Lattice calculation of Collins-Soper Kernel

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arXiv: 2302.xxxxx

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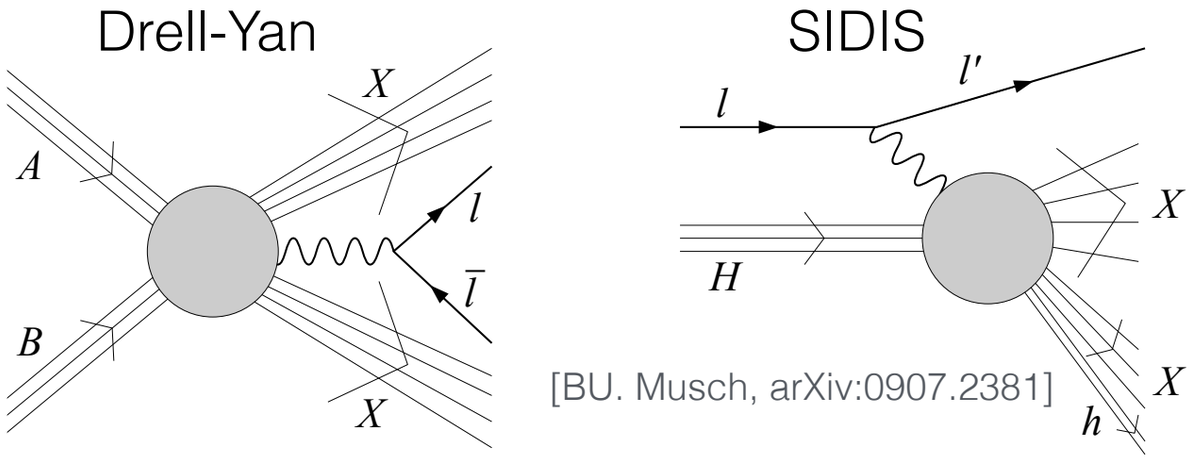
From TMDs to Collins-Soper kernel

- TMDs universal in Drell-Yan process and semi-inclusive deep inelastic scattering



[BU. Musch, arXiv:0907.2381]

- CS kernel encodes the rapidity scaling properties of transverse momentum dependent parton distribution functions (TMDPDFs) / wave functions (TMDWFs):

$$2\zeta\frac{\mathrm{d}}{\mathrm{d}\zeta}\ln f^{\mathrm{TMD}}(x,b_\perp,\mu,\zeta)=K(b_\perp,\mu)$$

- Extraction of CS kernel on the lattice:
  - From ratio of TMDWFs within LaMET [M. A. Ebert, L. W. Stewart and Y. Zhao, PRD 99, 034505]
  - From ratio of TMDPDFs using Mellin moments [A. A. Vladimirov and A. Schaefer, PRD 101, 074517]

- An application of CS kernel: from quasi TMDs to physical TMDs [X. Ji and Y. Liu, PRD105,076014]

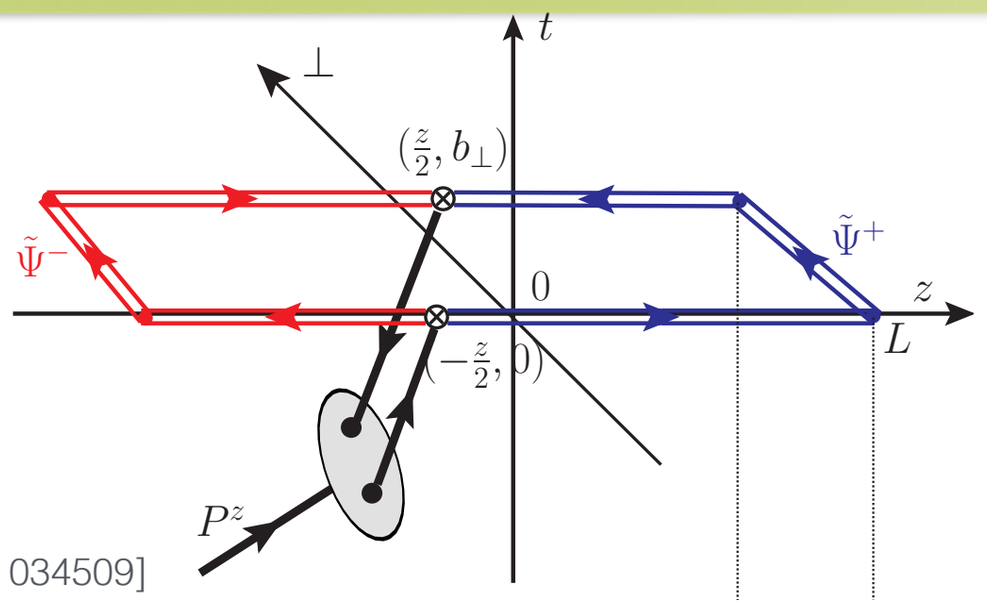
$$\tilde{\Psi}^+(x,b_\perp,\mu,\zeta_z)S_r^{1/2}(b_\perp,\mu)=H^+(\zeta_z,\bar{\zeta_z},\mu^2)\exp\left[\frac{1}{2}K(b_\perp,\mu)\ln\frac{-\zeta_z-i\epsilon}{\zeta}\right]\Psi^+(x,b_\perp,\mu,\zeta)+\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{\zeta_z},\cdots\right)$$

Determine CS kernel within LaMET

- Factorization based on LaMET using TMDWFs:

$$K(b_\perp,\mu,x,P_1^z,P_2^z)=\frac{1}{\ln(P_1^z/P_2^z)}\ln\frac{H^-(xP_2^z,\mu)\Psi(x,b_\perp,\mu,P_1^z)}{H^-(xP_1^z,\mu)\Psi(x,b_\perp,\mu,P_2^z)}$$

  - Leading order matching: [Q.A. Zhang et al., PRL125, 192001]
    [Yuan Li et al, PRL.128(2022)6,062002]
  - Next-to-leading order matching: [M.-H. Chu et al, PRD 106 (2022) 3, 034509]



Goal of this study ➡ A NLO CS kernel for CLS ensemble

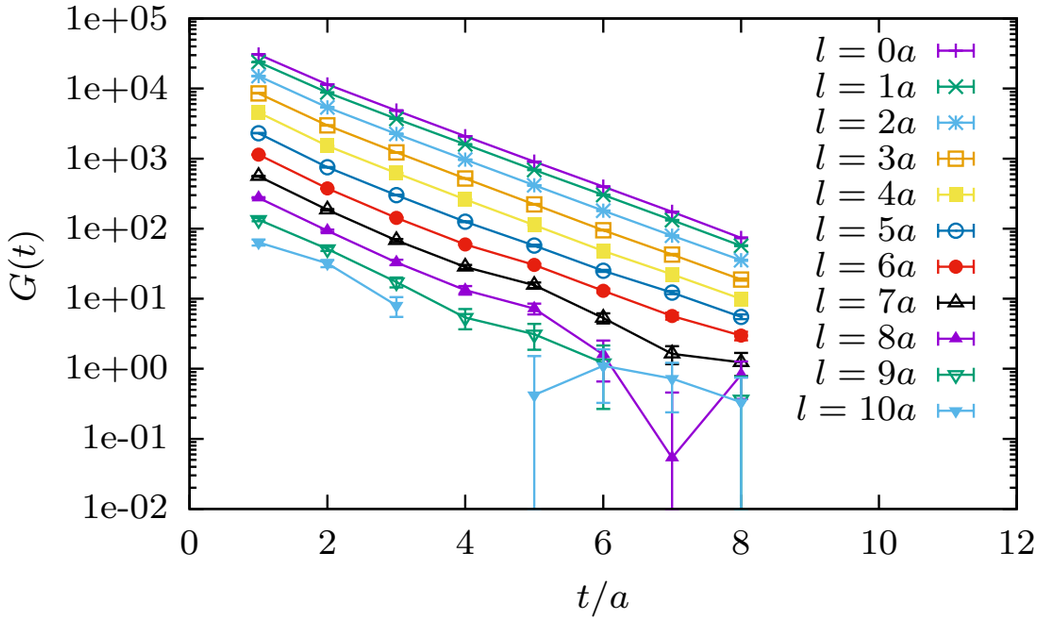
- Quasi TMD wave functions:

$$\phi_l(z,b_\perp,P^z,l)=\left\langle 0\middle|\bar{\psi}(\vec{z}/2+\vec{b})\Gamma\mathcal{W}(\vec{b},l)\psi(-\vec{z}/2)\middle|\pi(\vec{P})\right\rangle$$

- Quasi TMD wave functions from 2pt functions:

$$C^{2pt}_{\Gamma_\phi}(b_\perp,l,P^z,t)=\frac{Z_\phi}{L^3}\sum_{\vec{x}}e^{-iP^z x_z}\langle O_\phi(t,b_\perp,l)\,O_\pi^\dagger(0,P^z)\rangle$$
$$=\frac{A_w(p_z)A_p}{2E}e^{-Et}\phi_\ell(0,b_\perp,P^z,\ell)(1+c_0e^{-\Delta Et})$$

$P^z=6\frac{2\pi}{L},\ b_\perp=7a,\ z=0a$



Joint fit of all l&b (sharing energies) for each P^z

Lattice setup

- We use newly generated 2+1 flavor clover fermion CLS ensemble X650

β	$L^3 \times T$	a	m_π^{sea}	m_π^v	#conf
3.34	$48^3 \times 48$	0.098 fm	333 MeV	662 MeV	1000

- HYP smearing + Wall source
- 2 sources per configuration
- Transverse direction: x and y
- Momenta: P^z={0, 6,8,10,12} x 2pi/L

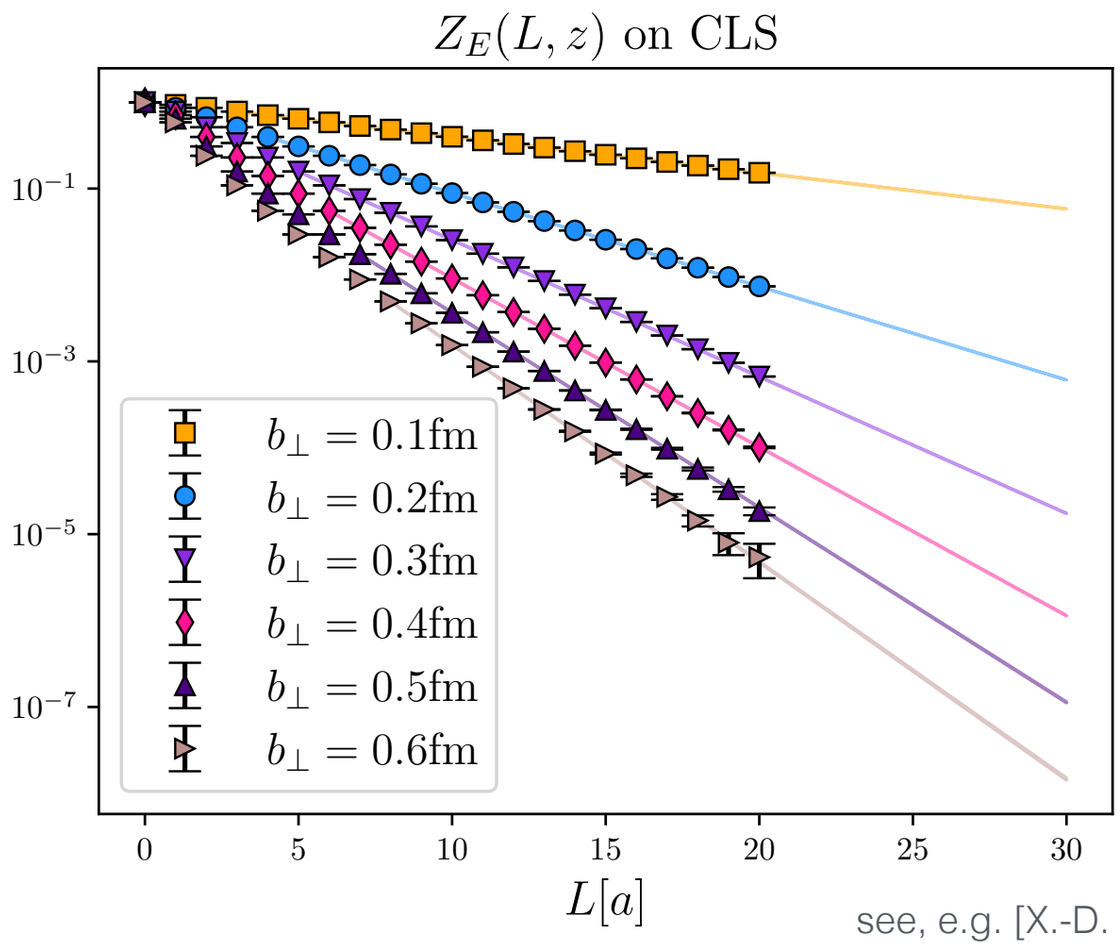
  (i.e. 0,1.6, 2.1, 2.6, 3.2 GeV)
- 0<=l/a<=10, 0<=b/a<=7, 0<=t/a<=9

Renormalization of the quasi TMDWFs (I)

[LPC, PRL 129 (2022) 082002]

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty} \int \frac{P^z dz}{2\pi} e^{ixzP^z} \frac{\tilde{\Psi}^{\pm}(z, b_\perp, \mu, P^z)}{\sqrt{Z_E(2L \pm z, b_\perp, \mu) Z_O(1/a, \mu, \Gamma)}}$$

- The linear divergence and heavy quark potential is removed by Wilson loop Z_E
- Residual logarithmic divergence is removed by Z_O



see, e.g. [X.-D. Ji, et al, PRL 120 (2018) 112001]

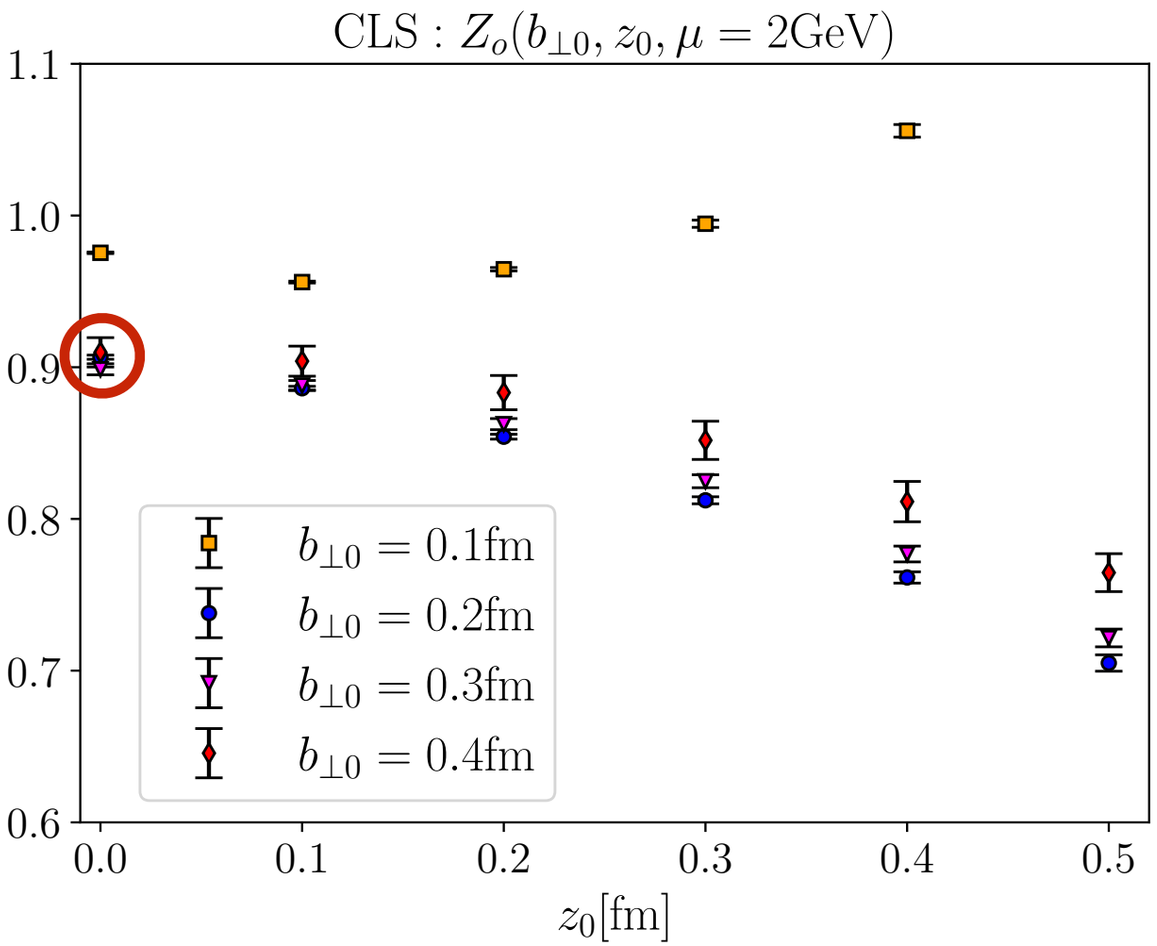
- Large L limit not available on the lattice
- Extrapolation to large L with 1-state fit

Renormalization of the quasi TMDWFs (II)

[LPC, PRL 129 (2022) 082002]

$$\tilde{\Psi}^{\pm}\left(x, b_{\perp}, \mu, \zeta^{z}\right)=\lim_{L \to \infty} \int \frac{P^{z} dz}{2\pi} e^{ixzP^{z}} \frac{\tilde{\Psi}^{\pm}(z, b_{\perp}, \mu, P^{z})}{\sqrt{Z_{E}\left(2L \pm z, b_{\perp}, \mu\right)} Z_{O}(1/a, \mu, \Gamma)}$$

- The linear divergence and heavy quark potential is removed by Wilson loop Z_E
- Residual logarithmic divergence is removed by Z_O [LPC, PRL 129 (2022) 082002]

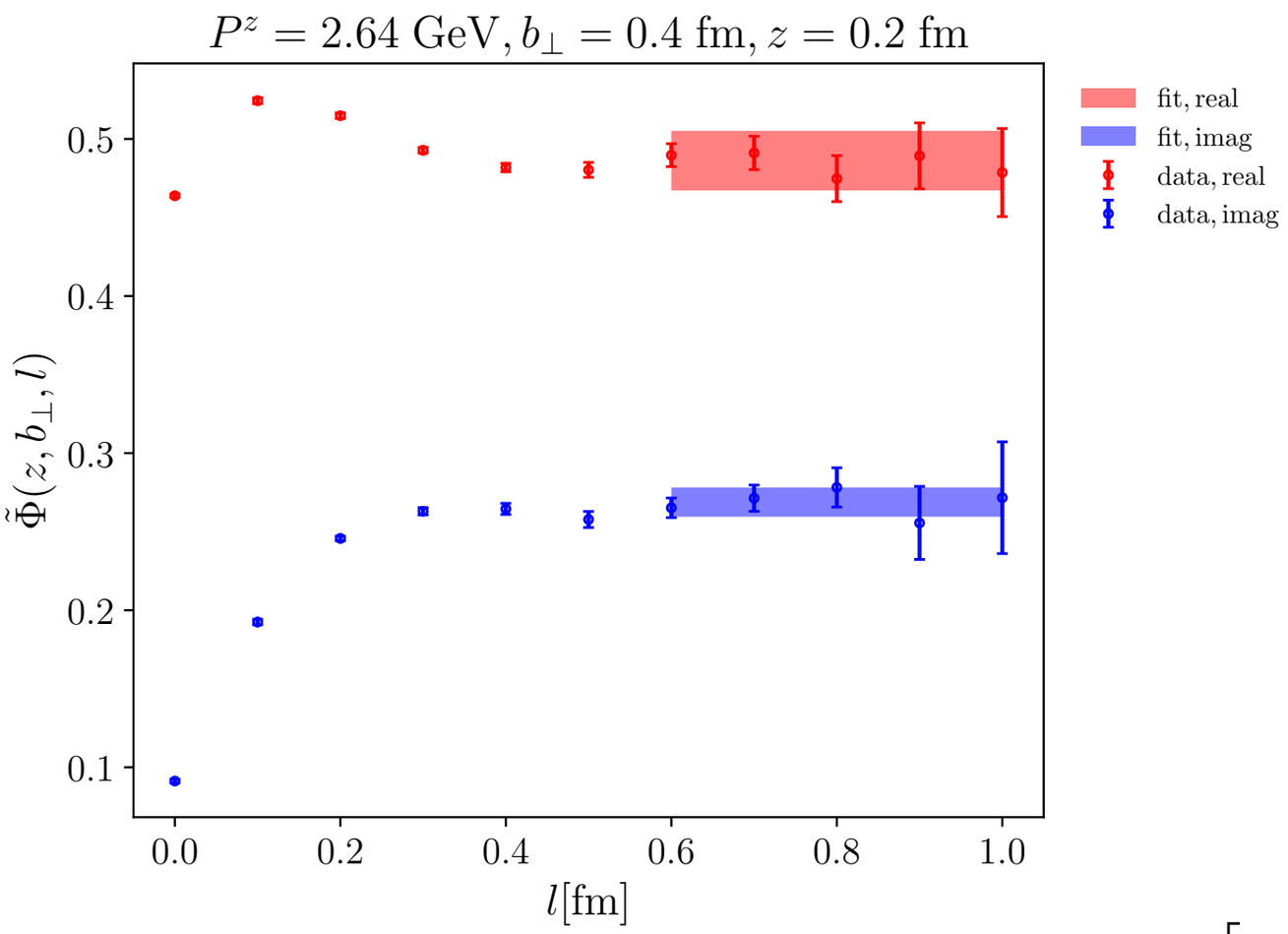


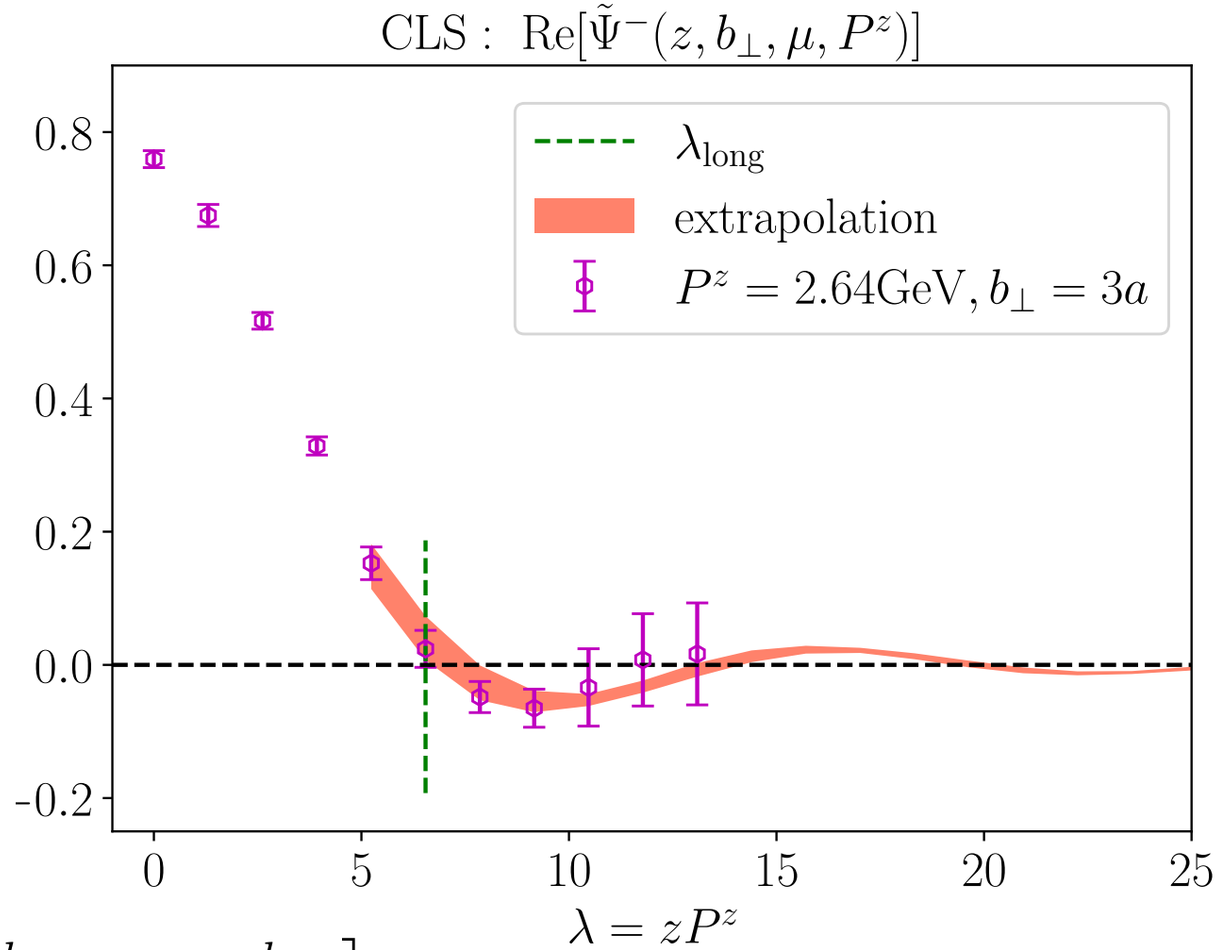
$$Z_{O}(1/a, \mu)=\frac{\tilde{\Psi}^{\pm,0}\left(z_{0}, b_{\perp 0}, \zeta^{z}=0, L\right)}{\sqrt{Z_{E}\left(2L \pm z_{0}, b_{\perp 0}, \mu\right)} \tilde{\psi}^{\overline{\mathrm{MS}}}\left(b_{\perp 0}, z_{0}, \mu\right)}$$

$$\tilde{\psi}^{\overline{\mathrm{MS}}}(b_{\perp 0}, z_{0}, \mu)=1+\frac{\alpha_{s} C_{F}}{2\pi}\left\{\frac{1}{2}+3\gamma_{E}-3\ln 2 + \frac{3}{2}\ln[\mu^{2}(b_{0}^{2}+z_{0}^{2})]-2\frac{z_{0}}{b_{0}}\arctan\frac{z_{0}}{b_{0}}\right\}+\mathcal{O}(\alpha_{s}^{2})$$

- Look for plateau in *b* where both discretization effects and higher twist contaminations are suppressed

From position space to momentum space



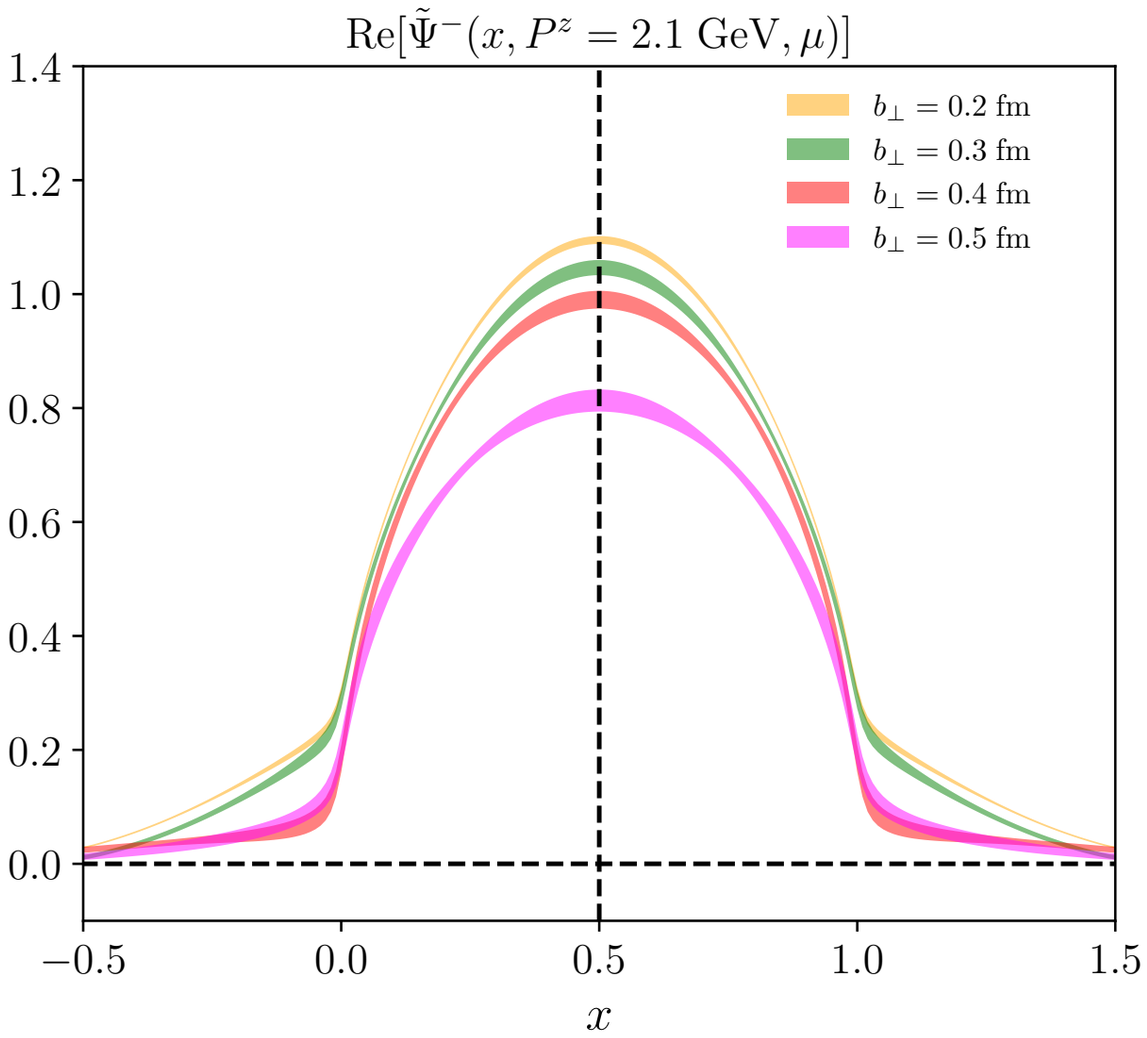


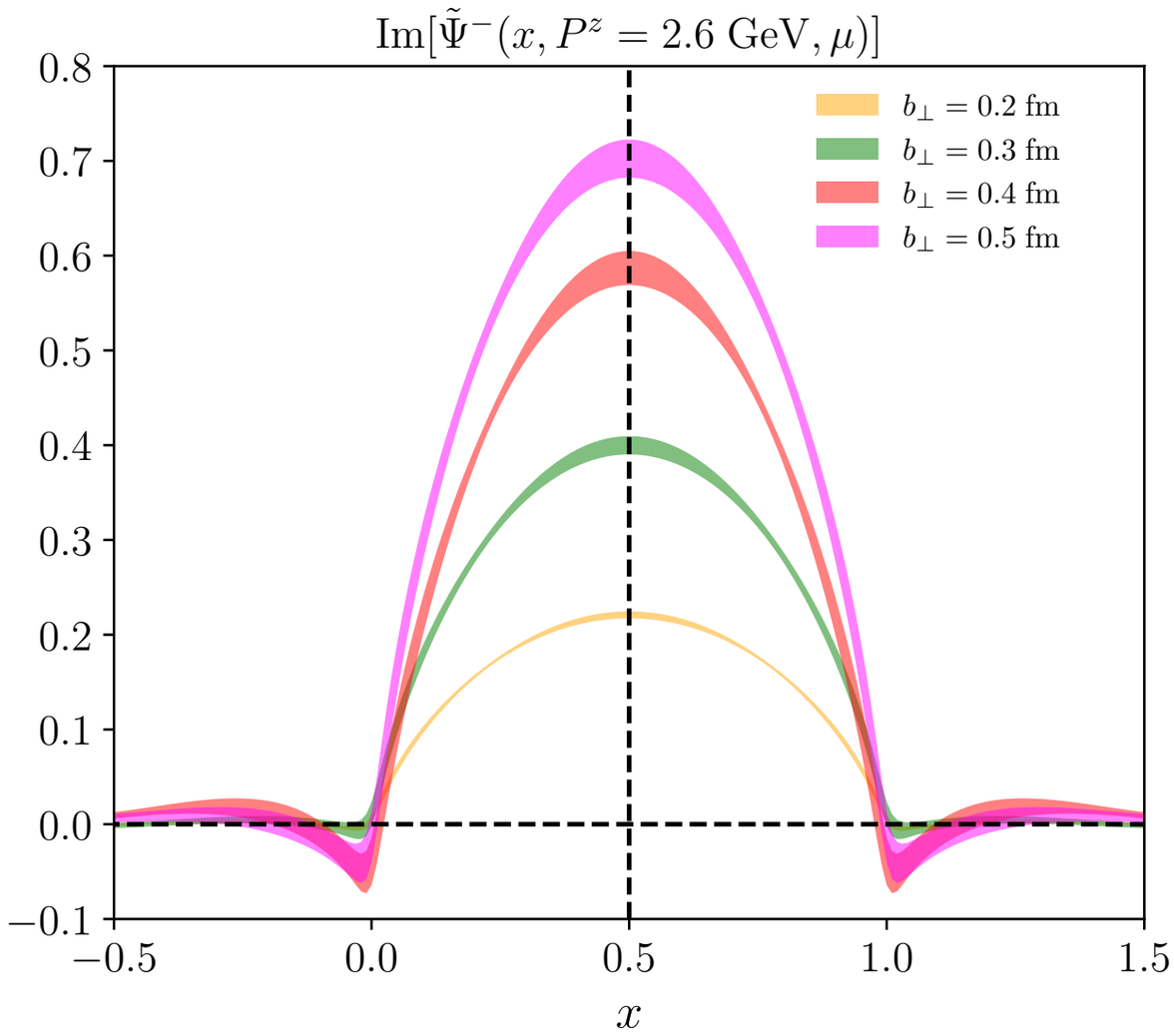
$$\tilde{\Psi}(z, b_\perp, \mu, \zeta^z) = f(b_\perp)\left[\frac{k_1}{(-i\lambda)^d} + e^{i\lambda}\frac{k_2}{(i\lambda)^d}\right]e^{-\frac{\lambda}{\lambda_0}}$$

- Large L limit via constant fit in plateau range (L>~0.7 fm)
- Extrapolation for large lambda range using theoretically-inspired Ansatz [X.-D. Ji, et al., NPB 964 (2021) 115311]
- Fourier transformation going to momentum space

$$\tilde{\Psi}^{\pm}(x, b_\perp, \mu, \zeta^z) = \lim_{L\to\infty}\int \frac{P^z dz}{2\pi} e^{ixzP^z}\cdots$$

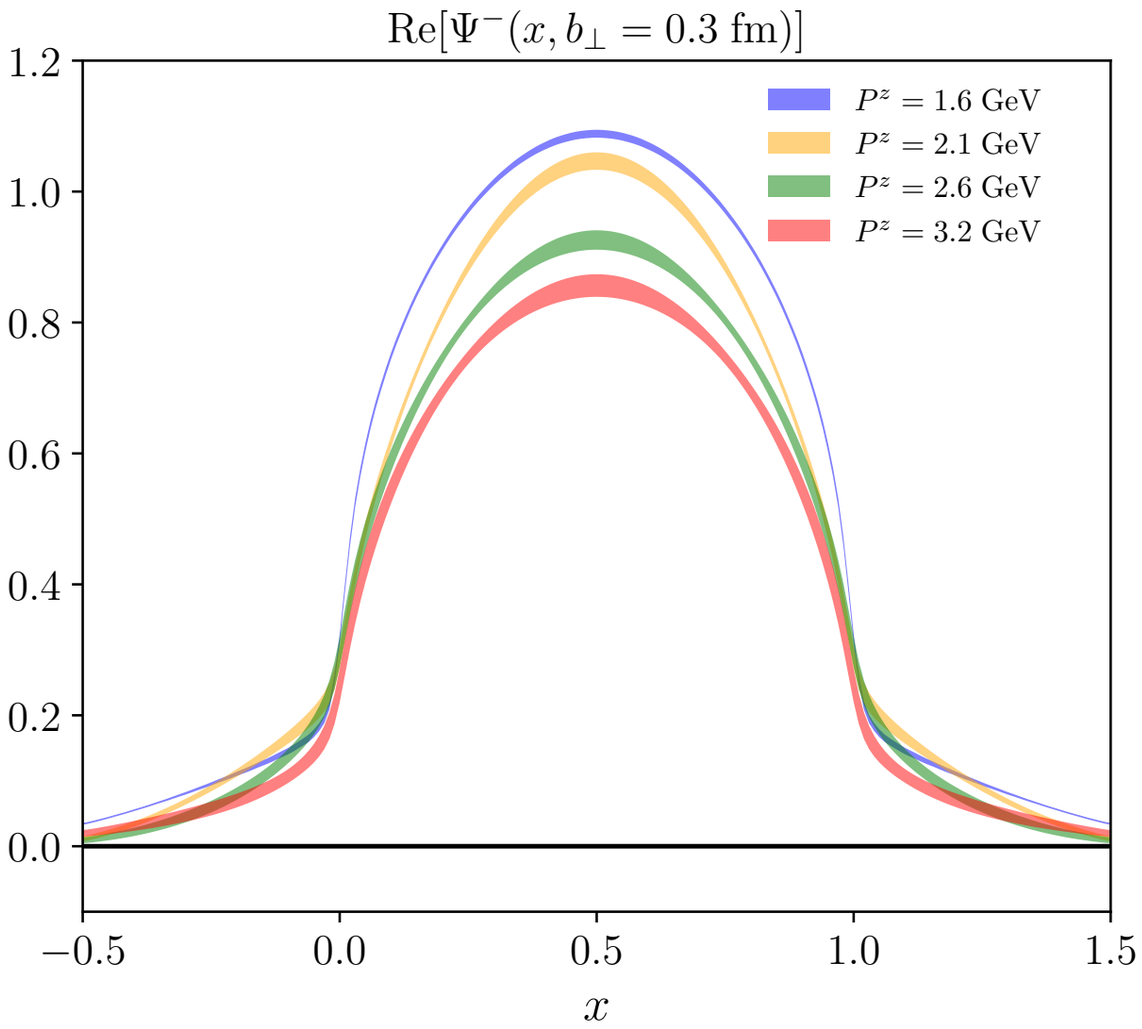
Quasi TMDWFs in momentum space (I)

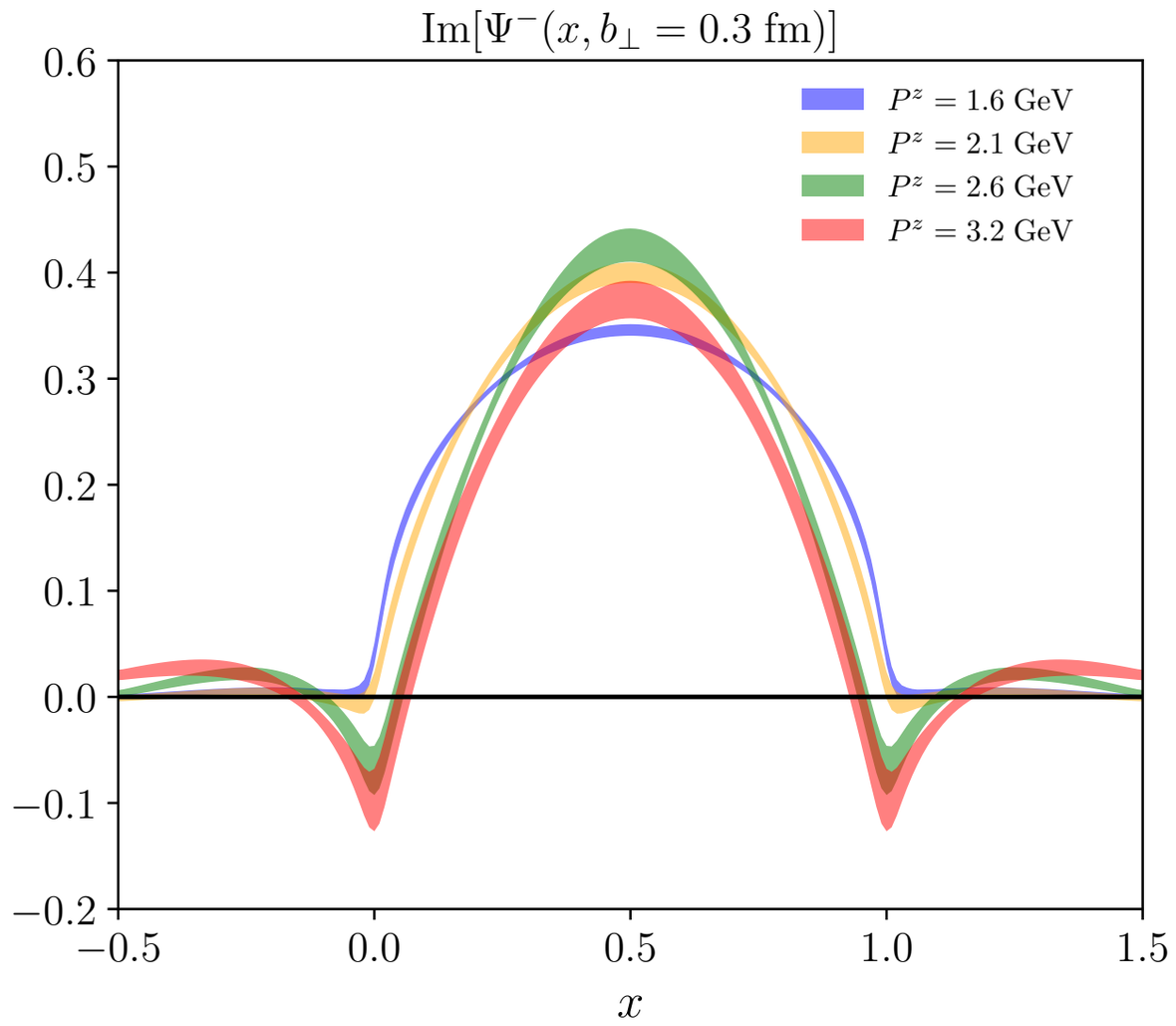




- Real part has larger amplitude at same *b*
- Clear dependence on *b* for both real and imaginary parts
- Stronger dependence on b for imaginary part

Quasi TMDWFs in momentum space (II)





- Real part has larger amplitude at same momentum
- Clear dependence on momentum for both real and imaginary parts
- Stronger dependence on momentum for real part

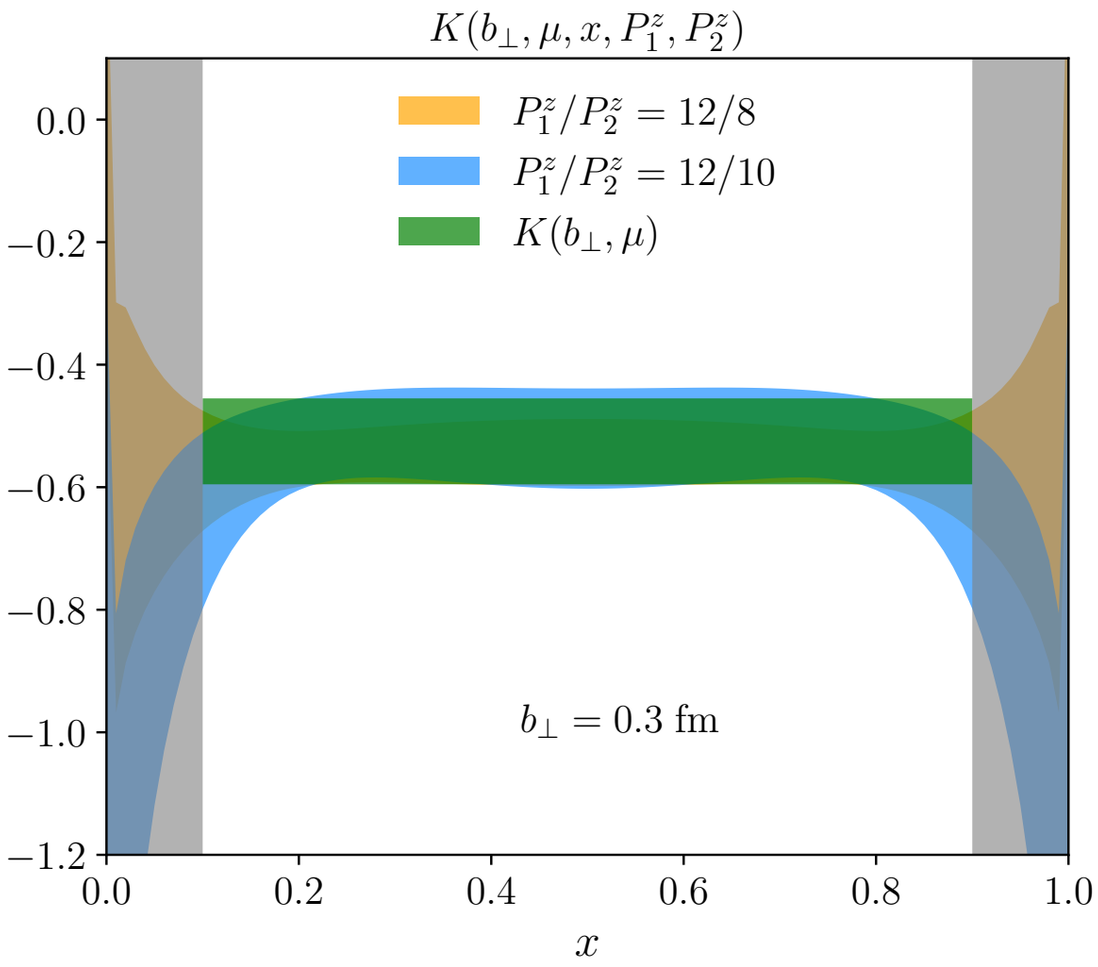
Collins-Soper kernel from LaMET

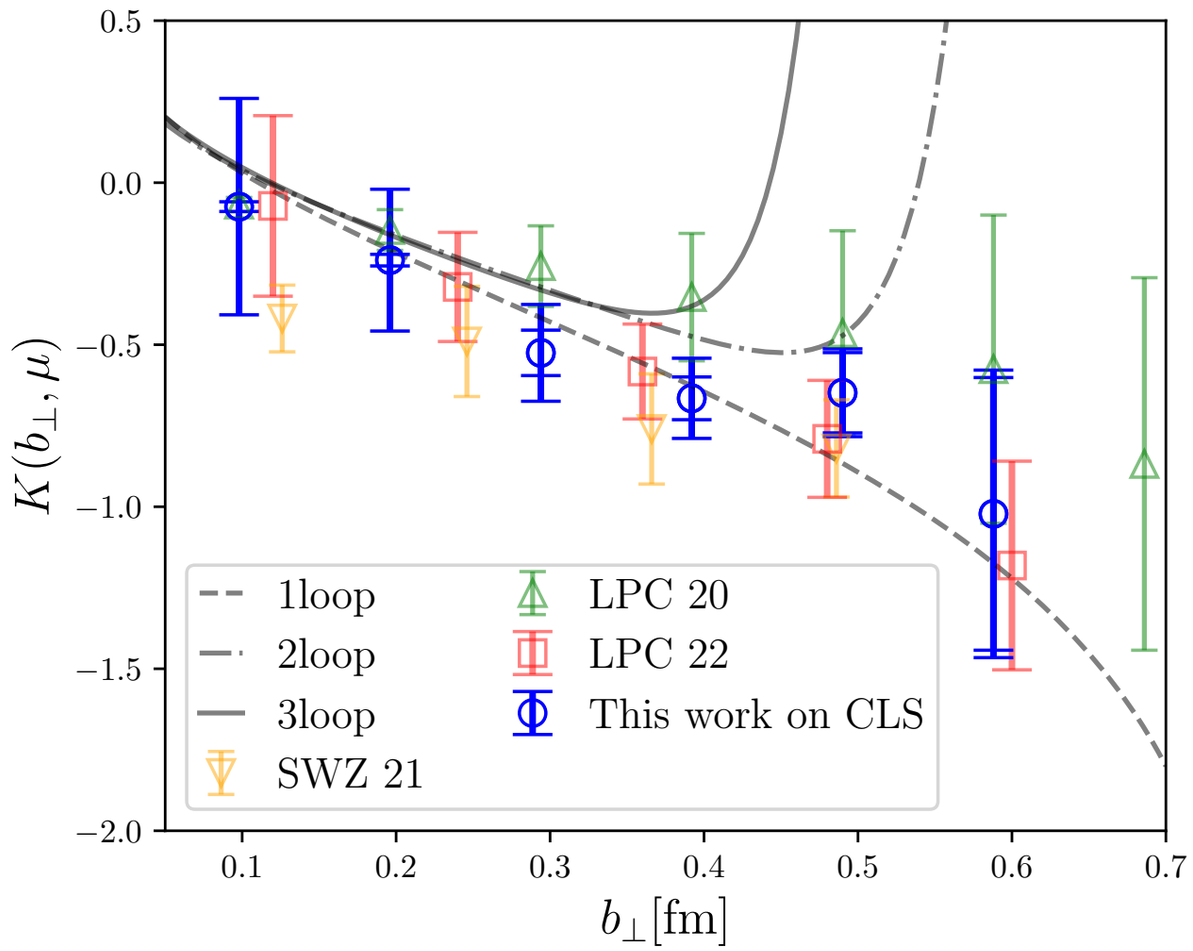
- 1-loop matching kernel *H* from: [X.-D. Ji and Y.-Z. Liu, PRD105, 076014]

$$K(b_\perp,\mu,x,P_1^z,P_2^z)=\frac{1}{\ln(P_1^z/P_2^z)}\ln\frac{H^-(xP_2^z,\mu)\Psi(x,b_\perp,\mu,P_1^z)}{H^-(xP_1^z,\mu)\Psi(x,b_\perp,\mu,P_2^z)}$$

- Extract leading power contribution via joint fit: [M.-H. Chu et al, PRD 106 (2022) 3, 034509]

$$K(b_\perp,\mu,x,P_1^z,P_2^z)=K(b_\perp,\mu)+A\left[\frac{1}{x^2(1-x)^2(P_1^z)^2}-\frac{1}{x^2(1-x)^2(P_2^z)^2}\right]$$





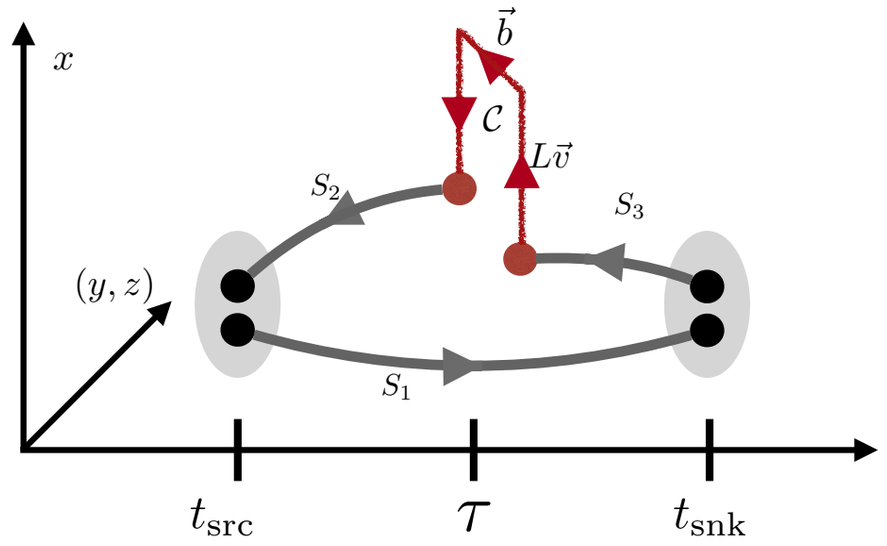
- Good agreement with previous lattice calculations

CS kernel from Mellin moments

- CS kernel from (first) Mellin moments of ratio of TMDPDFs:

A. Vladimirov and A. Schaefer, PRD 101, 074517]

$$R^{[\Gamma]}(x,b,\mu;P_1,P_2) = \frac{W^{[\Gamma]}_{f/h}(x,b;P_1,S;\mu)}{W^{[\Gamma]}_{f/h}(x,b;P_2,S;\mu)} = \left(\frac{P_1^+}{P_2^+}\right)^{K(b,\mu)} \frac{\mathbb{C}_H(xP_1^+,\mu)}{\mathbb{C}_H(xP_2^+,\mu)} + \mathcal{O}(\lambda^2)$$



3pt of pion on the lattice

- Construction of TMDPDF matrix element:

$$W^{[\Gamma]}_{f/h}(b;\ell,L;v,P,S;\mu) = \langle h(P,S)|\bar{q}_f(b+\ell v)\Gamma\,\mathcal{U}[\mathcal{C}(\ell,v,b,L)]\;q_f(0)|h(P,S)\rangle$$

- Large separation limit achieved by constant fit: $W^{[\Gamma]} = 2E_P \lim_{0\ll\tau\ll t} \frac{C_{3pt}(P,\mathcal{C},t,\tau,\Gamma)}{C_{2pt}(P,t)}$

- Parametrization via (Lorentz) invariant amplitudes: [B. U. Musch, et al, PRD 85, 094510]

$$W^{[\gamma^\mu]}(b;L;v,P,S) = P^\mu \tilde{A}_2 + \frac{M^2}{v\cdot P} v^\mu \tilde{B}_1 - iM\epsilon^{\mu\nu\alpha\beta}P_\nu b_\alpha S_\beta \tilde{A}_{12} - iM^3\epsilon^{\mu\nu\alpha\beta}b_\nu(Lv_\alpha)S_\beta\tilde{B}_8 - iM^2 b^\mu \tilde{A}_3 + \cdots$$

- Combine invariant amplitudes to build twist-2 and twist-3 TMDPDFs:

$$f_1(b^2,P^+) = P^+\big(\tilde{A}_2(b^2) + M^2\frac{v^+}{(v\cdot P)P^+}\tilde{B}_1(b^2)\big),$$
$$e(b^2,P^+) = P^+\tilde{A}_1(b^2)$$

f_1 and *e* for both pion and proton !

Lattice setup

- Calculations on CLS ensemble H101

name	β	$L^3 \times T$	a[fm]	$\kappa_l = \kappa_s$	m_π	m_K	$m_\pi L$	L[fm]	conf
H101	3.4	$32^3 \times 96$	0.0854	0.13675962	422	422	5.8	2.7	2016

Table 1: Parameters of the H101 CLS ensemble used in the present study.

- The hierarchy needs to be respected for the factorization theorem

$$\frac{P^-}{P^+} \ll 1,\ \frac{1}{|b|P^+} \ll 1,\ \frac{|b|}{L} \ll 1,\ \frac{\ell}{L} \ll 1,\ \ell\Lambda_{\rm QCD} \ll 1$$

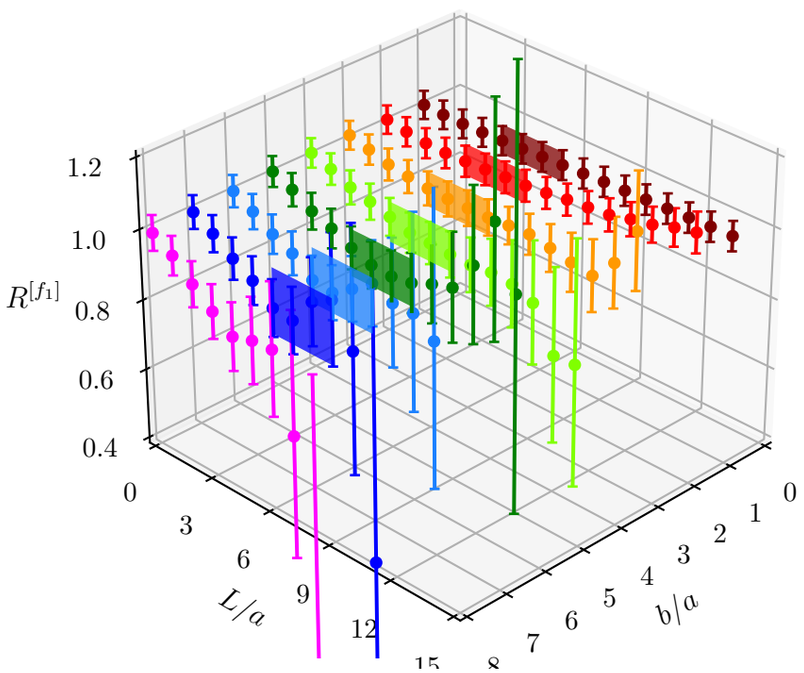
- HYP smearing + Momentum smearing
- Multiple source per configuration
- Momenta: P^z={0, 1, 2, 3, 4, 5} x 2pi/L
- l=0, 0<=L/a<=9~20, 0<=b/a<=8, 0<=t/a<=9/11

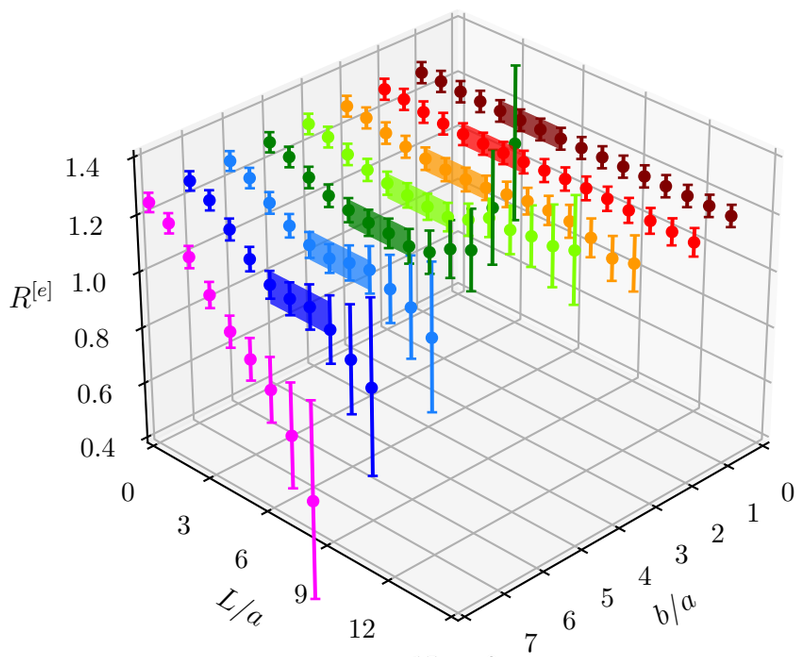
Constant fit for the ratio at P_1/P_2=3/2

f_1:

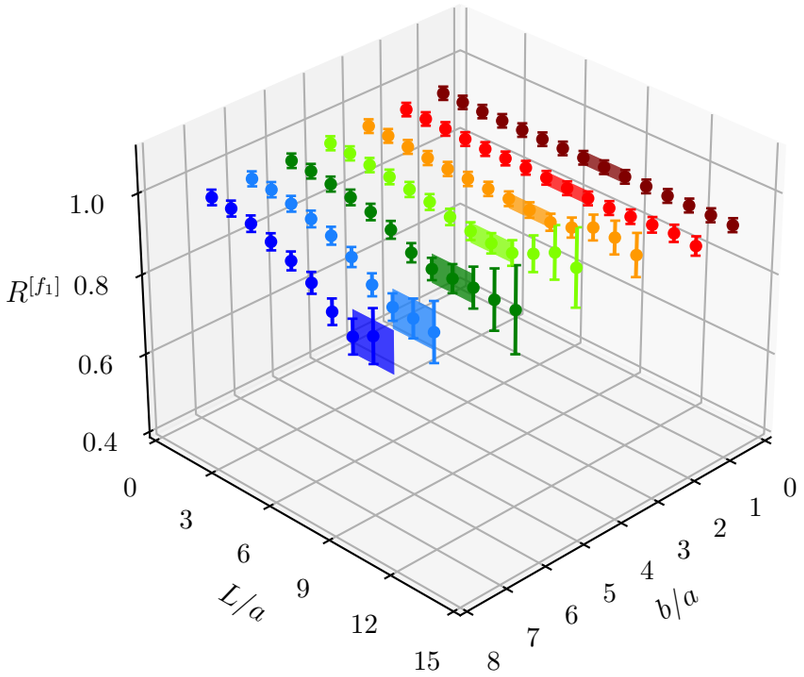
e:

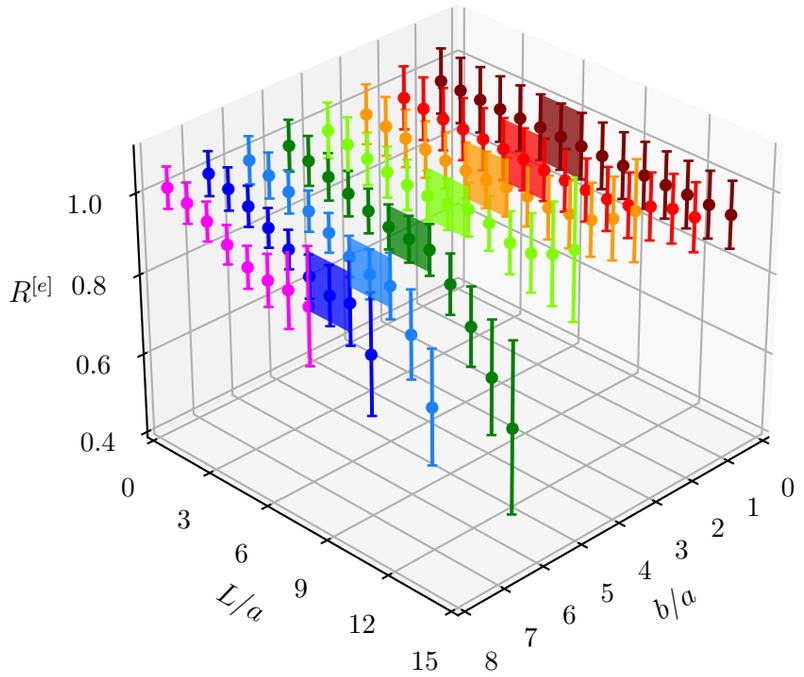
pion:



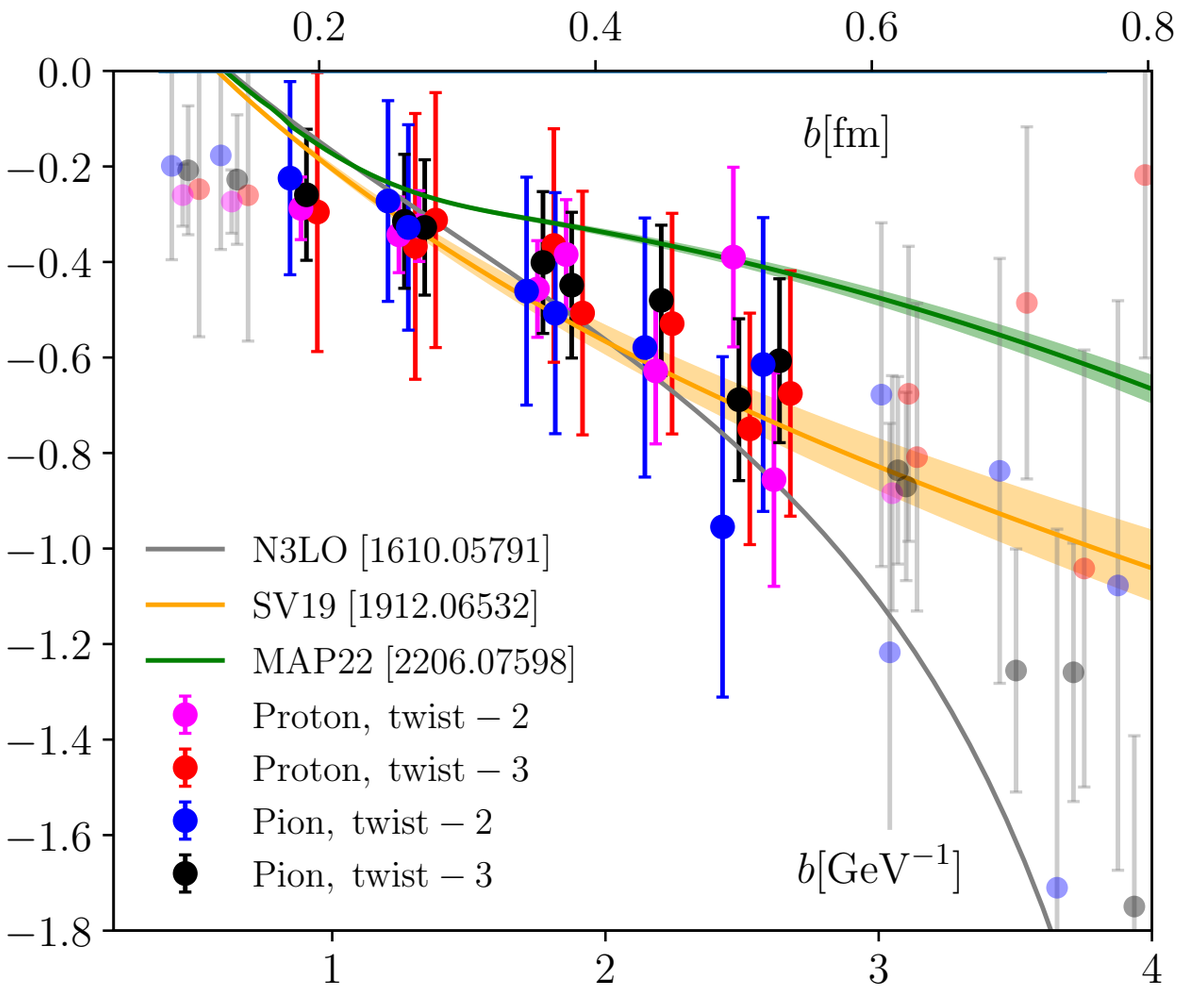


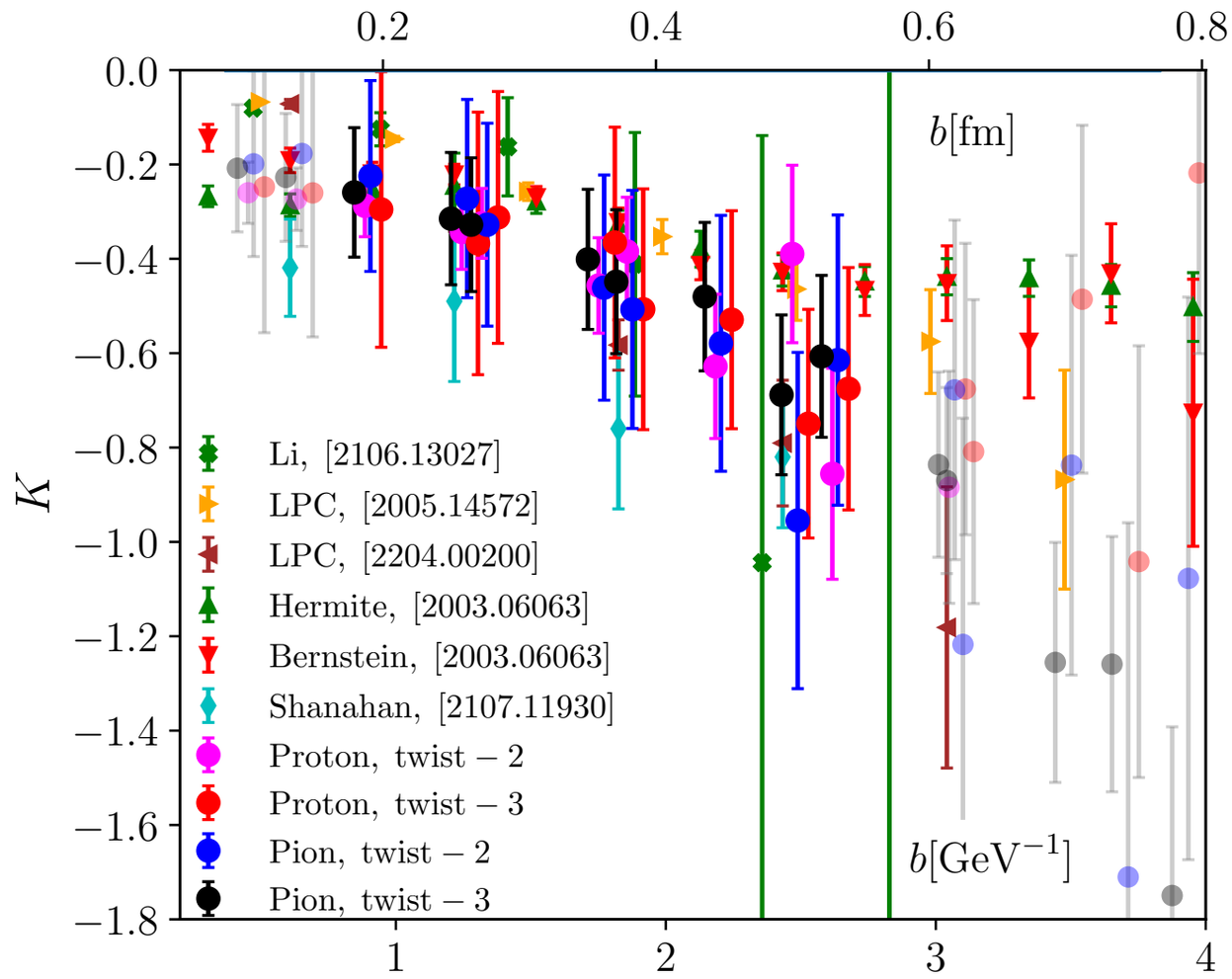
proton:





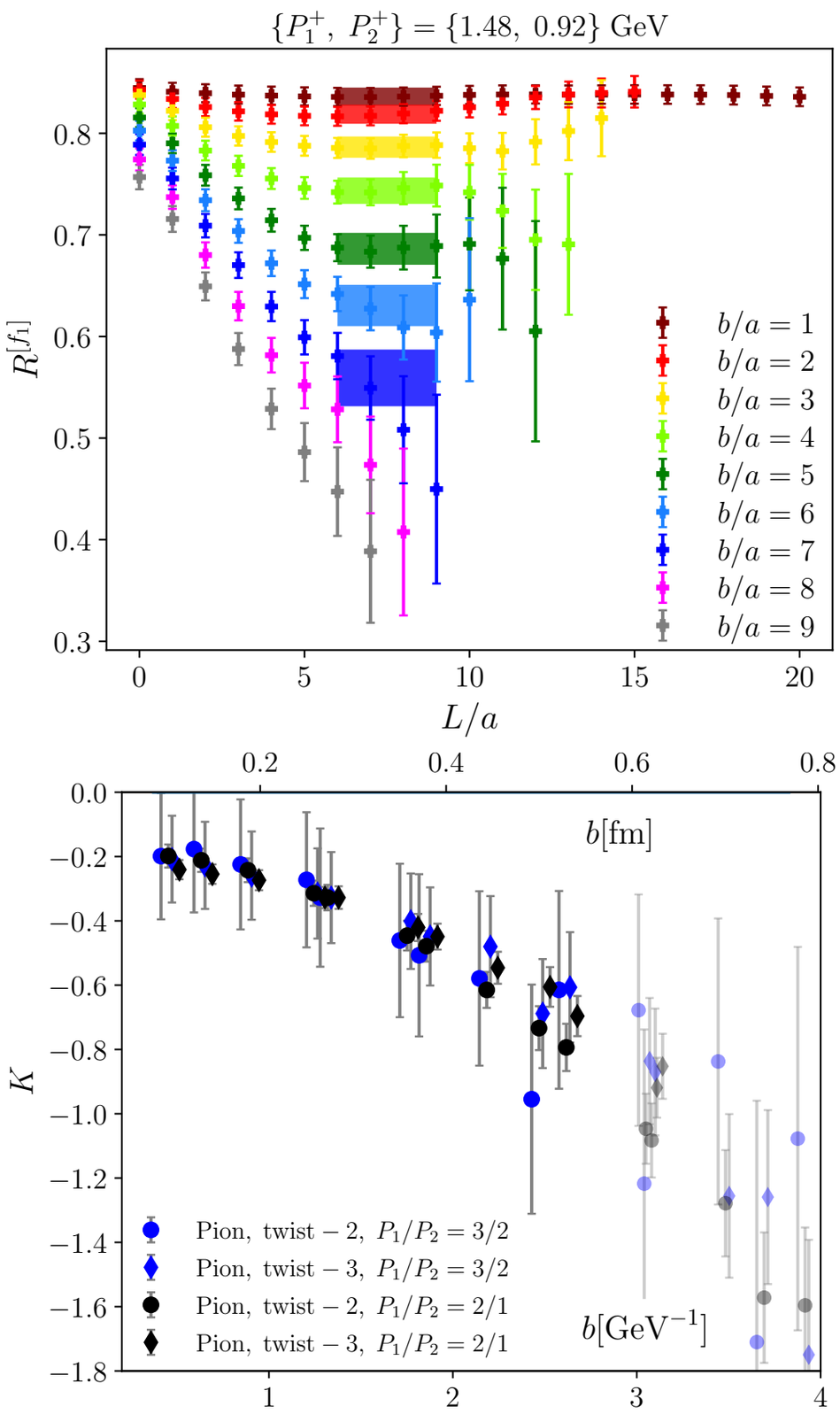
Collins-Soper kernel from P_1/P_2=3/2

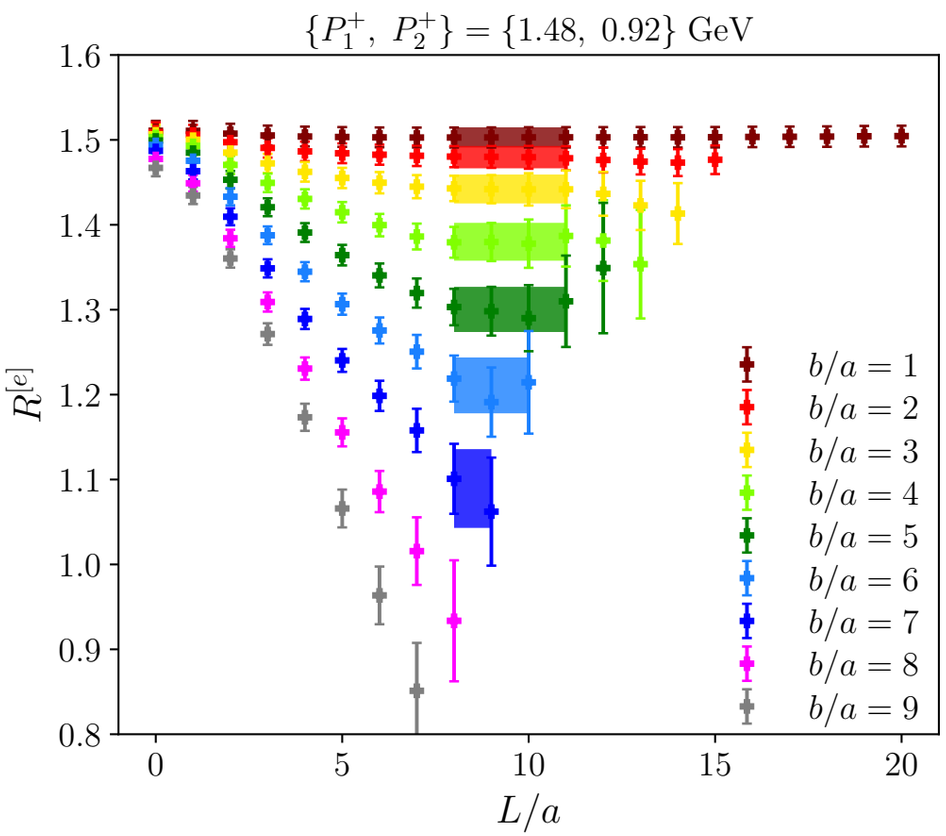




- Consistent results for pion and proton, twist-2 and twist-3
- This study agrees with SV19 phenomenal calculations and is lower than MAP22
- Lattice computations display qualitatively similar behavior
- The discrepancies can be understood as systematic effects

Collins-Soper kernel from P_1/P_2=2/1





- Smaller statistical errors but probably larger systematical errors
- Better identified plateaus
- Very close central values for CS kernel from different momentum pairs (power corrections suppressed)

Summary

- CS kernel calculated using lattice QCD in different ways on two different CLS ensembles
- Good agreement among different lattice extractions
- Universality of the lattice-determined CS kernel was observed
- High-precision determination of CS kernel is in reach