1, x_2, y)$$

$$T_{\tilde{\Delta}_N}^{\vec{\mu}_N}(x_1, x_2, x_3) = \frac{\Lambda^{\mu_1}(x_1,x_2,x_3)\ldots\Lambda^{\mu_N}(x_1,x_2,x_3)\ -\ \text{traces}}{|x_{12}|^{\Delta_1+\Delta_2-\tilde{\Delta}_N+N}|x_{13}|^{\Delta_1+\tilde{\Delta}_N-N-\Delta_2}|x_{23}|^{\Delta_2+\tilde{\Delta}_N-N-\Delta_1}}$$

- Subtlety: contributions of $y \lesssim x_{12}$ must be excluded

How to take the integral?

$$T^{n\dots n}_{\tilde\Delta_N}(x_1,x_2,p) = n_{\mu_1}\dots n_{\mu_N}\int d^d y\, e^{ip\cdot y}\, T^{\mu_1\dots\mu_N}_{\Delta_N}(x_1,x_2,y) = ? \qquad n^2=0$$

- Useful representation

$$T^{n\dots n}_{\tilde\Delta_N}(x_1,x_2,y) = \frac{2^{-N}}{|x_{12}|^{\Delta_1+\Delta_2-\tilde\Delta_N+N}}\int_{\mathfrak{D}} D_{j_1}z_1\int_{\mathfrak{D}} D_{j_2}z_2\,\frac{\bar z_{12}^N}{|x_1-y-z_1n|^{4j_1}|x_2-y-z_2n|^{4j_2}},$$

$$4j_1=\Delta_1+\tilde\Delta_N-N-\Delta_2\,, \qquad 4j_2=\Delta_2+\tilde\Delta_N-N-\Delta_1$$

- Integration over z_1, z_2 goes over the unit disks $|z_k|\le 1$ in the complex plane

$$\mathfrak{D}=\{z\in\mathbb{C},|z|<1\} \qquad\qquad D_j z=\frac{2j-1}{\pi}(1-|z|^2)^{2j-2}d^2z$$

- Identity: "Reproducing operator"

$$\forall f(w) \qquad f(w)=\int_{\mathfrak{D}} D_j z\,(1-w\bar z)^{-2j}f(z) \tag{1}$$

OPE of two scalar operators

$$\bar{u} = 1-u\,, \quad x_{12} = x_1 - x_2, \quad x_{21}^u = \bar{u}x_2 + ux_1$$

$$\mathcal{O}_\Delta(x_1)\mathcal{O}_\Delta(x_2) = \sum_N \frac{c_N}{|x_{12}|^{2\Delta - t_N}} \sum_{k=0}^{N} \frac{N!}{(N-k)!}\Gamma(\varkappa_N - k)\left(\frac{x_{12}^2}{8}\right)^k \times \int_0^1 du\,(u\bar{u})^{j_N-1} C_k^{\varkappa_N-k}(2u-1) \times \sum_{m=0}^{\infty} \frac{(-u\bar{u}x_{12}^2\partial^2/4)^m}{m!\Gamma(\Delta_N + k - d/2 + m + 1)} \mathcal{O}_N^{(k)}(x_{21}^u)\,.$$

$$\Delta_N = N + d - 2 + \gamma_N$$

$$j_N = \tfrac{1}{2}(\Delta_N + N), \quad t_N = \Delta_N - N, \quad \varkappa_N = \tfrac{1}{2}(d - t_N - 1)$$

where

$$\mathcal{O}_N^{(k)}(y) = \partial_y^{\mu_1}\ldots\partial_y^{\mu_k}\mathcal{O}_{\mu_1\ldots\mu_k\mu_{k+1}\ldots\mu_N}(y)x_{12}^{\mu_{k+1}}\ldots x_{12}^{\mu_N}\,.$$

- The coefficients c_N and scaling dimensions Δ_N are not fixed by symmetry
- k counts applications of the divergence to the leading-twist operator, m counts applications of the Laplace operator, ∂^2
- Result originally derived in [Ferrara:1971vh] in a different form.

Vector currents

$$J^\mu(x_1)\, J^\nu(x_2) =?$$

- Four Lorentz structures consistent with conformal symmetry
- Two relations from current conservation $\partial_\mu J^\mu = 0$

$\Rightarrow$

- Two independent structures, coefficients fixed by the two CFs C_2 and C_L in DIS (in $d = 4 - \epsilon$)

Leading order

- Sums are truncated

$$J^\mu(x)J^\nu(0) \sim A\frac{1}{x^4} + B\frac{1}{x^2} + \cancel{C + Dx^2 + \ldots}$$

- Polynomials produce delta-functions in momentum space, can be dropped

Final result (LO): symmetric part

$$T\{j_\mu(x)j_\nu(0)\}^S = \sum_{N,\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N$$
$$\times\left\{g^{\mu\nu}\mathbb{T}_1 + x^\mu x^\nu \mathbb{T}_2 + (x^\mu\partial^\nu + x^\nu\partial^\mu)\mathbb{T}_3 + \frac{(x^\mu\mathbb{T}_4^\nu + x^\nu\mathbb{T}_4^\mu)}{u\bar{u}}\right\}$$

$$\mathbb{T}_1 = \frac{1}{2}(N+1)\mathcal{O}^N_{x\ldots x} + \frac{x^2}{8}\left[N(2u-1)\partial^\alpha\mathcal{O}^N_{\alpha x\ldots x} - u\bar{u}\,\partial^2\mathcal{O}^N_{x\ldots x}\right]$$

Final result (LO): symmetric part

$$T\{j_\mu(x)j_\nu(0)\}^S = \sum_{N,\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N$$
$$\times \left\{ g^{\mu\nu}\mathbb{T}_1 + x^\mu x^\nu \mathbb{T}_2 + (x^\mu\partial^\nu + x^\nu\partial^\mu)\mathbb{T}_3 + \frac{(x^\mu\mathbb{T}_4^\nu + x^\nu\mathbb{T}_4^\mu)}{u\bar{u}} \right\}$$

$$\mathbb{T}_2 = \frac{1}{2(N+1)}\Big[N(2u-1)\partial^\alpha\mathcal{O}^N_{\alpha x\ldots x} - u\bar{u}\,\partial^2\mathcal{O}^N_{x\ldots x}\Big]$$
$$+ \frac{x^2}{8(N+2)(N+1)}\bigg\{\Big[2+N(N+1)-4N(N-1)u\bar{u}\Big]\partial^\alpha\partial^\beta\mathcal{O}^N_{\alpha\beta x\ldots x}$$
$$- 2Nu\bar{u}(2u-1)\partial^2\partial^\alpha\mathcal{O}^N_{\alpha x\ldots x} + (u\bar{u})^2\partial^4\mathcal{O}^N_{x\ldots x}\bigg\}$$

Final result (LO): symmetric part

$$T\{j_\mu(x)j_\nu(0)\}^S = \sum_{N,\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N$$
$$\times\left\{g^{\mu\nu}\mathbb{T}_1 + x^\mu x^\nu \mathbb{T}_2 + (x^\mu\partial^\nu + x^\nu\partial^\mu)\mathbb{T}_3 + \frac{(x^\mu\mathbb{T}_4^\nu + x^\nu\mathbb{T}_4^\mu)}{u\bar{u}}\right\}$$

$$\mathbb{T}_3 = \frac{1}{4}(2u-1)\mathcal{O}^N_{x\ldots x}$$
$$+\frac{x^2}{16(N+2)(N+1)}\Big\{\Big[N(N-1) - 2 - 4N(N+2)u\bar{u}\Big]\partial^\alpha\mathcal{O}^N_{\alpha x\ldots x}$$
$$-(N+2)u\bar{u}(2u-1)\,\partial^2\mathcal{O}^N_{x\ldots x}\Big\}$$

Final result (LO): symmetric part

$$T\{j_\mu(x)j_\nu(0)\}^S = \sum_{N,\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N \times \Big\{ g^{\mu\nu}\mathbb{T}_1 + x^\mu x^\nu \mathbb{T}_2 + (x^\mu\partial^\nu + x^\nu\partial^\mu)\mathbb{T}_3 + \frac{(x^\mu \mathbb{T}_4^\nu + x^\nu \mathbb{T}_4^\mu)}{u\bar{u}}\Big\}$$

$$\mathbb{T}_4^\nu = \frac{1}{4}N\Big[2u\bar{u}-1\Big]\mathcal{O}^N_{\nu x\ldots x} + \frac{x^2}{16(N+2)(N+1)}\Big\{N(2u-1)\Big[2+N(N+1)-2(N+2)(N-1)u\bar{u}\Big]\partial^\alpha\mathcal{O}^N_{\nu\alpha x\ldots x} - (N+2)u\bar{u}\Big[N+1-2Nu\bar{u}\Big]\partial^2\mathcal{O}^N_{\nu x\ldots x}\Big\}$$

Final result (LO): antisymmetric part

$$T\{j_\mu(x)j_\nu(0)\}^A = \sum_{N.\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N$$
$$\times\left\{(\partial^\mu \mathbb{T}_5^\nu - \partial^\nu \mathbb{T}_5^\mu) + (x^\mu\partial^\nu - x^\nu\partial^\mu)\mathbb{T}_6 + \frac{(x^\mu \mathbb{T}_7^\nu - x^\nu \mathbb{T}_7^\mu)}{u\bar{u}}\right\}$$

$$\mathbb{T}_5^\nu = -\frac{x^2}{4}\mathcal{O}^N_{\nu x\ldots x}$$

Final result (LO): antisymmetric part

$$T\{j_\mu(x)j_\nu(0)\}^A = \sum_{N.\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N \times \Big\{(\partial^\mu \mathbb{T}_5^\nu - \partial^\nu \mathbb{T}_5^\mu) + (x^\mu\partial^\nu - x^\nu\partial^\mu)\mathbb{T}_6 + \frac{(x^\mu \mathbb{T}_7^\nu - x^\nu \mathbb{T}_7^\mu)}{u\bar{u}}\Big\}$$

$$\mathbb{T}_6 = -\frac{1}{4}\mathcal{O}^N_{x\ldots x} - \frac{x^2}{16(N+1)(N+2)}\Big[(N+1)(N-2)(2u-1)\partial^\alpha \mathcal{O}^N_{\alpha x\ldots x} - (N+2)u\bar{u}\,\partial^2\mathcal{O}^N_{x\ldots x}\Big]$$

Final result (LO): antisymmetric part

$$T\{j_\mu(x)j_\nu(0)\}^A = \sum_{N.\text{even}} \frac{4(2N+1)}{(N+1)!}\frac{1}{\pi^2 x^4}\int_0^1 du\, e^{u(x\cdot\partial)}(u\bar{u})^N \times \Big\{(\partial^\mu \mathbb{T}_5^\nu - \partial^\nu \mathbb{T}_5^\mu) + (x^\mu\partial^\nu - x^\nu\partial^\mu)\mathbb{T}_6 + \frac{(x^\mu \mathbb{T}_7^\nu - x^\nu \mathbb{T}_7^\mu)}{u\bar{u}}\Big\}$$

$$\mathbb{T}_7^\nu = \frac{1}{4}N(2u-1)\mathcal{O}^N_{\nu x\ldots x} + \frac{x^2}{16(N+2)(N+1)}\Big\{\Big[N(2+N+N^2) - 2(N-1)(2+3N+2N^2)u\bar{u}\Big]\partial^\alpha \mathcal{O}^N_{\alpha\nu x\ldots x} - (N+1)(N+2)u\bar{u}(2u-1)\,\partial^2\mathcal{O}^N_{\nu x\ldots x}\Big\}$$

Outlook

Done:

- All-order (in twist) resummation of the descendants of twist-2 operators
- All-order (in α_s) in conformal limit
- Leading-order (in α_s) in full QCD for vector and axial-vector

To-Do:

- Calculate DVCS observables (Compton form factors)
- Verify/prove factorization of kinematic corrections to all powers
- Verify consistency with dispersion relations and threshold behavior
- Prove that target mass corrections only enter as $(\xi m)^2/Q^2$ (DVCS on nuclei)
- Numerical studies in JLAB/EIC kinematics

Eventually:

- NLO in full QCD, gluon GPDs