NLO Drell-Yan at low q_T

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Outline

1. Angular distributions in DY
2. Perturbative NLO calculation
3. Systematic small q_T-expansion
4. Results for gluon-fusion process
5. Summary and Outlook

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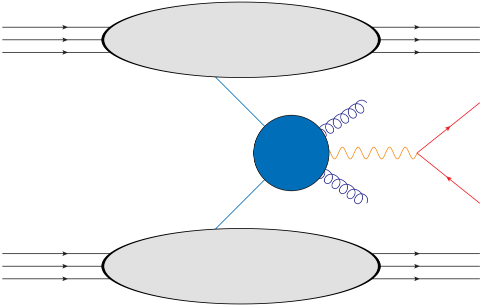
Angular distributions in DY

Drell-Yan process



Differential DY cross section

$$\frac{d\sigma_{pp\to\ell\bar{\ell}X}}{d^4q\, d\Omega} = \frac{\alpha^2}{2(2\pi)^4 s^2 Q^4} L_{\mu\nu} W^{\mu\nu}$$

Reference frames

kinematic variables

$$s = (P_1 + P_2)^2, \; q^2 = Q^2, \; q = x_1 P^+ + x_2 P^- + q_\mathrm{T}, \; \rho^2 = Q_\mathrm{T}^2 / Q^2 \, .$$

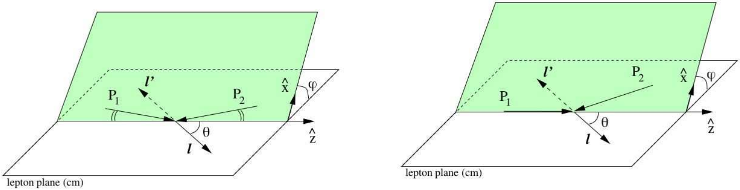
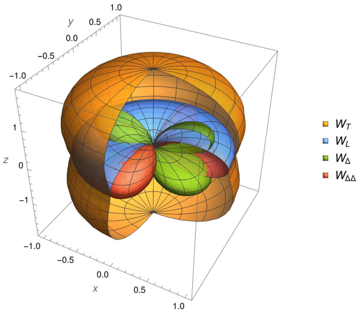


Figure: Collin-Soper frame (l.) and Gottfried-Jackson frame (r.) [Boer & Vogelsang, 2006]

Angular distributions

Helicity structure functions for $pp \to \gamma^* \to \ell\bar{\ell}$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^4 q\,\mathrm{d}\Omega} = \frac{\alpha^2}{2(2\pi)^4 s^2 Q^4}\Big[W_{\mathrm{T}}(1+\cos^2\theta) + W_{\mathrm{L}}(1-\cos^2\theta) + W_{\Delta}\sin 2\theta\cos\phi + W_{\Delta\Delta}\sin^2\theta\cos 2\phi\Big]$$



Basis of covariant projectors

$$W_g \equiv g_{\mu\nu} W^{\mu\nu}\,,$$
$$W_1 \equiv P_{1,\mu} P_{1,\nu} W^{\mu\nu}\,,$$
$$W_2 \equiv P_{2,\mu} P_{2,\nu} W^{\mu\nu}\,,$$
$$W_{12} \equiv (P_{1,\mu} P_{2,\nu} + P_{2,\mu} P_{1,\nu})\, W^{\mu\nu}\,.$$

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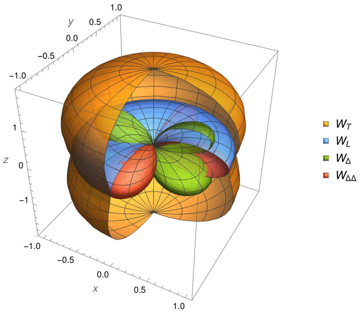

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Helicity structure functions in terms of covariant projectors

$$W_{\mathrm{T}} = -\frac{1}{2}\left[W_g + \frac{1}{Q^2 + Q_{\mathrm{T}}^2}\left(x_1^2 W_1 + x_2^2 W_2 - x_1 x_2 W_{12}\right)\right]$$

$$W_{\mathrm{L}} = \frac{1}{Q^2 + Q_{\mathrm{T}}^2}\left[x_1^2 W_1 + x_2^2 W_2 - x_1 x_2 W_{12}\right]$$

$$W_{\Delta} = \frac{1}{\rho(Q^2 + Q_{\mathrm{T}}^2)}\left[x_1^2 W_1 - x_2^2 W_2\right]$$

$$W_{\Delta\Delta} = -\frac{1}{2}\left\{W_g + \frac{1}{\rho^2(Q^2 + Q_{\mathrm{T}}^2)} \times \left[(2+\rho^2)x_1^2 W_1 + (2+\rho^2)x_2^2 W_2 + (2-\rho^2)x_1 x_2 W_{12}\right]\right\}$$

Collinear factorization

Collinear factorization for DY differential cross section

$$\frac{\mathrm{d}\sigma_{pp\to\ell\bar{\ell}X}(P_1,P_2)}{\mathrm{d}^4q\,\mathrm{d}\Omega} = \sum_{a,b}\int_0^1 \mathrm{d}\xi_1 \int_0^1 \mathrm{d}\xi_2\, f_{a/p}(\xi_1) f_{b/p}(\xi_2) \times \frac{\mathrm{d}\hat{\sigma}_{ab\to\ell\bar{\ell}X}(\xi_1 P_1, \xi_2 P_2)}{\mathrm{d}^4q\,\mathrm{d}\Omega}$$

Collinear factorization for hadronic tensor; $z_i = x_i/\xi_i$

$$x_1 x_2 W^{\mu\nu}_{pp} = \sum_{a,b}\int_{x_1}^1 \mathrm{d}z_1 \int_{x_2}^1 \mathrm{d}z_2\, f_{a/p}\left(\frac{x_1}{z_1}\right) f_{b/p}\left(\frac{x_2}{z_2}\right) \hat{W}^{\mu\nu}_{ab}(z_1,z_2)$$

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Partonic helicity structure functions in CS-frame

$$\hat{W}_T = -\frac{1}{2}\left[\hat{W}_g + \frac{1}{Q^2 + Q_\mathrm{T}^2}\left(z_1^2 \hat{W}_1 + z_2^2 \hat{W}_2 - z_1 z_2 \hat{W}_{12}\right)\right]$$

$$\hat{W}_L = \frac{1}{Q^2 + Q_\mathrm{T}^2}\left[z_1^2 \hat{W}_1 + z_2^2 \hat{W}_2 - z_1 z_2 \hat{W}_{12}\right]$$

$$\hat{W}_\Delta = \frac{1}{\rho(Q^2 + Q_\mathrm{T}^2)}\left[z_1^2 \hat{W}_1 - z_2^2 \hat{W}_2\right]$$

$$\hat{W}_{\Delta\Delta} = -\frac{1}{2}\left\{\hat{W}_g + \frac{1}{\rho^2(Q^2 + Q_\mathrm{T}^2)} \times \left[(2+\rho^2) z_1^2 \hat{W}_1 + (2+\rho^2) z_2^2 \hat{W}_2 + (2-\rho^2) z_1 z_2 \hat{W}_{12}\right]\right\}$$

LO QCD calculation [Boer & Vogelsang, 2006]

- contributions from annihilation process $q\bar{q} \to \gamma^* g$ and Compton-like process $qg \to \gamma^* q$
- simple tree-level results, functions of z_1, z_2 and ρ^2
- phase-space proportional to $\int_{x_1}^{1} \mathrm{d}z_1 \int_{x_2}^{1} \mathrm{d}z_2 \, \delta\left(\frac{\hat{s}_2}{Q^2}\right)$, with $\hat{s}_2 = \hat{s} + \hat{t} + \hat{u} - Q^2$
- small Q_T-expansion achieved by expansion of delta function

Delta function expansion

$$\delta\left((1-z_1)(1-z_2) - \frac{z_1 z_2 \rho^2}{1+\rho^2}\right) = \frac{\delta(1-z_1)}{(1-z_1)_+} + \frac{\delta(1-z_2)}{(1-z_2)_+} - \delta(1-z_1)\delta(1-z_2)\log\rho^2 + \mathcal{O}(\rho^2)$$

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LO results for small Q_T; Sudakov logarithms

$$W_\mathrm{T} = \frac{\alpha_s}{2\pi}\frac{1}{\rho^2}[C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P_{qq}\otimes\bar{q})(x_2) + (P_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(P_{qg}\otimes g)(x_2) + (P_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)]$$

$$W_\mathrm{L} = \frac{\alpha_s}{2\pi}[C_F(2\ln\rho^2+3)q(x_1)\bar{q}(x_2) + q(x_1)(P'_{qq}\otimes\bar{q})(x_2) + (P'_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(P'_{qg}\otimes g)(x_2) + (P'_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)]$$

$$W_\Delta = \frac{\alpha_s}{2\pi}\frac{1}{\rho}[q(x_1)(\tilde{P}_{qq}\otimes\bar{q})(x_2) - (\tilde{P}_{qq}\otimes q)(x_1)\bar{q}(x_2) + q(x_1)(\tilde{P}_{qg}\otimes g)(x_2) - (\tilde{P}_{qg}\otimes g)(x_1)\bar{q}(x_2) + \mathcal{O}(\rho^2)]$$

$W_{\Delta\Delta} = W_\mathrm{L}/2$ **(Lam-Tung relation)**

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Effects of resummation on small-Q_T behvior

$$W_\mathrm{T} = \int \frac{\mathrm{d}^2 b}{4\pi}\, \mathrm{e}^{\mathrm{i}\vec{q}_\mathrm{T}\cdot\vec{b}} \sum_a e_a^2 q_a(x_1, b_0/b)\bar{q}_a(x_2, b_0/b)\, \mathrm{e}^{S(b,Q^2)}$$

with Sudakov form factor

$$S(b,Q) = -\int_{b_0^2/b^2}^{Q^2} \frac{\mathrm{d}k_\mathrm{T}^2}{k_\mathrm{T}^2}\left[A(\alpha_\mathrm{s}(k_\mathrm{T})\ln\left(\frac{Q^2}{k_\mathrm{T}^2}\right) + B(\alpha_\mathrm{s}(k_\mathrm{T}))\right]$$

$$\sim \frac{1}{\rho^2} \longrightarrow \text{CSS-Resummation} \longrightarrow$$

[1]Figure from [Ebert et al., 2021]

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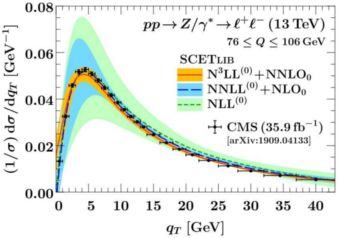

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Perturbative NLO calculation

NLO calculation

- NLO for Drell-Yan differential in Q_T calculated by Mirkes in 1992 [Mirkes, 1992]
- real corrections slightly wrong caused by erroneous recursion relation for one class of angular integrals
- independent recalculation using `Mathematica` and `FeynCalc`
- new general results for angular integrals, including double massive integral, all-order ε-expansions [Lyubovitskij et al., 2021]
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- $gg \to \gamma^* q\bar{q}$ subprocess as first example for Q_T expansion

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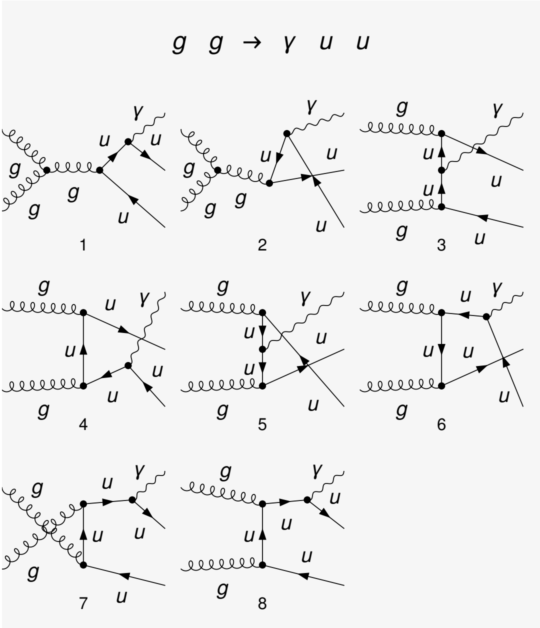
g g → γ u u
1
2
3
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7
8

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$$s = \frac{(1+\rho^2)Q^2}{z_1 z_2}, \quad t = -\frac{Q^2(1-z_1+\rho^2)}{z_1},$$
$$u = -\frac{Q^2(1-z_2+\rho^2)}{z_2}, \quad s_2 = \frac{Q^2((1-z_1)(1-z_2)+\rho^2(1-z_1-z_2))}{z_1 z_2}.$$

Example for Angular integral

$$\int \mathrm{d}\Omega_{k_1 k_2} \frac{1}{(p_1-k_1)^2(p_2-k_1)^2}$$
$$= -\frac{4\pi z_1 z_2}{\varepsilon Q^4 \rho^2 (1+\rho^2)} + \frac{4\pi z_1 z_2 \log\left(\frac{\rho^2 z_1 z_2}{(1+\rho^2)(1-z_1)(1-z_2)}\right)}{Q^4 \rho^2 (1+\rho^2)} + O\left(\varepsilon^1\right)$$

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Results – general structure

$$
\begin{aligned}
\hat{W}_i = {} & c_1 + c_2 \log\left(\frac{Q^2}{\mu_F^2}\right) + c_3 \log\left(\frac{\rho^2 z_1 z_2}{(1+\rho)^2(1-z_1)(1-z_2)}\right) \\
& + c_4 \log\left(\frac{(1+\rho^2)(1-z_1)^2 z_1}{z_2(1-z_1+\rho^2)^2}\right) + c_5 \log\left(\frac{(1+\rho^2)(1-z_2)^2 z_2}{z_1(1-z_2+\rho^2)^2}\right) \\
& + c_6 \log\left(1 - \frac{\rho^2 z_1 z_2}{(1+\rho^2)(1-z_1)(1-z_2)}\right) \\
& + c_7 \log\left(\frac{(1+\rho^2)(1-z_1)^2 z_2}{z_1(1-(1+\rho^2)z_1+(1-z_2)\rho^2}\right) \\
& + c_8 \log\left(1 + \frac{(1+\rho^2)(1-z_1-z_2}{z_1 z_2}\right) \\
& + c_9 \log\left(\frac{(1+\rho^2)(1-z_2)^2 z_1}{z_2(1-z_2(1+\rho^2)+\rho^2(1-z_1))^2}\right) \\
& + c_{10} \log\left(\frac{1+\sqrt{1-\frac{4z_1 z_2}{(1+\rho^2)(z_1+z_2)^2}}}{1-\sqrt{1-\frac{4z_1 z_2}{(1+\rho^2)(z_1+z_2)^2}}}\right)
\end{aligned}
$$

Naive small Q_T limit; W_T and W_Δ

Just expand partonic helicity structure functions in ρ^2 and keep only the leading term.

$$\hat{W}_\mathrm{T} \stackrel{?}{=} \frac{4\pi e_q^2 \alpha_s^2}{C_A z_1 z_2} \frac{1}{\rho^2} \left[(1 - 2z_1 + 2z_1^2)(1 - 2z_2 + 2z_2^2) \log\left(\frac{Q_\mathrm{T}^2}{\mu_\mathrm{F}^2}\right) - (2z_1 z_2 - z_1 - z_2)(2z_1 z_2 - z_1 - z_2 + 1) \right] + \mathcal{O}(\rho^0)$$

$$\hat{W}_\Delta \stackrel{?}{=} \frac{4\pi (z_1 - z_2) e_q^2 \alpha_s^2}{C_A z_1 z_2} \frac{1}{\rho} \left[(-1 + 2(z_1 + z_2) - 2z_1 z_2) \log\left(\frac{Q_\mathrm{T}^2}{\mu_\mathrm{F}^2}\right) + 1 - z_1 - z_2 + 2z_1 z_2 \right] + \mathcal{O}(\rho)$$

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Naive small Q_T limit; W_L and $W_{\Delta\Delta}$

$$\hat{W}_L \stackrel{?}{=} \frac{4\pi e_q^2 \alpha_s^2}{C_A z_1 z_2} \left[(8z_1^2 z_2^2 - 4z_1 z_2 + 2) \log\left(\frac{Q^2}{\mu_F^2}\right) - (2z_1^2 - 1)(2z_2^2 - 1) \log\left(\frac{Q_T^2}{\mu_F^2}\right) + f(z_1, z_2) \right] + \frac{4\pi e_q^2 \alpha_s^2}{C_F} g(z_1, z_2) + \mathcal{O}(\rho^2)$$

$$\hat{W}_{\Delta\Delta} \stackrel{?}{=} -\frac{2\pi e_q^2 \alpha_s^2}{C_A z_1 z_2} \left[(-1 - 2z_1^2 - 2z_2^2 + 4z_1 z_2 - 4z_1^2 z_2^2) \log\left(\frac{Q_T^2}{\mu_F^2}\right) + 12 z_1^2 z_2^2 + z_1^2 + z_2^2 - 8 z_1 z_2 + z_1 + z_2 \right] + \frac{2\pi e_q^2 \alpha_s^2}{C_F} + \mathcal{O}(\rho^2)$$

Lam-Tung relation $2\hat{W}_{\Delta\Delta} = \hat{W}_L$ only holds for the $\log \mu_F^2$ part.

Naive small Q_T limit; W_L and $W_{\Delta\Delta}$

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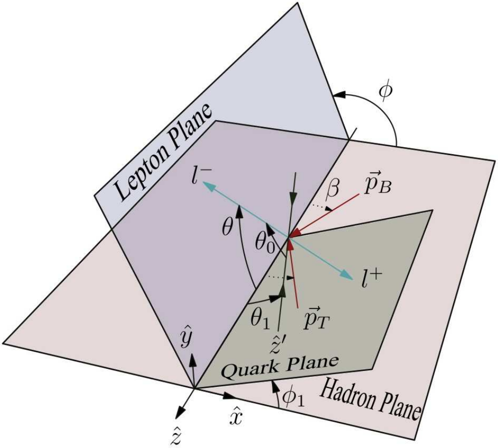
Geometry behind Lam-Tung relation



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$2W_{\Delta\Delta} = W_{\mathrm{L}}$,
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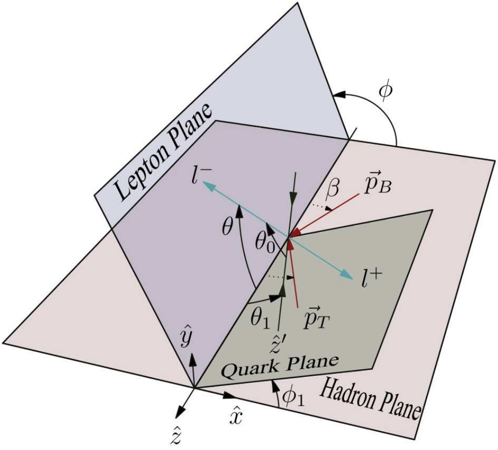

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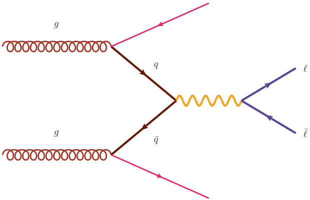

Figure: Contribution to Lam-Tung violation

Systematic small q_T-expansion

Setup

We have results of the form

$$x_1 x_2 W_i^{pp} = \sum_{a,b} \int_{x_1}^{1} \mathrm{d}z_1 \int_{x_2}^{1} \mathrm{d}z_2 f_{a/p}\left(\frac{x_1}{z_1}\right) f_{b/p}\left(\frac{x_2}{z_2}\right) \hat{W}_i^{ab}(z_1, z_2, \rho^2)\,,$$

where

$$\hat{W}_i^{ab}(z_1, z_2, \rho^2) \sim \Theta\left(\frac{s_2}{Q^2}\right) \sum_i \underbrace{\mathcal{S}_i(z_1, z_2, \rho^2)}_{\text{singular for } \rho^2 \to 0} \underbrace{\mathcal{R}_i(z_1, z_2, \rho^2)}_{\text{regular for } \rho^2 \to 0}$$

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Singular parts

- Regular parts can be directly expanded in ρ^2 in the integrand; integration boundaries x_1, x_2 into regular term as θ functions
- Singular parts $\mathcal{S}_i(z_1, z_2, \rho^2)$ e.g. $\frac{1}{1-z_1}$, $\frac{1}{1-z_2+\rho^2}$, $\frac{\log(1-z_1+\rho^2)}{1-z_2}$ and products thereof
- Number of different singular parts can be drastically reduced by partial fraction decomposition w.r.t. z_1 and z_2; total of 44 $\mathcal{S}_i(z_1, z_2, \rho^2)$

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Singular factors can not simply be expanded about ρ^2.

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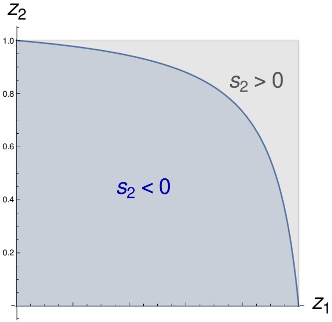
Example

$$\frac{1}{1 - z_1 + \rho^2} = \sum_{n=0}^{\infty} \frac{(-1)^n \rho^{2n}}{(1 - z_1)^{n+1}}$$



When we take the limit of the integration area, each term gives a contribution from the area where $1 - z_1 \sim \rho^2$. This is of the order

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for every term.

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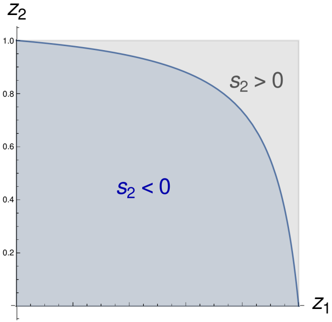

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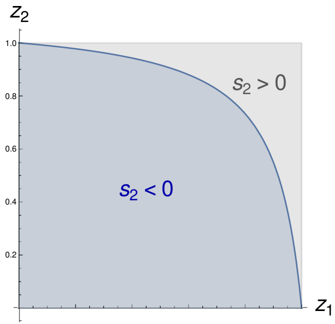

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What if we really want to drop some terms?

Idea

Introduce subtractions to the regular part s.t. the contribution from the $s_2 > 0$ area is negligible. Then drop terms in the series.

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \frac{\Theta(s_2)\phi(z_1,z_2)}{1-z_1+\rho^2} = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \Theta(s_2) \frac{\phi(z_1,z_2)-\phi(1,z_2)}{1-z_1+\rho^2} + \int_0^1 \mathrm{d}z_1 \phi(1,z_2) \underbrace{\int_0^1 \mathrm{d}z_2 \frac{\Theta(s_2)}{1-z_1+\rho^2}}_{\log(1-z_2+\rho^2)-\log(\rho^2)}$$

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$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \frac{\Theta(s_2)\phi(z_1,z_2)}{1-z_1+\rho^2} = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \Theta(s_2) \frac{\phi(z_1,z_2)-\phi(1,z_2)}{1-z_1+\rho^2} + \int_0^1 \mathrm{d}z_1 \phi(1,z_2) \underbrace{\int_0^1 \mathrm{d}z_2 \frac{\Theta(s_2)}{1-z_1+\rho^2}}_{\log(1-z_2+\rho^2)-\log(\rho^2)}$$

In the first integral the denominator is $\sim (1-z_1)$, hence the overall integral is $\sim \rho^0$ in the $s_2 < 0$ region.

$\rightarrow$ we can put $\rho^2 = 0$ and $\Theta(s_2) \rightarrow 1$; error of $\mathcal{O}(\rho^2)$

What if we really want to drop some terms?

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Result in terms of distributions

$$\frac{\theta(s_2)}{1-z_1+\rho^2} = \left[\frac{1}{1-z_1}\right]_+ + \delta(1-z_1)\left[\log(1-z_2) - \log(\rho^2)\right] + \mathcal{O}(\rho^2)\,,$$

where $\int_0^1 \mathrm{d}z \frac{f(z)}{[1-z]_+} \equiv \int_0^1 \mathrm{d}z\, \frac{f(z)-f(1)}{1-z}\,.$

Question

Can we do better than $\mathcal{O}(\rho^2)$ errors?

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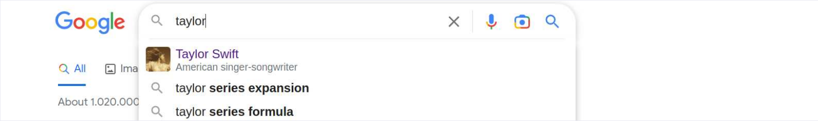

Taylor polynomial subtraction

Let $\phi(z_1, z_2)$ be a sufficiently regular function, $\mathcal{T}^n_{(1,z_2)}\phi(z_1, z_2)$, $\mathcal{T}^m_{(z_1,1)}\phi(z_1, z_2)$, and $\mathcal{T}^{n,m}_{(1,1)}\phi(z_1, z_2)$ its one- and two-fold Taylor polynomials. Then

$$\phi(z_1, z_2) - \mathcal{T}^n_{(1,z_2)}\phi(z_1, z_2) - \mathcal{T}^m_{(z_1,1)}\phi(z_1, z_2) + \mathcal{T}^{n,m}_{(1,1)}\phi(z_1, z_2)$$
$$\sim (1 - z_1)^{n+1}(1 - z_2)^{m+1}$$
$$\sim \rho^{2(n+1)} + \rho^{2(m+1)}$$

in the $s_2 < 0$ region.

Therefore...

Regularization to higher order makes singular part finite in $s_2 < 0$ region, which allows for expansion in ρ^2. Error from $\Theta(s_2) \to 1$ suppressed by powers of ρ^2.

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Overview of Q_T expansion algorithm

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \underbrace{\Theta(s_2)\mathcal{S}(z_1, z_2, \rho^2)} \phi(z_1, z_2)$$

Master formula for ρ^2 expansion

Calculate Mellin moments

Log-Laurent expansion

Reduce distributions

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \overbrace{\sum_i c_i \, \mathcal{S}_i^{\text{dist.}}(z_1, z_2)} \phi(z_1, z_2)$$

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Notation I - brace yourselves, lots of formulas are coming

Degree of divergence

n_1, n_2 are the smallest integers s.t. $\lim_{z_1,z_2\to 1}(1-z_1)^{n_1}(1-z_2)^{n_2}\mathcal{S}_{n_1,n_2}(z_1,z_2)$ is finite.

Generalized Plus distributions

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,m_1}^{+,m_2} \phi(z_1,z_2)$$

$$= \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}$$

$$\times\left[\phi(z_1,z_2) - \mathcal{T}^{m_1}_{(1,z_2)}\phi(z_1,z_2) - \mathcal{T}^{m_2}_{(z_1,1)}\phi(z_1,z_2) + \mathcal{T}^{m_1,m_2}_{(1,1)}\phi(z_1,z_2)\right]$$

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Notation II: Mellin moments

Single moments

$$\mathcal{M}(f)(k_1, z_2) = \int_0^1 \mathrm{d}z_1 (1 - z_1)^{k_1} f(z_1, z_2)\,,$$

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Master formula for distributional expansion

Theorem

$$\mathcal{S}_{n_1,n_2}(z_1,z_2,\rho^2) = \sum_{n=0}^{N} \rho^{2n} \left[\mathcal{S}^{(n)}_{n_1^{(n)},n_2^{(n)}}(z_1,z_2)\right]_{+,m_1}^{+,m_2}$$
$$+ \sum_{k_1=0}^{m_1} \frac{\delta^{(k_1)}(1-z_1)}{k_1!} \left[\mathcal{M}\left(\mathcal{S}_{n_1,n_2}\right)(k_1,z_2,\rho^2)\right]^{+,m_2}$$
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Log-Laurent expansion

$$[\mathcal{S}(z_1, z_2)]_{+,m_1}^{+,m_2} = \sum_{k_1=1}^{m_1+1} \sum_{k_2=1}^{m_2+1} \sum_{l_1=0}^{l_1^{\max}} \sum_{l_2=0}^{l_2^{\max}} c_{k_1 k_2 l_1 l_2}^{(\mathcal{S})} \left[\frac{\log^{l_1}(1-z_1) \log^{l_2}(1-z_2)}{(1-z_1)^{k_1}(1-z_2)^{k_2}} \right]_{+,m_1}^{+,m_2}$$

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$$+ \left[\mathcal{S}^{\text{finite}}(z_1, z_2) \right]_{+,m_1}^{+,m_2}$$

with coefficients $c_{k_1 k_2 l_1 l_2}^{(\mathcal{S})}$ and finite functions $c_{k_1 l_1}^{(\mathcal{S})}(z_2)$, $c_{k_2 l_2}^{(\mathcal{S})}(z_1)$, and $\mathcal{S}^{\text{finite}}(z_1, z_2)$.

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$$+ \sum_{k_1=1}^{m_1+1} \sum_{l_1=0}^{l_1^{\max}} \left[c^{(\mathcal{S})}_{k_1 l_1}(z_2) \frac{\log^{l_1}(1-z_1)}{(1-z_1)^{k_1}} \right]^{+,m_2}_{+,m_1}$$

$$+ \sum_{k_2=1}^{m_2+1} \sum_{l_2=0}^{l_2^{\max}} \left[c^{(\mathcal{S})}_{k_2 l_2}(z_1) \frac{\log^{l_2}(1-z_2)}{(1-z_2)^{k_2}} \right]^{+,m_2}_{+,m_1}$$

$$+ \left[\mathcal{S}^{\text{finite}}(z_1, z_2) \right]^{+,m_2}_{+,m_1}$$

with coefficients $c^{(\mathcal{S})}_{k_1 k_2 l_1 l_2}$ and finite functions $c^{(\mathcal{S})}_{k_1 l_1}(z_2)$, $c^{(\mathcal{S})}_{k_2 l_2}(z_1)$, and $\mathcal{S}^{\text{finite}}(z_1, z_2)$.

Log-Laurent expansion

$$
\begin{aligned}
[\mathcal{S}(z_1, z_2)]^{+,m_2}_{+,m_1} = & \sum_{k_1=1}^{m_1+1} \sum_{k_2=1}^{m_2+1} \sum_{l_1=0}^{l_1^{\max}} \sum_{l_2=0}^{l_2^{\max}} c^{(\mathcal{S})}_{k_1 k_2 l_1 l_2} \left[\frac{\log^{l_1}(1-z_1) \log^{l_2}(1-z_2)}{(1-z_1)^{k_1}(1-z_2)^{k_2}} \right]^{+,m_2}_{+,m_1} \\
& + \sum_{k_1=1}^{m_1+1} \sum_{l_1=0}^{l_1^{\max}} \left[c^{(\mathcal{S})}_{k_1 l_1}(z_2) \frac{\log^{l_1}(1-z_1)}{(1-z_1)^{k_1}} \right]^{+,m_2}_{+,m_1} \\
& + \sum_{k_2=1}^{m_2+1} \sum_{l_2=0}^{l_2^{\max}} \left[c^{(\mathcal{S})}_{k_2 l_2}(z_1) \frac{\log^{l_2}(1-z_2)}{(1-z_2)^{k_2}} \right]^{+,m_2}_{+,m_1} \\
& + \left[\mathcal{S}^{\text{finite}}(z_1, z_2)\right]^{+,m_2}_{+,m_1}
\end{aligned}
$$

with coefficients $c^{(\mathcal{S})}_{k_1 k_2 l_1 l_2}$ and finite functions $c^{(\mathcal{S})}_{k_1 l_1}(z_2)$, $c^{(\mathcal{S})}_{k_2 l_2}(z_1)$, and $\mathcal{S}^{\text{finite}}(z_1, z_2)$.

Log-Laurent expansion

$$[\mathcal{S}(z_1,z_2)]_{+,m_1}^{+,m_2} = \sum_{k_1=1}^{m_1+1}\sum_{k_2=1}^{m_2+1}\sum_{l_1=0}^{l_1^{\max}}\sum_{l_2=0}^{l_2^{\max}} c_{k_1k_2l_1l_2}^{(\mathcal{S})}\left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{k_1}(1-z_2)^{k_2}}\right]_{+,m_1}^{+,m_2}$$
$$+\sum_{k_1=1}^{m_1+1}\sum_{l_1=0}^{l_1^{\max}}\left[c_{k_1l_1}^{(\mathcal{S})}(z_2)\frac{\log^{l_1}(1-z_1)}{(1-z_1)^{k_1}}\right]_{+,m_1}^{+,m_2}$$
$$+\sum_{k_2=1}^{m_2+1}\sum_{l_2=0}^{l_2^{\max}}\left[c_{k_2l_2}^{(\mathcal{S})}(z_1)\frac{\log^{l_2}(1-z_2)}{(1-z_2)^{k_2}}\right]_{+,m_1}^{+,m_2}$$
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Reduction of distributions I

$$\left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,m_1}^{+,m_2} = \left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,n_1-1}^{+,n_2-1}$$
$$-\sum_{k_1=n_1}^{m_1}\frac{(-1)^{l_1}l_1!}{k_1!(k_1-n_1+1)}\delta^{(k_1)}(1-z_1)\left[\frac{\log^{l_2}(1-z_2)}{(1-z_2)^{n_2}}\right]^{+,n_2-1}$$
$$-\sum_{k_2=n_2}^{m_2}\frac{(-1)^{l_2}l_2!}{k_2!(k_2-n_2+1)}\delta^{(k_2)}(1-z_2)\left[\frac{\log^{l_1}(1-z_1)}{(1-z_1)^{n_1}}\right]^{+,n_1-1}$$
$$+\sum_{k_1=n_1}^{m_1}\sum_{k_2=n_2}^{m_2}\frac{(-1)^{l_1+l_2}l_1!l_2!}{k_1!k_2!(k_1-n_1+1)(k_2-n_2+1)}\delta^{(k_1)}(1-z_1)\delta^{(k_2)}(1-z_2)$$

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$$\left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,m_1}^{+,m_2} = \left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,n_1-1}^{+,n_2-1}$$

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$$\left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,m_1}^{+,m_2} = \left[\frac{\log^{l_1}(1-z_1)\log^{l_2}(1-z_2)}{(1-z_1)^{n_1}(1-z_2)^{n_2}}\right]_{+,n_1-1}^{+,n_2-1}$$

$$-\sum_{k_1=n_1}^{m_1}\frac{(-1)^{l_1}l_1!}{k_1!(k_1-n_1+1)}\delta^{(k_1)}(1-z_1)\left[\frac{\log^{l_2}(1-z_2)}{(1-z_2)^{n_2}}\right]^{+,n_2-1}$$

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Reduction of distributions II

$$\left[\frac{f(z_2)\log^{l_1}(1-z_1)}{(1-z_1)^{n_1}}\right]_{+,m_1}^{+,m_2} = f(z_2)\left[\frac{\log^{l_1}(1-z_1)}{(1-z_1)^{n_1}}\right]_{+,n_1-1}$$

$$-\sum_{k_1=n_1}^{m_1}\frac{(-1)^{l_1}l_1!}{k_1!(k_1-n_1+1)}\delta^{(k_1)}(1-z_1)f(z_2)$$

$$-\sum_{k_2=0}^{n_2}\frac{\mathcal{M}(f)(k_2)}{k_2!}\delta^{(k_2)}(1-z_2)\left[\frac{\log^{l_1}(1-z_1)}{(1-z_1)^{m_1}}\right]_{+,m_1-1}$$

$$+\sum_{k_1=n_1}^{m_1}\sum_{k_2=0}^{m_2}\frac{(-1)^{l_1}l_1!\mathcal{M}(f)(k_2)}{k_1!k_2!(k_1-n_1+1)}\delta^{(k_1)}(1-z_1)\delta^{(k_2)}(1-z_2)$$

Reduction of distributions III

$$
\begin{aligned}
[f(z_1,z_2)]_{+,m_1}^{+,m_2} =& f(z_1,z_2) - \sum_{k_1=0}^{m_1} \frac{\mathcal{M}(f)(k_1,z_2)}{k_1!}\delta^{(k_1)}(1-z_1) \\
& - \sum_{k_2=0}^{m_2} \frac{\mathcal{M}(f)(z_1,k_2)}{k_2!}\delta^{(k_2)}(1-z_2) \\
& + \sum_{k_1=0}^{m_1}\sum_{k_2=0}^{m_2} \frac{\mathcal{M}(f)(k_1,k_2)}{k_1!k_2!}\delta^{(k_1)}(1-z_1)\delta^{(k_2)}(1-z_2)
\end{aligned}
$$

Overview of Q_T expansion algorithm

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \underbrace{\Theta(s_2)\mathcal{S}(z_1, z_2, \rho^2)}\, \phi(z_1, z_2)$$

↓

Master formula for ρ^2 expansion

Calculate Mellin moments

Log-Laurent expansion

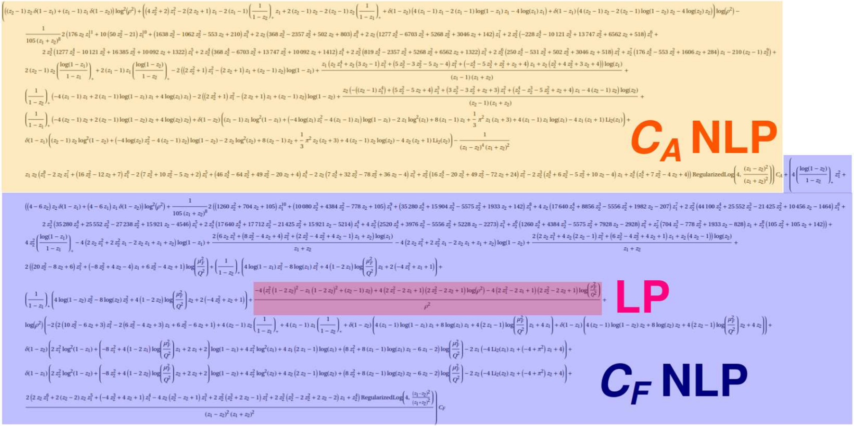
Reduce distributions

↓

$$\int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2 \overbrace{\sum_i c_i\, \mathcal{S}_i^{\text{dist.}}(z_1, z_2)}\, \phi(z_1, z_2)$$

Results for gluon-fusion process

NLP results



Next-to-leading power contributions to Helicity structure functions

- result in terms of distributions

$$\delta^{(n_1)}(1-z_1)\delta^{(n_2)}(1-z_2) \quad \text{and} \quad \left[\frac{\log^{m_1}(1-z_1)\log^{m_2}(1-z_2)}{(1-z_1)^{n_1+1}(1-z_2)^{n_2+1}}\right]_{+,n_1}^{+,n_2}$$

- phase-space effects essential for NLP expansion
- LP non-trivial phase-space effects only in $W_{\Delta\Delta}$, not logarithmic
- Agreement with resummation prediction for $W_{\Delta\Delta}$ for logarithmic term at LP (Feng Yuan)
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$W_{\Delta\Delta}$ result

$$\hat{W}_{\Delta\Delta}^{\text{naive limit}} \stackrel{?}{=} -\frac{2\pi e_\mathrm{q}^2\alpha_s^2}{C_A z_1 z_2}\Bigg[\left(-1-2z_1^2-2z_2^2+4z_1z_2-4z_1^2z_2^2\right)\log\left(\frac{Q_\mathrm{T}^2}{\mu_\mathrm{F}^2}\right) + 12z_1^2z_2^2+z_1^2+z_2^2-8z_1z_2+z_1+z_2\Bigg] + \frac{2\pi e_\mathrm{q}^2\alpha_s^2}{C_F} + \mathcal{O}(\rho^2)$$

Inlcuding phase space effects

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Summary and Outlook

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- We developed an algorithmic way to perform small-Q_T expansion of DY-Helicity structure functions
- We calculated the gluon fusion contribution to DY to NLP in Q_T
- Higher powers in the Q_T expansion, improve analytic calculation of Mellin moments
- complete DY calculation; expansion of $\left[\frac{1}{s_2}\right]_+$ in $q\bar{q}$-channel
- development of resummation formalism for W_L, W_Δ, and $W_{\Delta\Delta}$ (extension of [Berger et al., 2007])
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References I

D. Boer and W. Vogelsang. *Drell-Yan lepton angular distribution at small transverse momentum*. Physical Review D 74.1 (2006): 014004.

E. Mirkes, *Angular decay distribution of leptons from W-bosons at NLO in hadronic collisions*. Nuclear Physics B 387.1 (1992): 3-85.

V. E. Lyubovitskij, F. Wunder, and A. S. Zhevlakov. *New ideas for handling of loop and angular integrals in D-dimensions in QCD*. Journal of High Energy Physics 2021.6 (2021): 1-128.

J.-C. Peng et al. *On the rotational invariance and non-invariance of lepton angular distributions in Drell–Yan and quarkonium production*. Physics Letters B 789 (2019): 356-359.

M. A. Ebert et al. *Drell-Yan qT resummation of fiducial power corrections at N3LL*. Journal of High Energy Physics 2021.4 (2021): 1-102.

References II

E. L. Berger, Edmond L., J. W. Qiu, and R. A. Rodriguez-Pedraza. *Transverse momentum dependence of the angular distribution of the Drell-Yan process*. Physical Review D 76.7 (2007): 074006.