Universität Tübingen

The polarized photon content of the nucleon

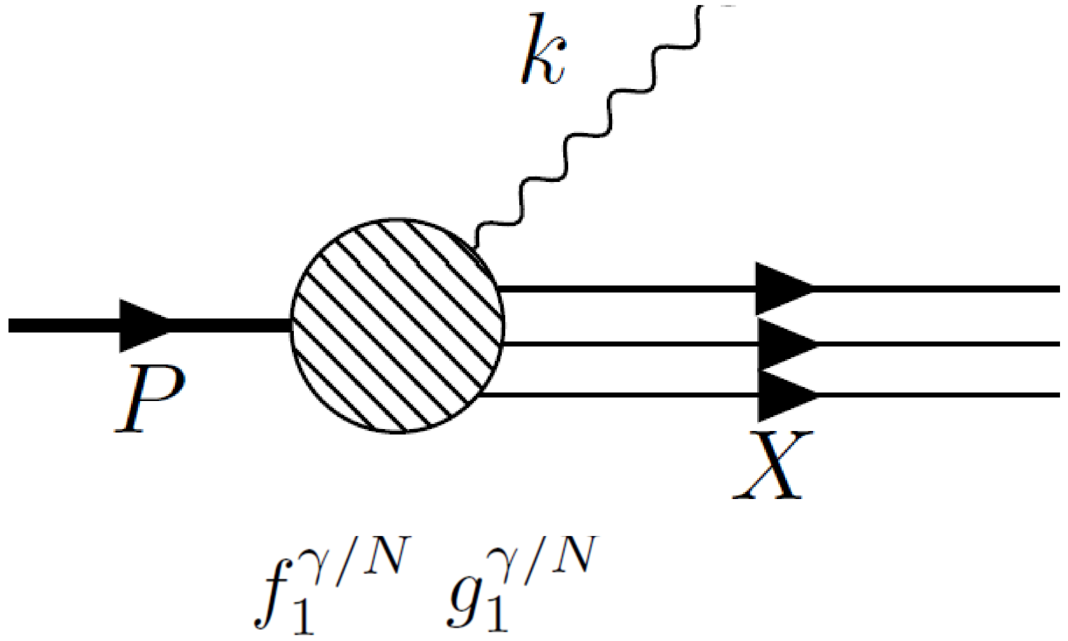
Single-inclusive photon production in lepton-nucleon scattering

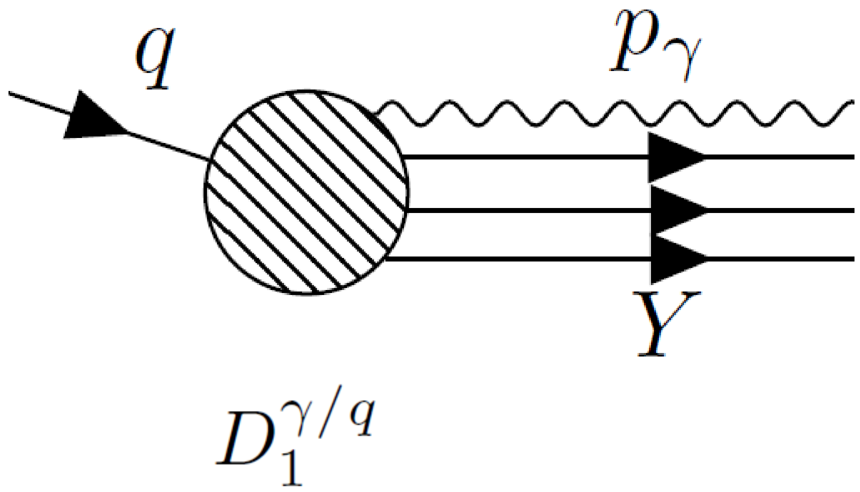
$$\ell(l)N(P) \to \gamma(p_\gamma)X$$

Table of Contents

Introduction/Motivation
Why $\ell N \to \gamma X$?

→ Access to (poorly known) soft photonic functions!





Introduction/Motivation
Why $\ell N \to \gamma X$?

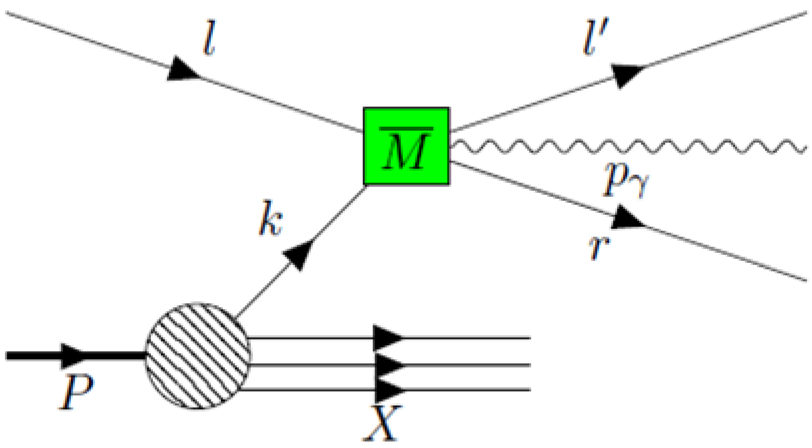
- c.o.m. energies of 100 GeV or higher are possible at a future Electron-Ion Collider (EIC)
- Fixed target experiments including an electron beam energy of 24 GeV with possible upgrade at Jefferson Lab (JLab)

→Very large range of kinematic setups can be tested in future experiments

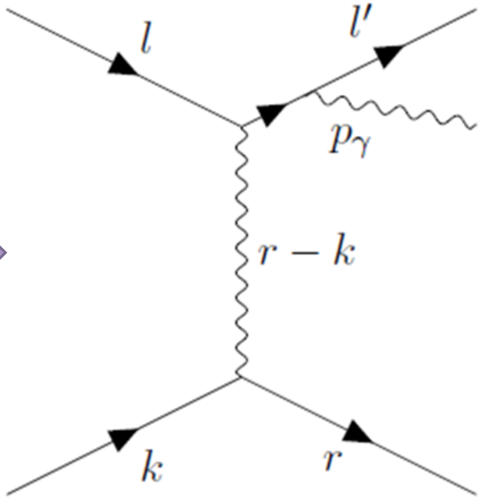
- Access to higher twist correlation functions through transverse spin observables, like the transverse single spin asymmetry (SSA)

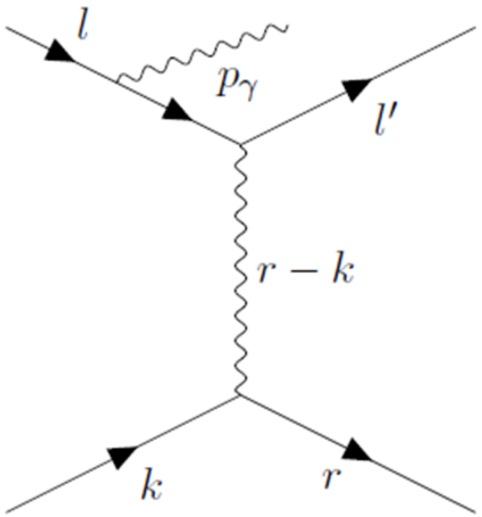
Calculation

- Fully analytic results for σ, $\Delta\sigma$ and A_{LL}
- Dimensional regularization with $d = 4 - 2\varepsilon$
- Calculation up to $\mathcal{O}({\alpha_{em}}^3)$ (LO)
- Light-cone gauge $n \cdot A = 0$
- All external particles are assumed to be massless
- Cross section can be written as the sum of 3 parts characterized by what power of e_q they are proportional to



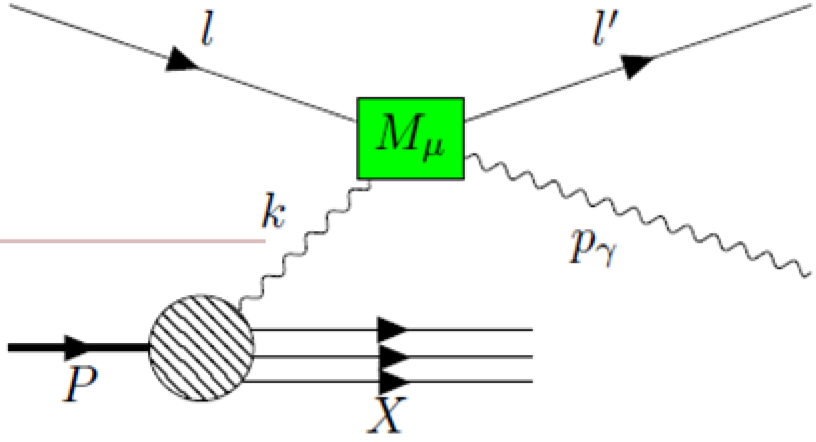
Direct channel





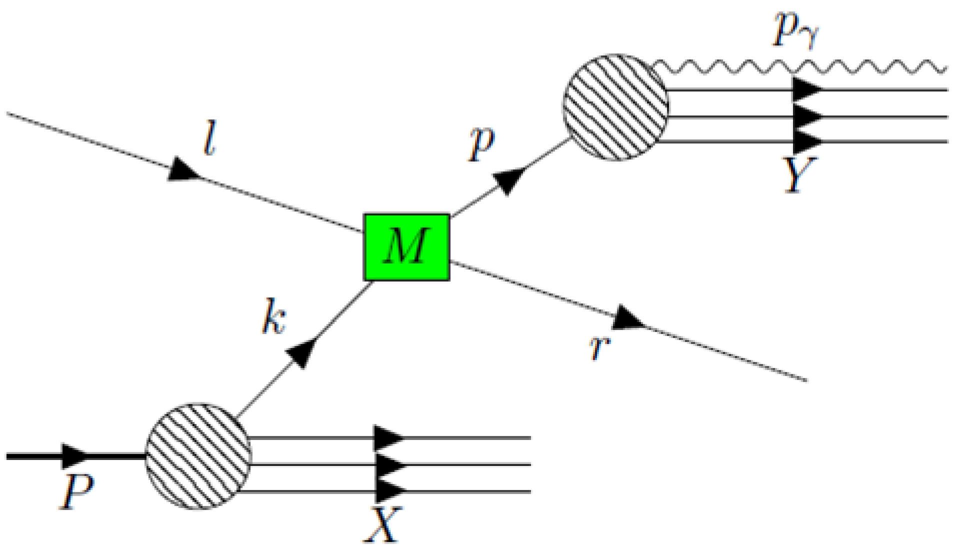
Bethe Heitler

$\propto e_q{}^2$

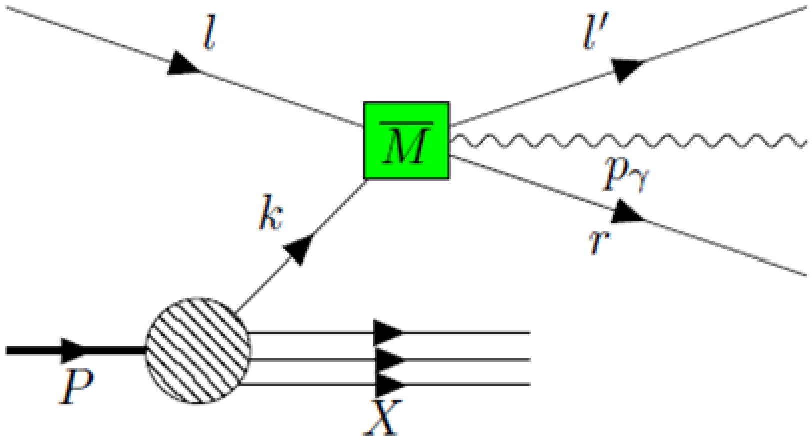


γPDF channel

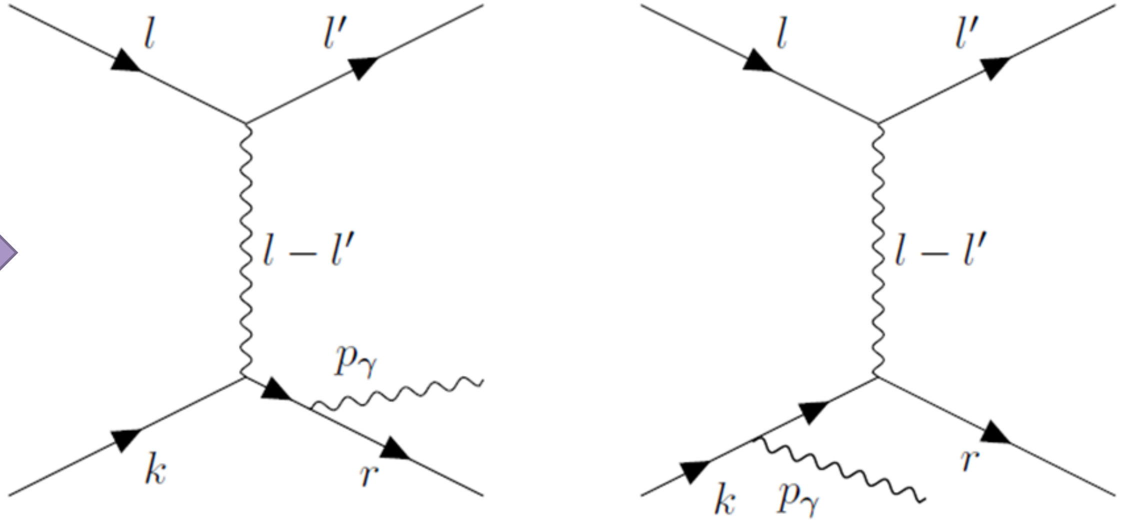
$f_1^{\gamma/N}$ $g_1^{\gamma/N}$



WW channel

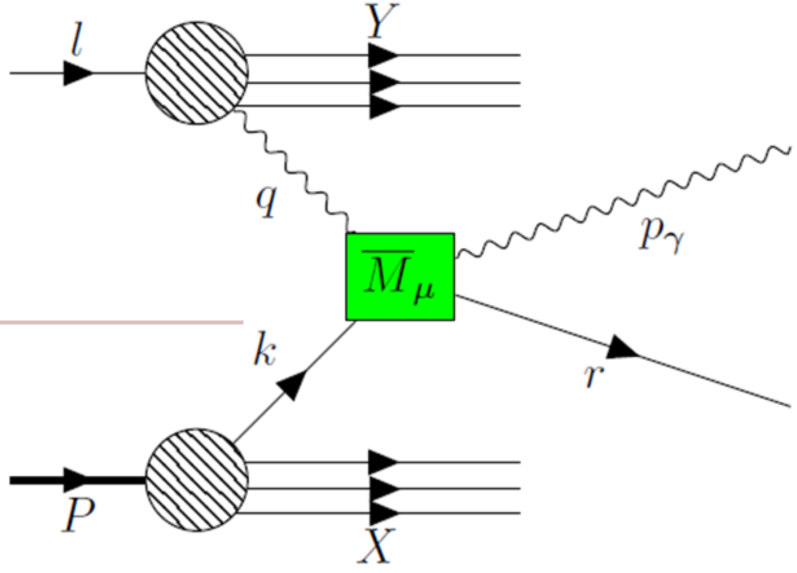


Direct channel

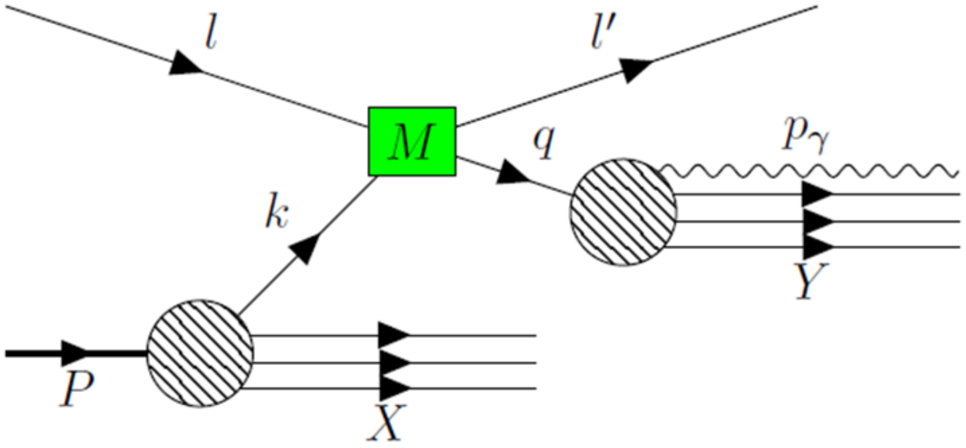


Compton

$\propto e_q{}^4$

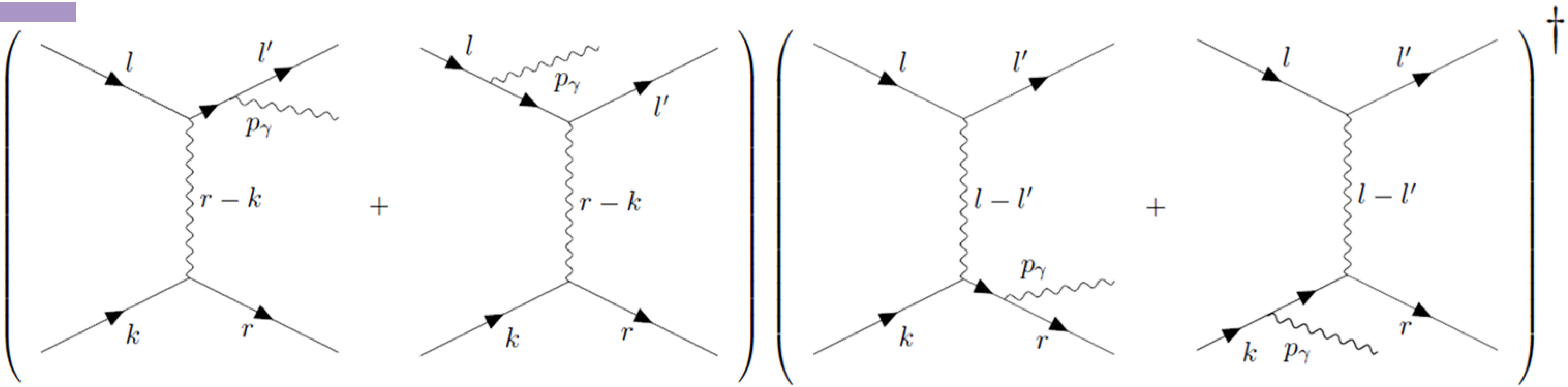


Compton WW channel



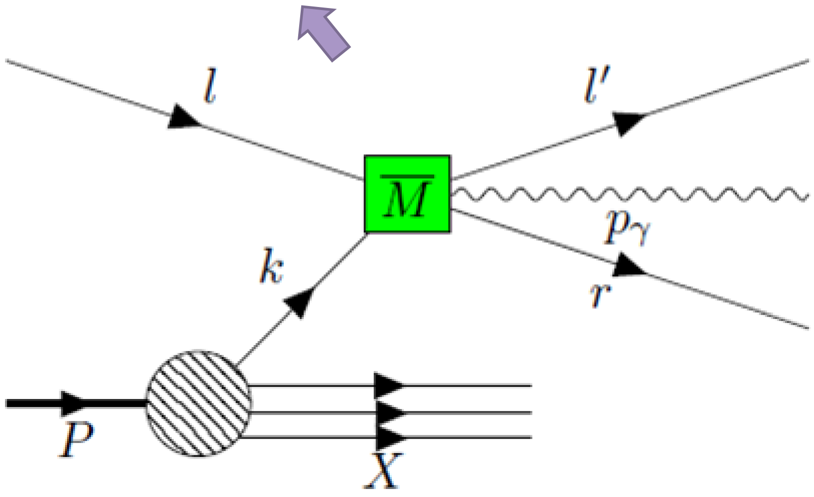
γFF channel

→ $D_1^{\gamma/q}$



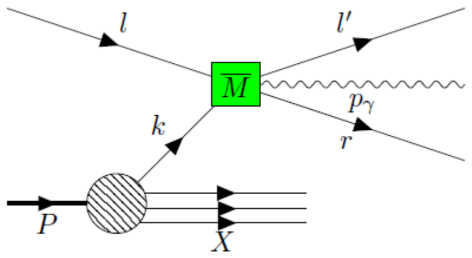
Interference

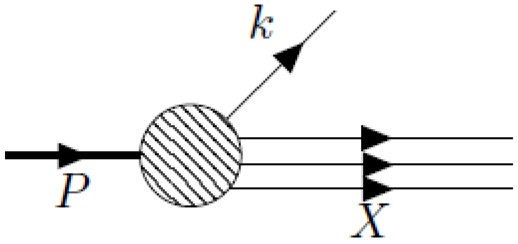
$\propto {e_q}^3$



Direct channel

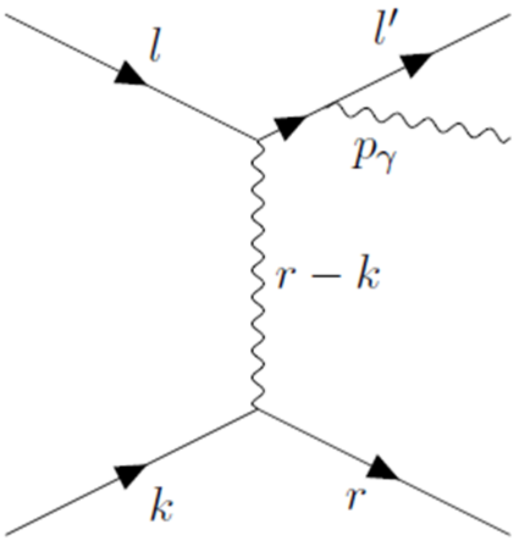
Direct BH Channel

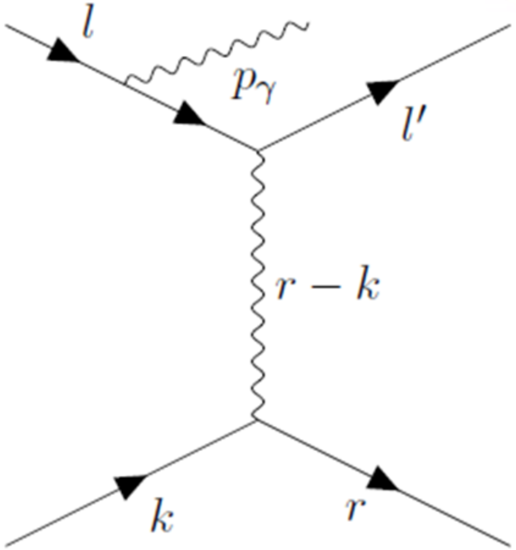




$$\Phi_{ij}(P,S;x) \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \langle N(P,S)|\, \bar{q}_j(0)\mathcal{W}[0;\lambda n] q_i(\lambda n)\, |N(P,S)\rangle$$

$$= \frac{1}{2}\left(\not{P} f_1^{q/N,\overline{\mathrm{MS}}}(x,\mu) - S_L \not{P}\gamma_5 g_1^{q/N,\overline{\mathrm{MS}}}(x,\mu) + \ldots \right)_{ij}$$

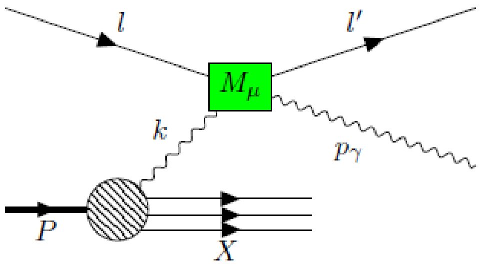


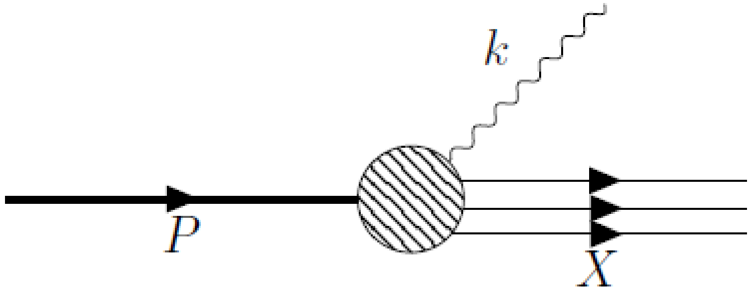


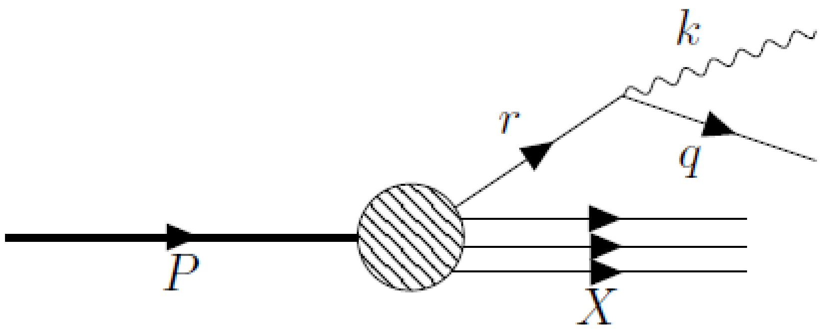
- The partonic scattering is purely QED
- The lepton propagator can produce a pole when the photon is emitted collinearly

→ Can be avoided by keeping a non-zero lepton mass

γPDF Channel
Soft Emission Part





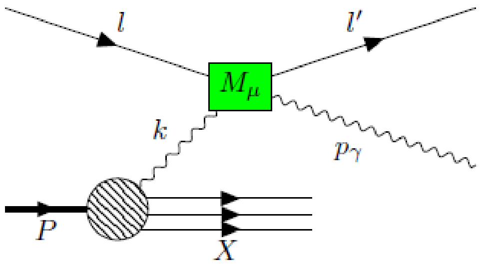


$$f_1^{\gamma/N} \quad g_1^{\gamma/N}$$

$$f_1^{\gamma/N} \longleftrightarrow f_1^{q/N}$$

$$g_1^{\gamma/N} \longleftrightarrow g_1^{q/N}$$

γPDF Channel
Soft Emission Part



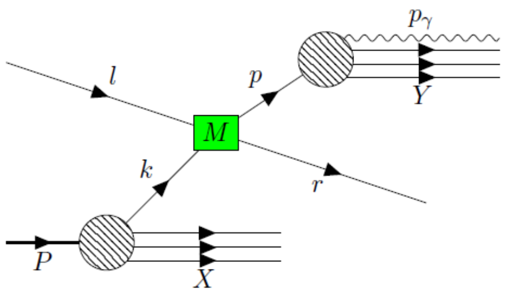
Translating the last slide into mathematical expressions yields a photon-photon correlator

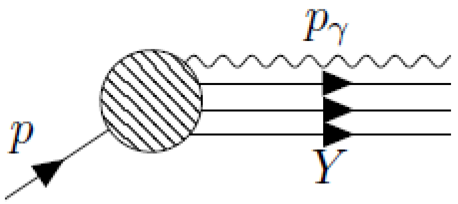
$$\phi^{\mu\nu}(x) \equiv \int_{-\infty}^{\infty} \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda x} \langle N(P,S)|F^{n\nu}(0)F^{n\mu}(\lambda n)|N(P,S)\rangle$$
$$= \frac{x}{2}\left(-\frac{g_\perp^{\mu\nu}(P)}{1-\varepsilon} f_{1,bare}^{\gamma/N}(x) + iS_L\epsilon^{Pn\nu\mu} g_{1,bare}^{\gamma/N}(x) + \ldots\right)$$

$$f_{1,bare}^{\gamma/N}(x,\mu) = \frac{\alpha_{em}}{2\pi}\frac{S_\varepsilon}{\varepsilon}\int_x^1 \frac{\mathrm{d}w}{w} P_{\gamma q}(w) f_1^{BH,\overline{\mathrm{MS}}}\left(\frac{x}{w},\mu\right) + f_1^{\gamma/N,\overline{\mathrm{MS}}}(x,\mu)$$

$$P_{\gamma q}(w) \equiv \frac{1+(1-w)^2}{w} \qquad f_1^{BH,\overline{\mathrm{MS}}}(x,\mu) \equiv \sum_q e_q^2 \left(f_1^{q/N,\overline{\mathrm{MS}}}(x,\mu) + f_1^{\bar{q}/N,\overline{\mathrm{MS}}}(x,\mu)\right)$$

Weizsäcker Williams Channel
Soft Fragmentation Part

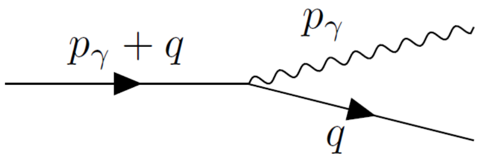




$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{-i\lambda/z} \sum\!\!\!\!\!\!\!\int_Y \langle 0|\psi_l(0)|\gamma(p_\gamma), Y\rangle \langle \gamma(p_\gamma), Y|\overline{\psi}_i(\lambda n)|0\rangle \equiv \frac{z^{2\varepsilon}}{z} \not{p}_{\gamma,li} D_{1,bare}^{\gamma/\ell}(z)$$

→This is a pure QED expression! Pole and finite part are computable using perturbation theory!

$$\langle \gamma(p_\gamma), \ell(q)|\overline{\psi}_i(0)|0\rangle =$$



$$= \left(\bar{u}(q)\,(-ie\not{\epsilon}^*(p_\gamma))\, i\frac{\not{p}_\gamma + \not{q} + m_\ell}{(p_\gamma + q)^2 - m_\ell^2} \right)_i$$

$$D_{1,bare}^{\gamma/\ell}(z) = \frac{\alpha_{em}}{2\pi} P_{\gamma\ell}(z) \frac{S_\varepsilon}{\varepsilon} + \underbrace{\frac{\alpha_{em}}{2\pi} P_{\gamma\ell}(z) \left(\log \frac{\mu^2}{z^2 m_\ell^2} - 1 \right)}_{D_1^{\gamma/\ell,\overline{\mathrm{MS}}}(z)} + \mathcal{O}(\varepsilon)$$

Some Remarks on the polarized and non-zero lepton mass cases

$$\not{P} f_1^{q/N} \to -\not{P}\gamma_5 g_1^{q/N}$$
$$u(l)\bar{u}(l) \to -\not{l}\gamma_5$$
$$-g_\perp^{\mu\nu}(P)/(1-\varepsilon) f_1^{\gamma/N} \to i\epsilon^{Pn\nu\mu} g_1^{\gamma/N}$$

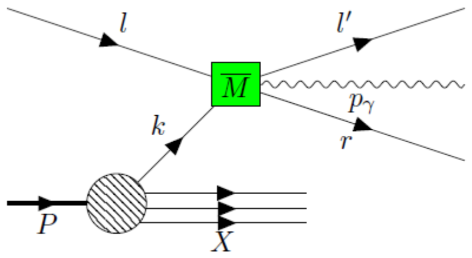
Use HVBM scheme for handling γ_5 and $\epsilon^{\mu\nu\rho\sigma}$
→also affects the relation between photon and quark distribution

$$g_{1,bare}^{\gamma/N}(x,\mu) = \frac{\alpha_{em}}{2\pi}\frac{S_\varepsilon}{\varepsilon}\int_x^1 \frac{\mathrm{d}w}{w}\Delta P_{\gamma q}(w) g_1^{BH,\overline{\mathrm{MS}}}\left(\frac{x}{w},\mu\right) + g_1^{\gamma/N,\overline{\mathrm{MS}}}(x,\mu) \qquad \Delta P_{\gamma q}(w) \equiv 2-w$$

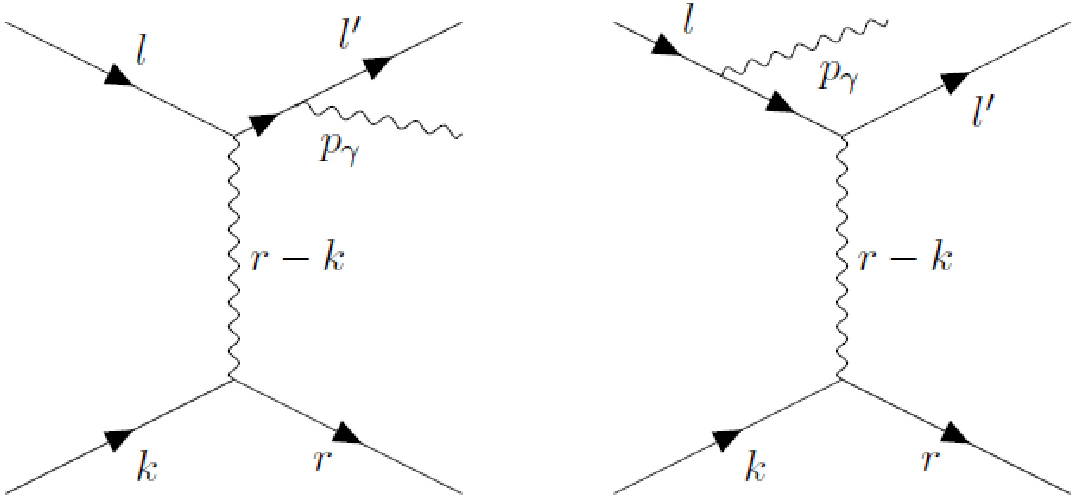
Repeating the calculation with $l^2 = l'^2 = m_\ell^2 \neq 0$ and expanding around $m_\ell = 0$ gives a good consistency check

Massless direct + WW channels ⟷ Massive direct channel

Direct Channel via Crossing



Bethe Heitler



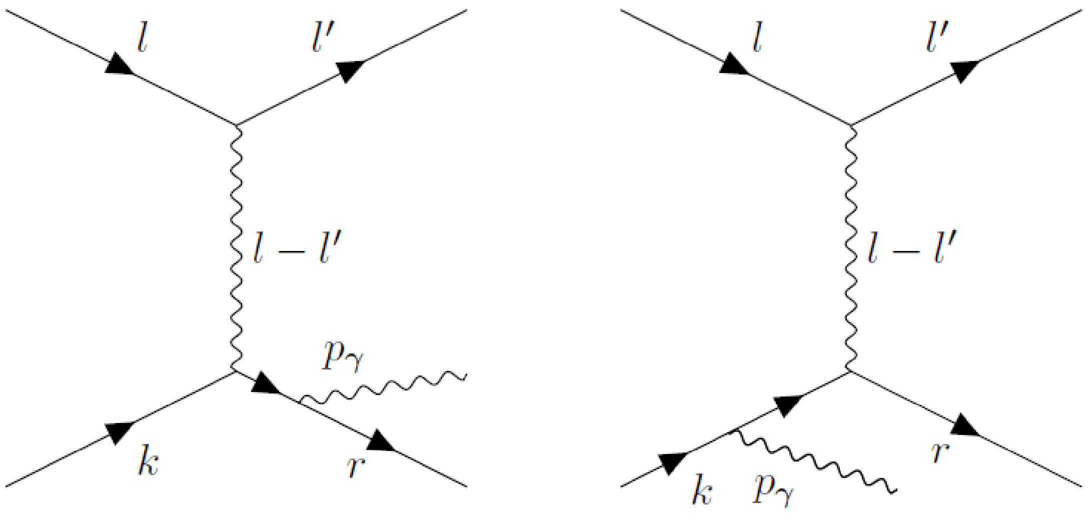
$$l \leftrightarrow k$$
$$r \leftrightarrow l'$$

or

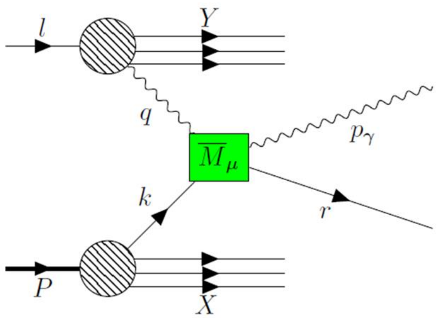
$$u \leftrightarrow xt$$

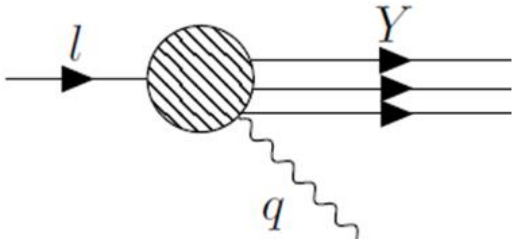
$$2k \cdot l \equiv xs$$
$$-2k \cdot p_\gamma \equiv xt$$
$$-2l \cdot p_\gamma \equiv u$$

Compton



CWW Channel
Soft Emission Part

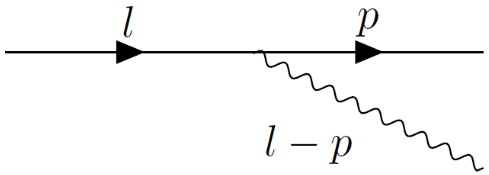




$$\int \frac{\mathrm{d}\lambda}{2\pi} e^{i\lambda y} \langle \ell(l,\lambda_\ell)|F^{n\nu}(0)F^{n\mu}(\lambda n)|\ell(l,\lambda_\ell)\rangle = \frac{y}{2}\left(-\frac{g_\perp^{\mu\nu}(l)}{1-\varepsilon} f_{1,bare}^{\gamma/\ell}(y) + i\lambda_\ell \epsilon^{ln\nu\mu} g_{1,bare}^{\gamma/\ell}(y)\right)$$

→Again, a pure QED expression computable in perturbation theory

$$\langle \ell(p,s)|A^\mu(0)|\ell(l,\lambda_\ell)\rangle =$$

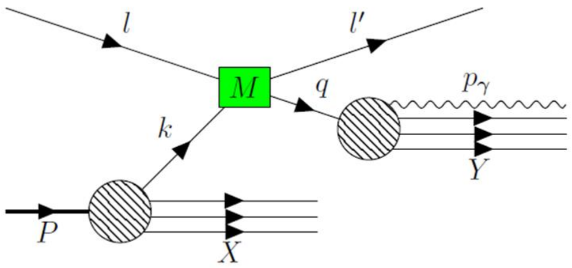


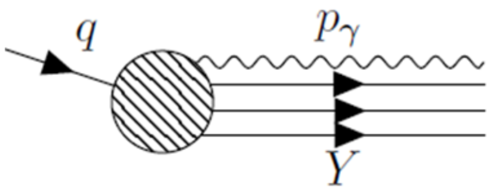
$$= \bar{u}(p,s)\,(-ie\gamma_\alpha)\,u(l,\lambda_\ell)\left(\frac{-ig_\perp^{\alpha\mu}(l-p)}{(l-p)^2}\right)$$

$$f_{1,bare}^{\gamma/\ell}(y,\mu) = \frac{\alpha_{em}}{2\pi} P_{\gamma\ell}(y)\frac{S_\varepsilon}{\varepsilon} + \frac{\alpha_{em}}{2\pi} P_{\gamma\ell}(y)\left(\log\left(\frac{\mu^2}{y^2m^2}\right) - 1\right) + \mathcal{O}(\varepsilon)$$

$$g_{1,bare}^{\gamma/\ell}(y,\mu) = \frac{\alpha_{em}}{2\pi} \Delta P_{\gamma\ell}(y)\frac{S_\varepsilon}{\varepsilon} + \frac{\alpha_{em}}{2\pi} \Delta P_{\gamma\ell}(y)\log\left(\frac{\mu^2}{y^2m^2}\right) + \mathcal{O}(\varepsilon)$$

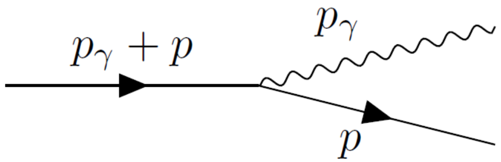
γFF Channel
Soft Fragmentation Part





$$\frac{z^{2\varepsilon}}{z}\not{p}_{\gamma,li} D^{\gamma/q}_{bare}(z) = \frac{1}{N_C}\sum_Y \int \frac{\mathrm{d}\lambda}{2\pi} e^{i\frac{\lambda}{z}} \langle 0|\mathcal{W}[\infty n;\lambda n]\, q_l(\lambda n)|\gamma(p_\gamma),Y\rangle \langle\gamma(p_\gamma),Y|\bar{q}_i(0)\mathcal{W}[0;\infty n]|0\rangle$$

$$\langle\gamma(p_\gamma),q(p)|\bar{q}_i(0)|0\rangle =$$



$$= -\frac{e e_q}{(p+p_\gamma)^2}\left(\bar{u}(p)\not{\epsilon}^*(p_\gamma)\left(\not{p}+\not{p}_\gamma\right)\right)_i$$

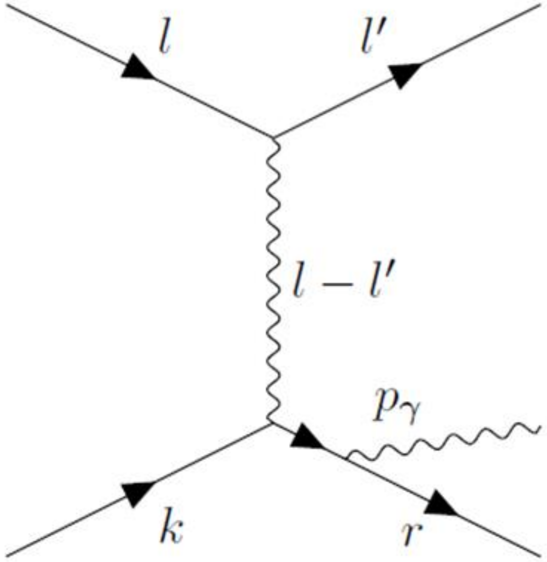
$$D^{\gamma/q}_{bare}(z,\mu) = \frac{\alpha_{em}}{2\pi}e_q^2 P_{\gamma q}(z)\frac{S_\varepsilon}{\varepsilon} + D_1^{\gamma/q,\overline{\mathrm{MS}}}(z,\mu)$$

↑ $\mathcal{O}\left(\alpha_{em}/\alpha_s\right)$

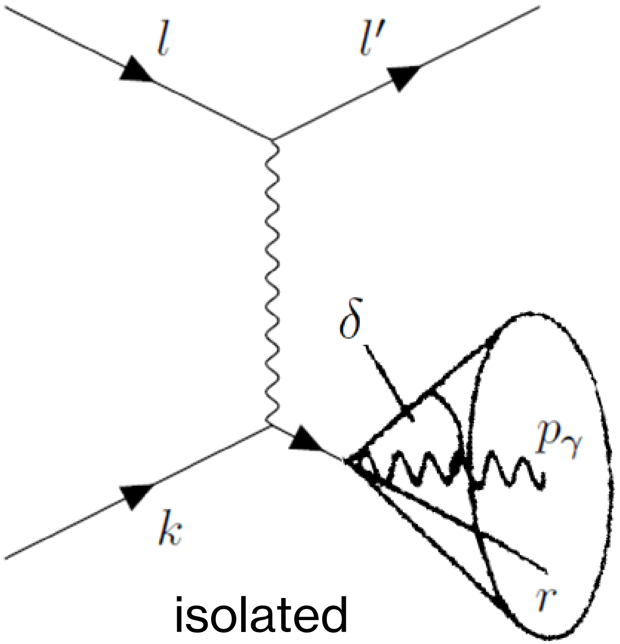
→Need NLO corrections to the tree level partonic scattering!!

[1505.06415] Single-Inclusive Production of Hadrons and Jets in Lepton-Nucleon Scattering at NLO (arxiv.org)
[1703.10872] Double-Longitudinal Spin Asymmetry in Single-Inclusive Lepton Scattering at NLO (arxiv.org)

Photon Isolation



inclusive



isolated

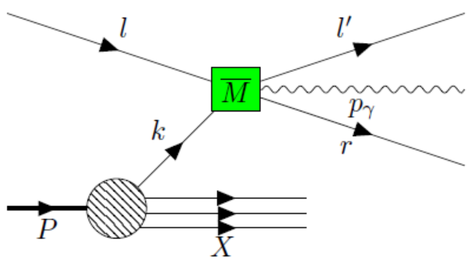
Inside the cone $\longleftrightarrow$ $\sqrt{(\Delta\eta)^2 + (\Delta\phi)^2} \leq R$

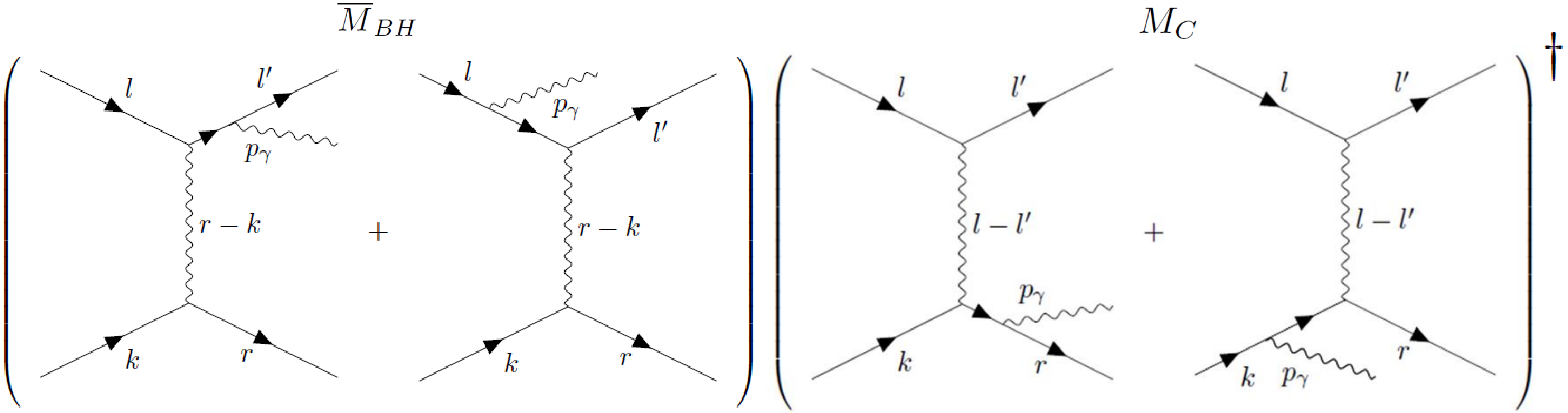
Event is vetoed if $E_h > \xi E_\gamma$

Subtract corresponding parts from the inclusive (full) cross section

- Can be done analytically for $R \ll 1$ (Small Cone Approximation)
- The direct Compton channel and the γFF channel get reduced, all other channels remain unaffected

Interference

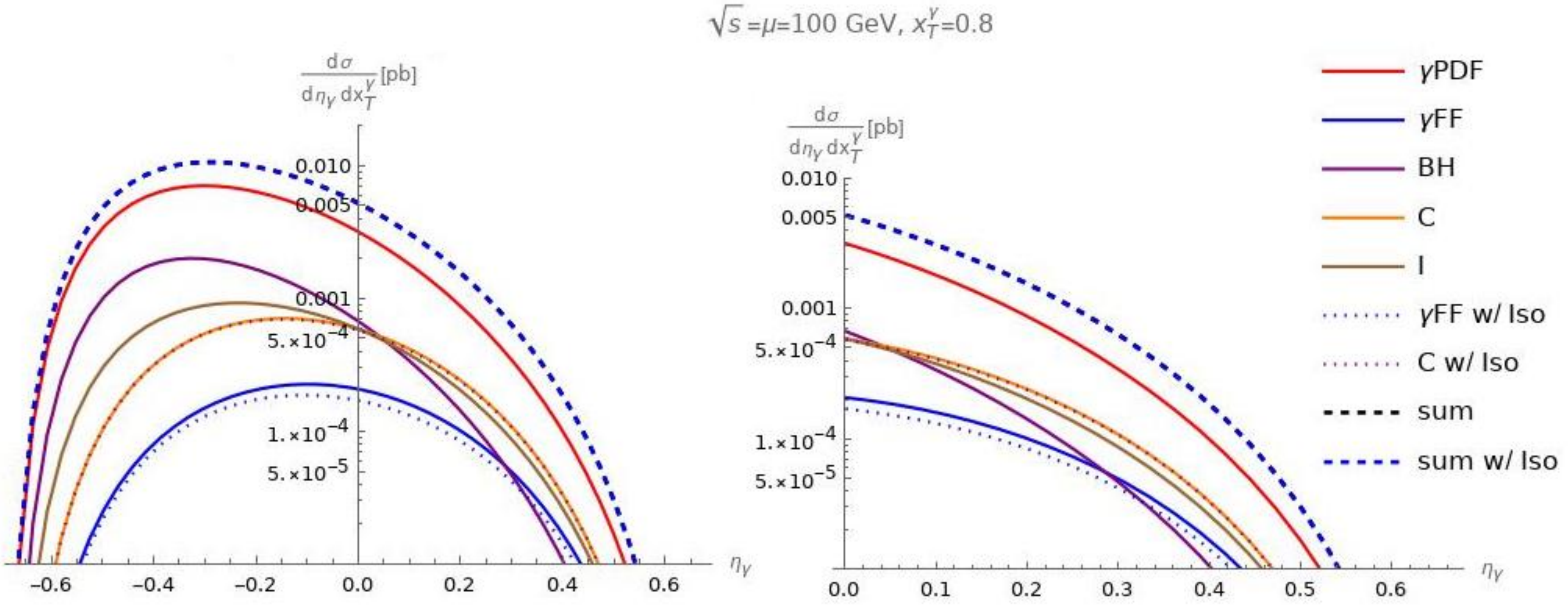




Four different denominators containing momenta l' , r which the phase space integral runs over
→Partial fraction decomposition to simplify the phase space integration
[2102.08943] New ideas for handling of loop and angular integrals in D-dimensions in QCD (arxiv.org)

Plots EIC

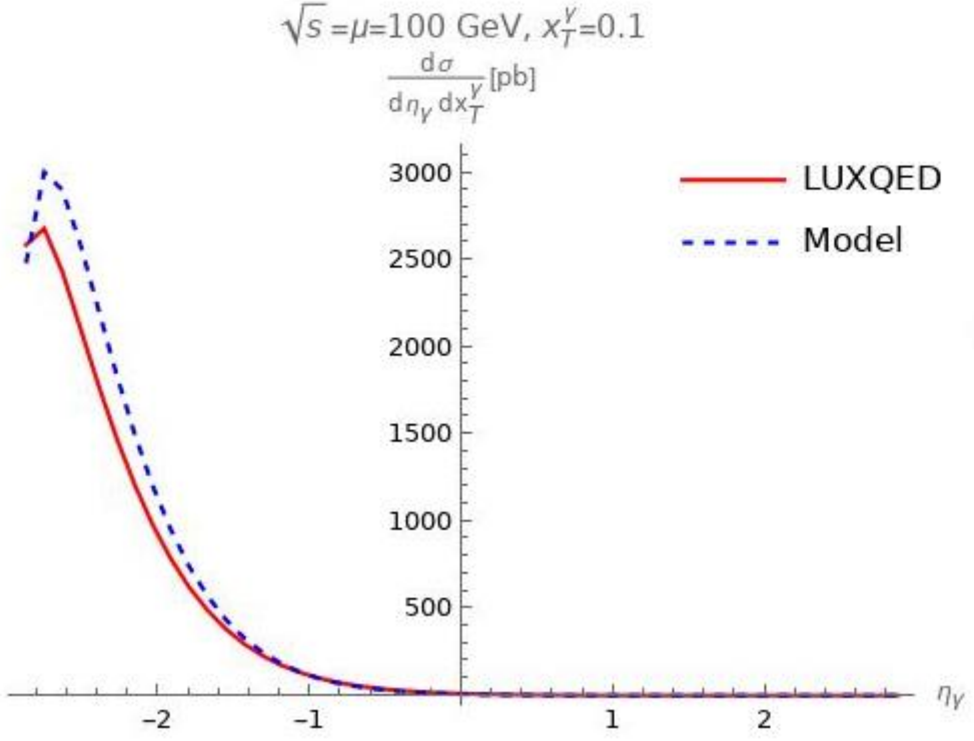
Unpolarized Vs. Pseudorapidity

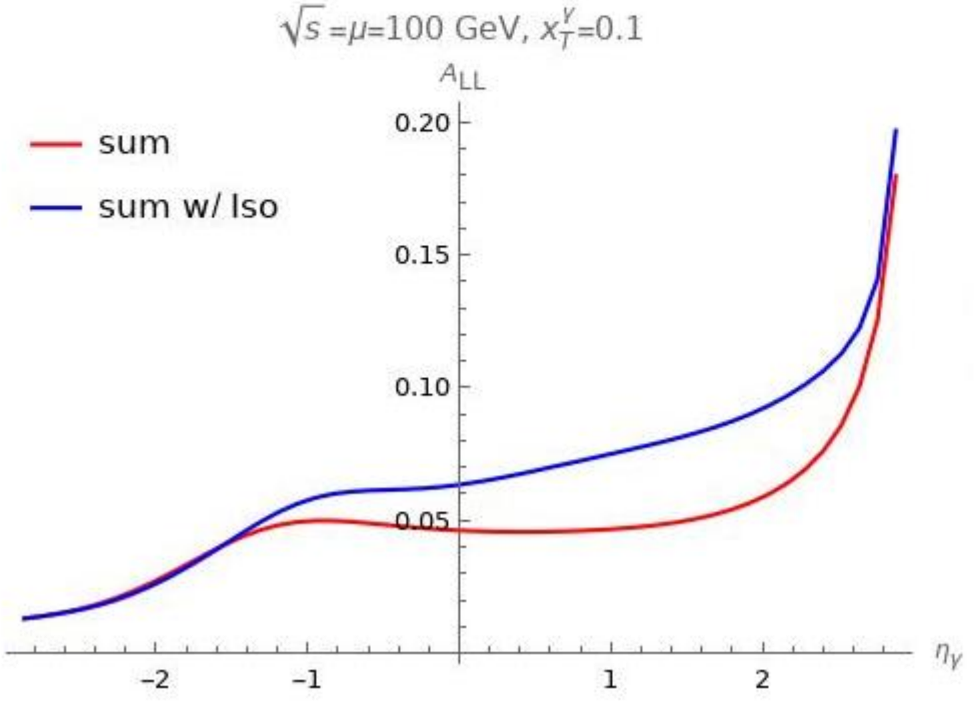


Plots EIC

Model and A_{LL} Vs. Pseudorapidity

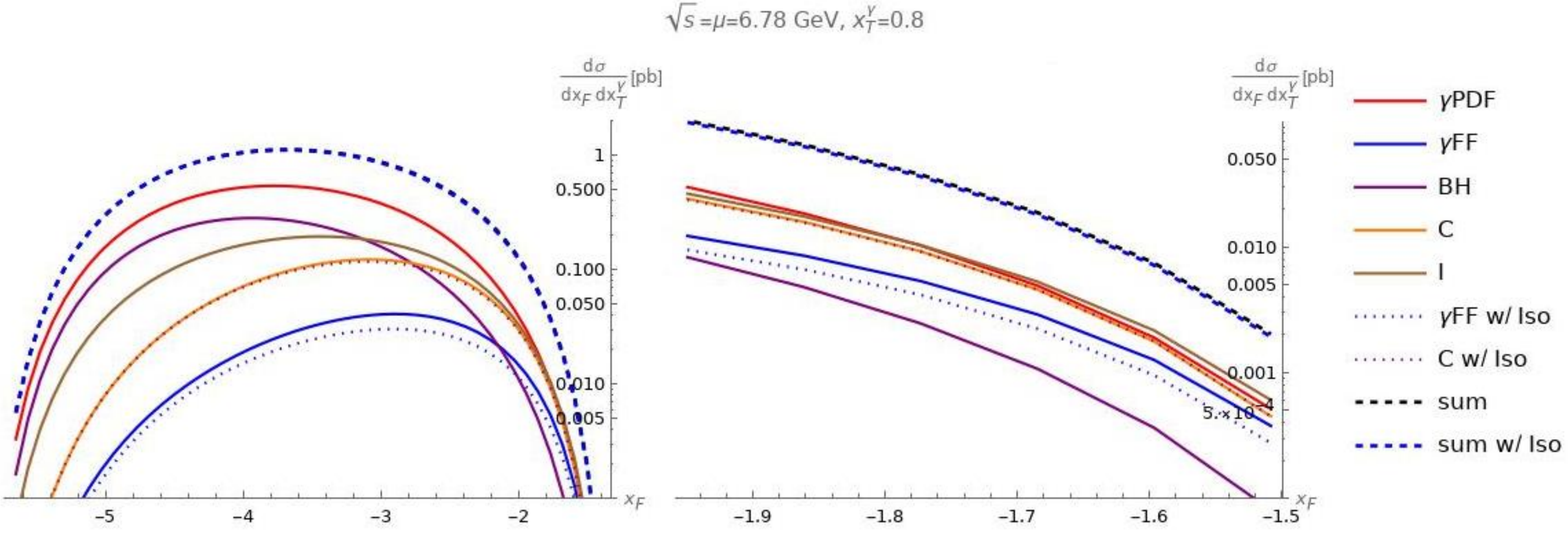
Assumption: $g_1^{\gamma/N} = \alpha_{em} g_1^{g/N}$





Plots JLab

Unpolarized Vs. Longitudinal Momentum



Summary

- (Almost) pure QED calculation featuring lesser known photonic soft functions $f_1^{\gamma/N}$ $g_1^{\gamma/N}$ $D_1^{\gamma/q}$
- Numeric predictions also show good accessibility in experiments and various favorable kinematic regions can be identified
- Next: Analysis of transverse spin effects / computation of the transverse spin asymmetry

$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$