# FACTORIZATION FOR GTMDs

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FOR2926-2023 16-17/2/2023 Regensburg

# GTMD

These distributions where postulated some years ago:

S. Meissner, A. Metz, M. Schlegel, and K. Goeke, JHEP 08 (2008) 038, arXiv:0805.3165 [hep-ph].
S. Meissner, A. Metz, and M. Schlegel, JHEP 08 (2009) 056, arXiv:0906.5323 [hep-ph].
C. Lorcé and B. Pasquini, JHEP 09 (2013) 138, arXiv:1307.4497 [hep-ph].
K. Kanazawa, C. Lorcé, A. Metz, B. Pasquini, and M. Schlegel,Phys. Rev. D 90 no. 1, (2014) 014028, arXiv:1403.5226 [hep-ph].

More recent work:

M. G. Echevarria, A. Idilbi, K. Kanazawa, C. Lorcé, A. Metz, B. Pasquini, and M. Schlegel, Phys. Lett. B 759 (2016) 336–341, arXiv:1602.06953 [hep-ph]. *[1-loop calculations and evolution]*
V. Bertone, *Eur.Phys.J.C* 82 (2022) 10, 941, arXiv:2207.09526 [hep-ph]. *[GTMD matching on GPD at one loop]*
S. Bhattacharya, A. Metz, and J. Zhou, Phys. Lett. B 771 (2017) 396–400, arXiv:1702.04387 [hep-ph]. *[Tree-level factorization]*
I. Scimemi, A. Tarasov, and A. Vladimirov, JHEP05 (2019) 125, arXiv:1901.04519 [hep-ph] *[Indirect checks]*
M. G. Echevarria, P. A. Gutierrez Garcia, I. Scimemi arXiv:2208.00021 [hep-ph] *[Factorization for a specific process]*

# GTMD

These distributions where postulated some years ago

$$w^{[\Gamma]}_{N'N}(x_P, z_\perp; \xi, \vec{\Delta}_\perp) = \int \frac{dz^-}{4\pi} e^{i\frac{1}{2}x_P P^+ z^-} \langle N' | \mathrm{T}\{\bar{\chi}'_n(0^+, -\frac{z^-}{2}, -\frac{z_\perp}{2}) \Gamma \chi_n(0^+, \frac{z^-}{2}, \frac{z_\perp}{2})\} | N \rangle$$

Same operator as for TMD

Initial and final states are different:
Momentum, spin, flavor can change,
**But** they are color singlets

Momentum Transfer $\Delta \equiv p_{N'} - p_N$

Momentum Average $P \equiv \frac{p_{N'} + p_N}{2}$

$$\Delta^\mu = (-2\xi P^+, 2\xi P^-, \vec{\Delta}_\perp), \quad \xi \equiv (p_N - p_{N'})^+ / (p_N + p_{N'})^+$$

# GTMD asymptotic limits

The asymptotic limits are obtained in the following cases

TMD limit $\lim_{\Delta^\mu \to 0} w_{NN}^{[\Gamma]}(x_P, z_\perp; \xi, \overrightarrow{\Delta}_\perp) = \int \frac{dz^-}{4\pi} e^{i\frac{1}{2}x_P P^+ z^-} \langle N | \mathrm{T}\{\bar{\chi}_n(0^+, -\frac{z^-}{2}, -\frac{\vec{z}_\perp}{2}) \Gamma \chi_n(0^+, \frac{z^-}{2}, \frac{\vec{z}_\perp}{2})\} | N \rangle = f_{q \leftarrow N}^{[\Gamma]}(x_P, z_\perp)$

GPD limit $\lim_{z_\perp \to 0} w_{N'N}^{[\Gamma]}(x_P, z_\perp; \xi, \overrightarrow{\Delta}_\perp) = C_{N' \leftarrow i}(x_P, z_\perp; \xi, \overrightarrow{\Delta}_\perp) \otimes G_{i \leftarrow N}^{[\Gamma]}(x_P; \xi, \overrightarrow{\Delta}_\perp)$

# GTMD EVOLUTION

The explicit calculation of GTMD shows both rapidity and UV divergences
(Because we have the same operator as in the TMD case).
The proper GTMD can be defined with the same soft factor of DY/SIDIS

$$W_{N'N}^{[\Gamma]}(x_P, \vec{z}_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu^2, \zeta) = w_{N'N}^{[\Gamma]}(x_P, \vec{z}_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu^2, \delta)\sqrt{S\left(\vec{z}_\perp; \mu^2, \delta^2\zeta/Q^2\right)}$$

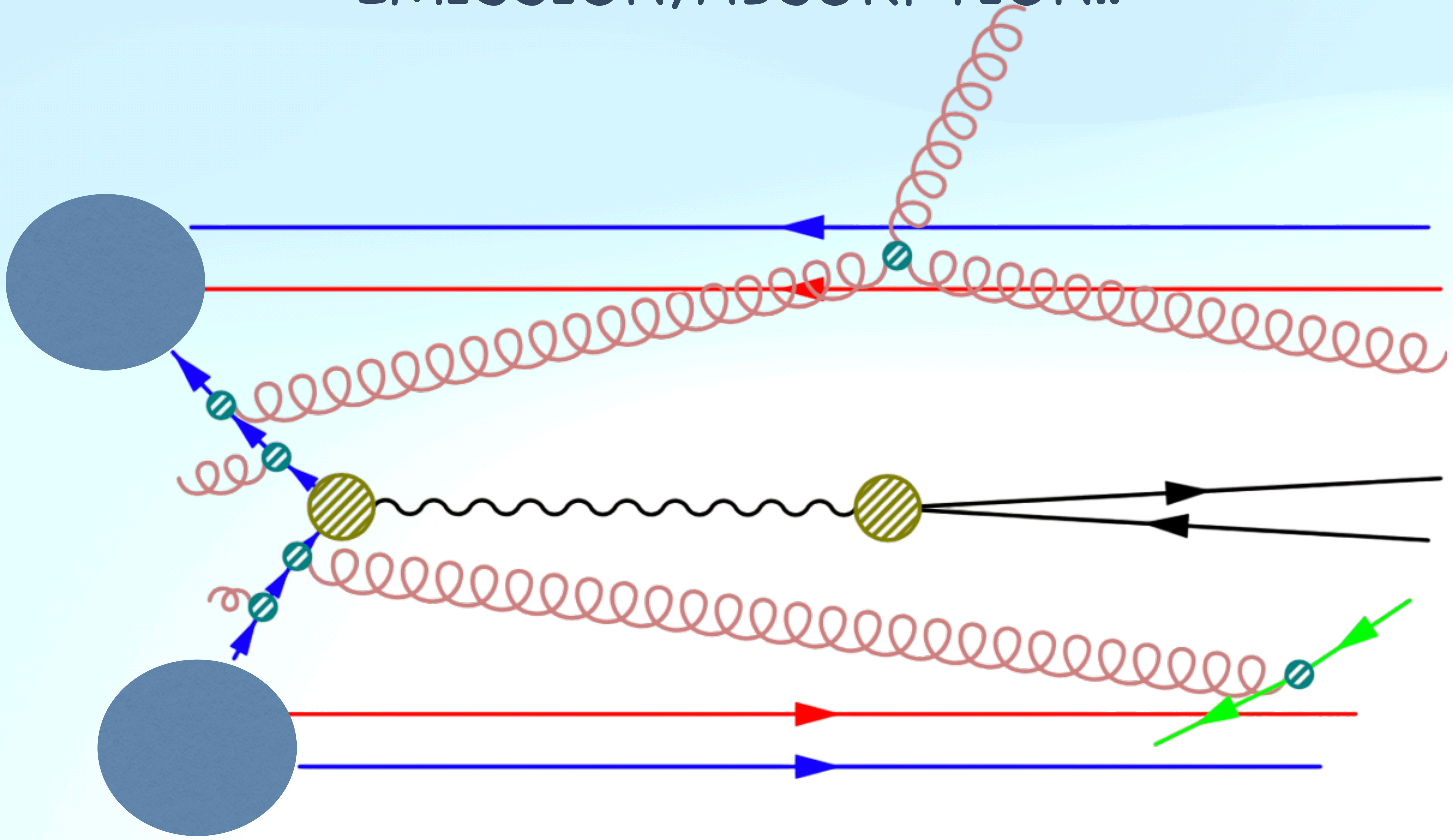
$$\frac{d}{d\ln\mu} \ln W_{N'N}^{[\Gamma]}(x_P, z_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu, \zeta) = \gamma_W(a_s, \mu, \zeta)$$

$$\frac{d}{d\ln\zeta} \ln W_{N'N}^{[\Gamma]}(x_P, \vec{z}_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu, \zeta) = -D(\vec{z}_\perp; \mu)$$

The same structure as for TMD

# From SPE to TPE

The TMD factorization theorem has been constructed for processes with A SINGLE PHOTON EMISSION/ABSORPTION..



..But interesting distributions can arise with TWO PHOTON EMISSIONS

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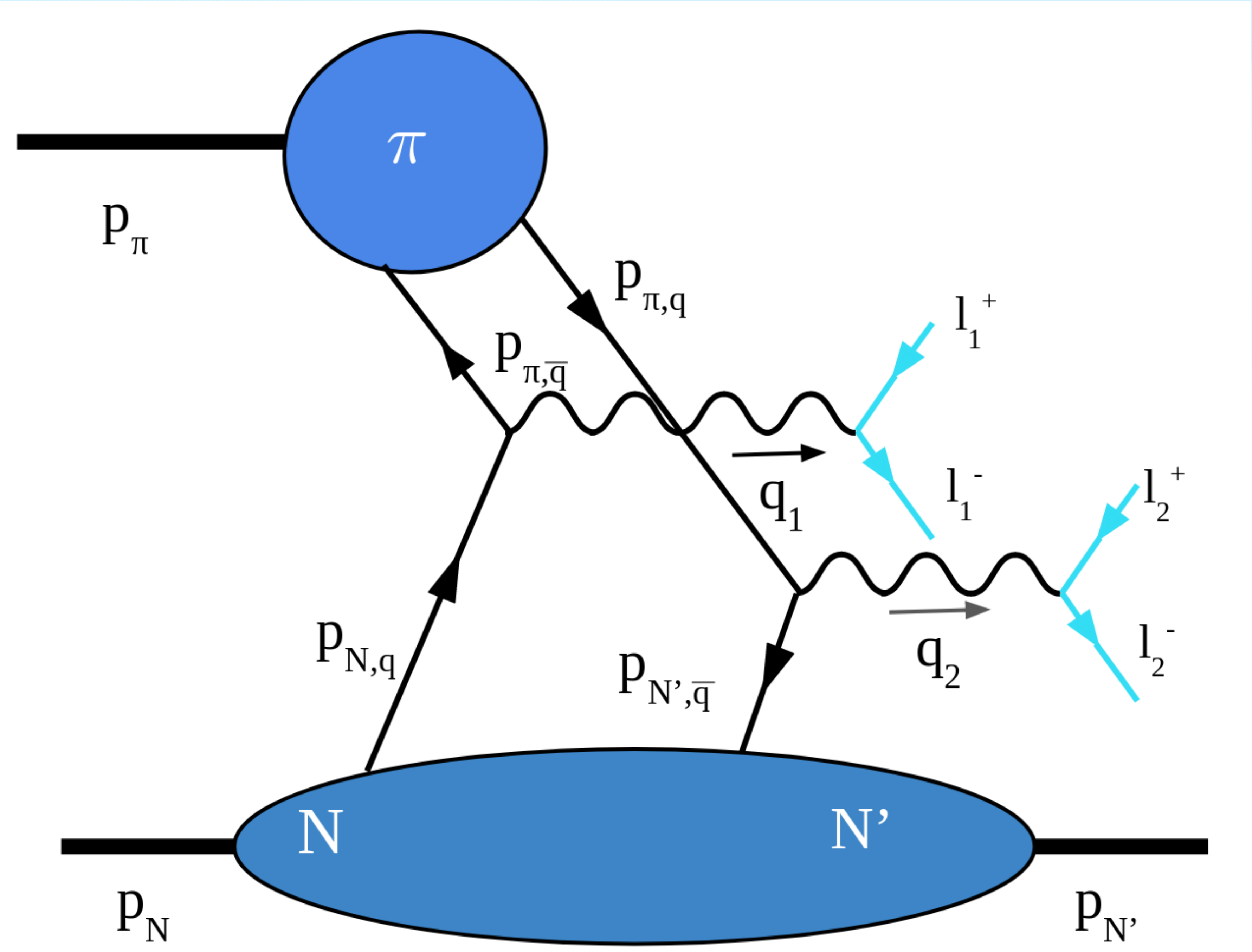
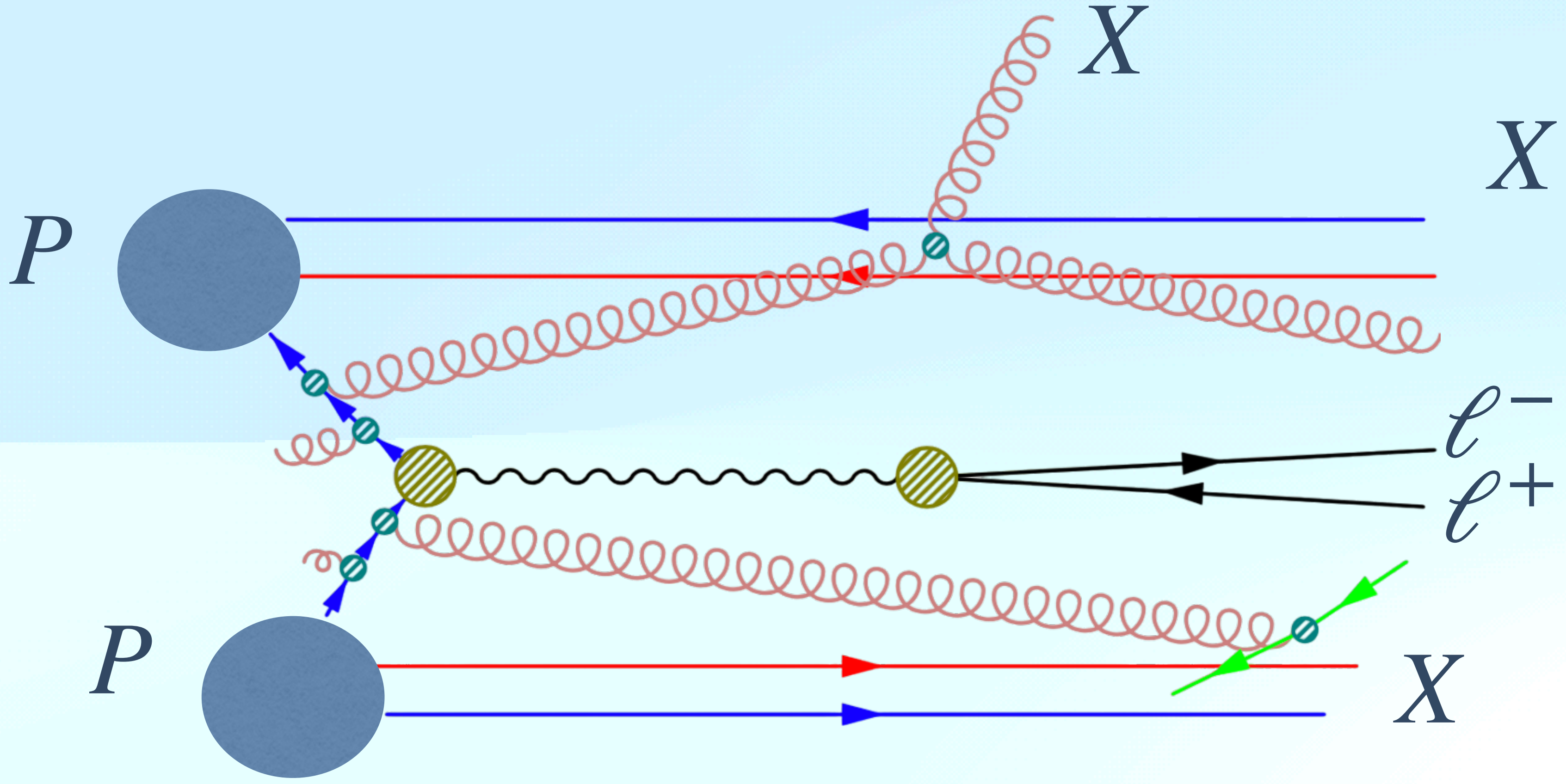
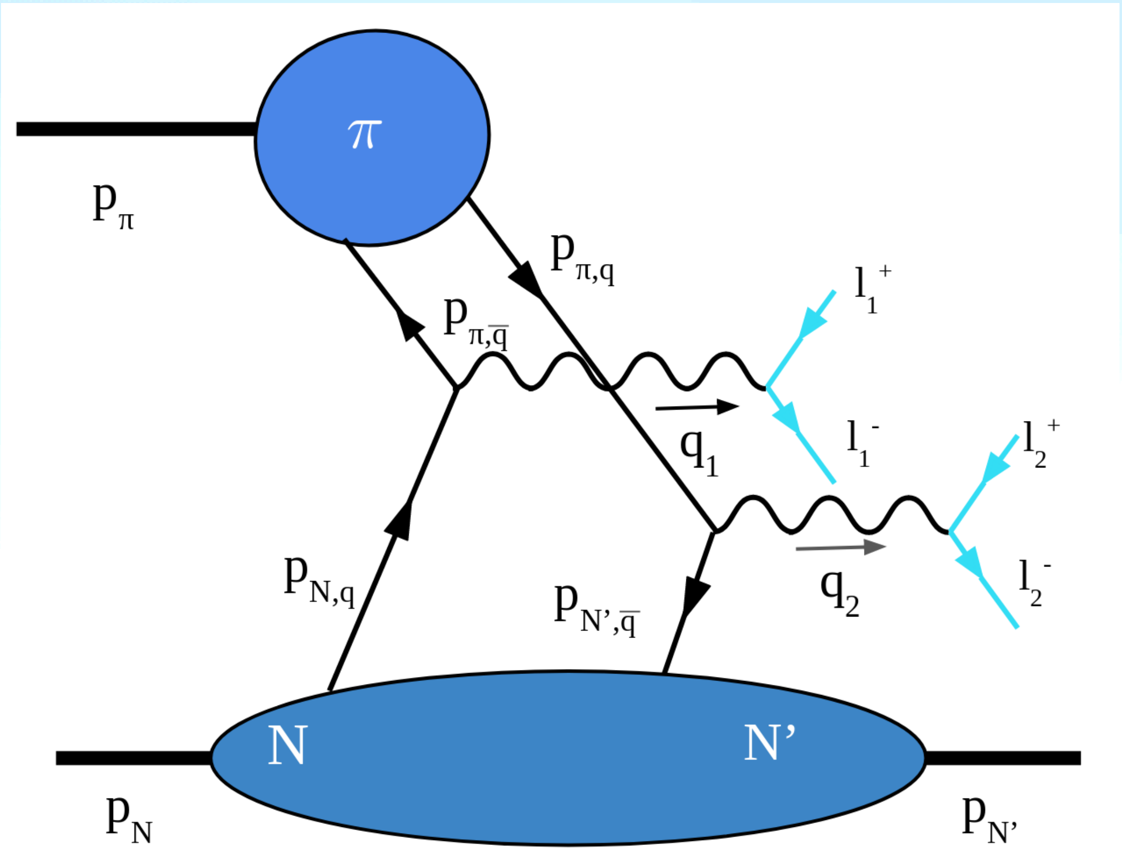



..But interesting distributions can arise with TWO PHOTON EMISSIONS

# From Single photon to Two Photons Exchange

In order to build a factorization theorem we need a power expansion rule




$$\frac{|q_\perp|}{\sqrt{q^2}} \ll 1$$

$$\frac{|q_{i\perp}|}{\sqrt{q_i^2}} \ll 1$$

Semi-inclusive $pp \to \ell^+\ell^- X$

Exclusive $\pi^- p \to n(\ell^+\ell^-)(\ell^+\ell^-)$

$$p_\pi + p_N = p_{N'} + q_1 + q_2$$

# Momentum Scaling and cross section

## Hadronic COM frame

$$p_N = (p_N^+, p_N^-, 0)$$

$$p_\pi = (p_\pi^+, p_\pi^-, 0)$$

$$p_{N'} = (p_{N'}^+, p_{N'}^-, \vec{p}_{N'\perp})$$

...massless case

$$p_\pi \simeq (0, p_\pi^-, 0) \quad p_N \simeq (p_N^+, 0, 0)$$

## Momenta scaling

$$p_\pi^- \sim p_{N,N'}^+$$

$$p_\pi^- \gg p_\pi^+$$

$$p_{N,N'}^+ \gg |\vec{p}_{N'\perp}| \gg p_{N,N'}^-$$

$$|t| \equiv |\Delta^2| \sim |P^2| \ll q_{1,2}^2$$

## Cross section

$$\frac{d\sigma}{dM_1^2 dy_1 d^2\vec{q}_{1\perp} dM_2^2 dy_2 d^2\vec{q}_{2\perp}} = d\hat{\sigma}_L^{\alpha\beta\mu\nu} W_{\alpha\beta\mu\nu}$$

$$M_{1,2}^2 = q_{1,2}^2, \quad y_{1,2} = \frac{1}{2} \ln \frac{q_{1,2}^+}{q_{1,2}^-}$$

# The GTMD factorization formula

$$W_{\alpha\beta\mu\nu} = \int \frac{d^3 p_{N'}}{2(2\pi)^3 E_{N'}} \int d^4 z_1 d^4 z_2 d^4 z_3 \; e^{-iq_1\cdot z_1 - iq_2\cdot(z_2 - z_3)} \langle \pi N \,|\, \overline{T}\{J^\dagger_\alpha(z_1) J^\dagger_\beta(z_2)\} \,|\, N'\rangle \, \langle N' \,|\, T\{J_\mu(z_3) J_\nu(0)\} \,|\, \pi N\rangle$$

Recall: each e.m. is a color and flavor singlet.

For a leading power factorization the SCET formalism is enough

$$\psi(z) \to \chi_n = W^\dagger_n(z^+, 0, z_\perp)\xi_n(z^+, 0, z_\perp), \quad \overline{\psi}(z) \to \overline{\chi}_{\bar{n}} = \bar{\xi}_{\bar{n}}(z^-, 0, z_\perp) W_{\bar{n}}(z^-, 0, z_\perp)$$

Wilson lines for initial states

$$W^\dagger_n(z) = P \exp\left[ ig \int_{-\infty}^{0} ds\; n \cdot A_n(z + sn)\right] \text{ and } W_{\bar{n}}(z) = \overline{P} \exp\left[ -ig \int_{-\infty}^{0} ds\; \bar{n} \cdot A_{\bar{n}}(z + s\bar{n})\right]$$

# The GTMD factorization formula

Next step is Fierzing to re-group collinear and anti-collinear currents in SCET formalism

$$\bar{\chi}'^{a}_{n}(z_3)\gamma_\mu \chi'^{a}_{\bar{n}}(z_3)\ \bar{\chi}^{b}_{\bar{n}}(0)\gamma_\nu \chi^{b}_{n}(0) = \sum_{\Gamma,\Gamma'} \tilde{C}^{\Gamma\Gamma'}_{\mu\nu}\ \bar{\chi}'^{a}_{n}(z_3)\Gamma\chi^{b}_{n}(0)\ \bar{\chi}^{b}_{\bar{n}}(0)\Gamma'\chi'^{a}_{\bar{n}}(z_3) =$$

$$= \sum_{\Gamma,\Gamma'} C^{\Gamma\Gamma'}_{\mu\nu}\ \bar{\chi}'^{a}_{n}(z_3)\Gamma\chi^{a}_{n}(0)\ \bar{\chi}^{b}_{\bar{n}}(0)\Gamma'\chi'^{b}_{\bar{n}}(z_3) + \sum_{\Gamma,\Gamma'} C^{(8)\Gamma\Gamma'}_{\mu\nu}\ \bar{\chi}'^{a}_{n}(z_3)\Gamma T^{A}_{ab}\chi^{b}_{n}(0)\ \bar{\chi}^{b}_{\bar{n}}(0)\Gamma' T^{A}_{ba}\chi'^{a}_{\bar{n}}(z_3)$$

Only color singlet matrix elements survive, states are color singlets

$$W_{\alpha\beta\mu\nu} = H \sum_{\Gamma_1,\Gamma'_1,\Gamma_2,\Gamma'_2} C^{\Gamma_1\Gamma'_1}_{\alpha\beta} C^{\Gamma_2\Gamma'_2}_{\mu\nu} \sum_{\{q\}} e_{q_1} e_{q_2} e_{q_3} e_{q_4} W$$

$\Gamma, \Gamma' = \Gamma_q, \Gamma_{\Delta q}, \Gamma_{\delta q}$ with $\Gamma_q = \gamma^+$, $\Gamma_{\Delta q} = \gamma^+\gamma_5$, $\Gamma^j_{\delta q} = i\sigma^{j+}\gamma_5$, and $j = 1,2$

# The GTMD factorization formula

$$W_{\alpha\beta\mu\nu} = H \sum_{\Gamma_1,\Gamma_1',\Gamma_2,\Gamma_2'} C_{\alpha\beta}^{\Gamma_1\Gamma_1'} C_{\mu\nu}^{\Gamma_2\Gamma_2'} \sum_{\{q\}} e_{q_1} e_{q_2} e_{q_3} e_{q_4} W$$

$$W = \frac{1}{N_c^4} \int \frac{d^3 p_{N'}}{2(2\pi)^3 E_{N'}} \int d^4 z_1 d^4 z_2 d^4 z_3 e^{-iq_1\cdot z_1 - iq_2 \cdot z_2 + iq_2 \cdot z_3}$$

LCWF (bare)

$$\times \langle \pi | \overline{\mathrm{T}}\{\bar{\chi}_{\bar{n}}(z_2^+, 0^-, \vec{z}_{2,\perp})\Gamma_1' \chi_{\bar{n}}(z_1^+, 0^-, \vec{z}_{1,\perp})\} | 0 \rangle \langle 0 | \mathrm{T}\{\bar{\chi}_{\bar{n}}(0)\Gamma_2' \chi_{\bar{n}}(z_3^+, 0^-, \vec{z}_{3,\perp})\} | \pi \rangle$$

$$\times \langle 0 | \overline{\mathrm{T}}\{[S_n^\dagger S_{\bar{n}}](\vec{z}_{1\perp})[S_{\bar{n}}^\dagger S_n](\vec{z}_{2,\perp})\} | 0 \rangle \langle 0 | \mathrm{T}\{[S_n^\dagger S_{\bar{n}}](\vec{z}_{3\perp})[S_{\bar{n}}^\dagger S_n](0)\} | 0 \rangle$$

$$\times \langle N | \overline{\mathrm{T}}\{\bar{\chi}_n(0^+, z_1^-, \vec{z}_{1,\perp})\Gamma_1 \chi_n(0^+, z_2^-, \vec{z}_{2,\perp})\} | N' \rangle \times \langle N' | \mathrm{T}\{\bar{\chi}_n(0^+, z_3^-, \vec{z}_{3,\perp})\Gamma_2 \chi_n(0)\} | N \rangle$$

LCWF (1-loop)

Ji, X. *Phys.Rev.D* 105 (2022) 7, 076014

GTMD (bare)

# Algebraic changes and polished formula

$J_\mu(z) = e^{iP(z/2)} J_\mu(\frac{z}{2}) e^{-iP(z/2)},$ and $e^{-iP\cdot(z^-/2)}|N\rangle = e^{-ip_N^+(z^-/2)/2}|N\rangle$

making the change of variables $z_1 - z_2 \to z_1,\ z_3 \to -z_2$

$$q_1^+ + q_2^+ = -\Delta^+ + p_\pi^+ \simeq -\Delta^+ \equiv 2\xi P^+, \text{ and } q_1^+ - q_2^+ = 2x_P P^+, \text{ and } q_1^- + q_2^- = -\Delta^- + p_\pi^- \simeq p_\pi^-$$

.. and momentum fractions.. $q_1^+ + \dfrac{\Delta^+}{2} \simeq x_P P^+, \quad q_2^+ + \dfrac{\Delta^+}{2} \simeq -x_P P^+ \quad q_2^- \equiv x_\pi p_\pi^-, \quad q_1^- \simeq (1-x_\pi) p_\pi^-$

From kinematics of the process we get that our variables are used only in the ERBL region

$$-\xi \leq x_P \leq \xi$$

# Algebraic changes and polished formula

$$W = \frac{\pi}{4N_c^4 E_{N'}} \delta(q_1^0 + q_2^0 + E_{N'} - E_N - E_\pi) \int d^2 z_{2\perp}\, e^{-i\frac{1}{2}\Delta\vec{q}_\perp \cdot \vec{z}_{2\perp}} \phi^{[\Gamma_2']}(x_\pi, z_{2\perp})\, S(z_{2\perp})\, w^{[\Gamma_2]}(x_P, z_{2\perp}; \xi, \overrightarrow{\Delta}_\perp)$$

$$\times \int d^2 z_{1\perp}\, e^{i\frac{1}{2}\Delta\vec{q}_\perp \cdot \vec{z}_{1\perp}} \phi^{*[\Gamma_1']}(x_\pi, z_{1\perp})\, S(z_{1\perp})\, w^{*[\Gamma_1]}(x_P, z_{1\perp}; \xi, \overrightarrow{\Delta}_\perp)$$

LCWF (bare)

GTMD (bare)

$$w^{[\Gamma_2]}(x_P, z_{2\perp}; \xi, \overrightarrow{\Delta}_\perp) = \int \frac{dz_2^-}{4\pi} e^{i\frac{1}{2}x_P P^+ z_2^-} \langle N' | \mathrm{T}\{\bar{\chi}_n(0^+, -\frac{z_2^-}{2}, -\frac{\vec{z}_{2\perp}}{2})\Gamma_2 \chi_n(0^+, \frac{z_2^-}{2}, \frac{\vec{z}_{2\perp}}{2})\} | N \rangle$$

$$\phi_\pi^{[\Gamma_2']}(x_\pi, \vec{z}_{2\perp}) = \int \frac{dz_2^+}{4\pi} e^{-i\frac{1}{2}(x_\pi - \frac{1}{2})p_\pi^- z_2^+} \langle 0 | \mathrm{T}\{\bar{\chi}_{\bar{n}}(\frac{z_2^+}{2}, 0^-, \frac{\vec{z}_{2\perp}}{2})\Gamma_2' \chi_{\bar{n}}(-\frac{z_2^+}{2}, 0^-, -\frac{\vec{z}_{2\perp}}{2})\} | \pi \rangle$$

# Renormalization

$$S(\vec{z}_{1\perp};\mu^2,\delta^+\delta^-)\,S(\vec{z}_{2\perp};\mu^2,\delta^+\delta^-) = \sqrt{S\left(\vec{z}_{1\perp};\mu^2,(\delta^+)^2\frac{\zeta_1}{p^+_{N,q}p^+_{N',\bar{q}}}\right)}\sqrt{S\left(\vec{z}_{1\perp};\mu^2,(\delta^-)^2\frac{\bar{\zeta}_1}{p^-_{\pi,q}p^-_{\pi,\bar{q}}}\right)}$$

$$\times\sqrt{S\left(\vec{z}_{2\perp};\mu^2,(\delta^+)^2\frac{\zeta_2}{p^+_{N,q}p^+_{N',\bar{q}}}\right)}\sqrt{S\left(\vec{z}_{2\perp};\mu^2,(\delta^-)^2\frac{\bar{\zeta}_2}{p^-_{\pi,q}p^-_{\pi,\bar{q}}}\right)}$$

We have this product because the process is exclusive

With the (most general) condition

$$\zeta_1\bar{\zeta}_1\zeta_2\bar{\zeta}_2 = (p^-_{\pi,q}p^-_{\pi,\bar{q}}p^+_{N,q}p^+_{N',\bar{q}})^2$$

# Renormalization

The GTMD and the LCWF result to be double-scale dependent as TMD

$$W_{N'N}^{[\Gamma_2]}(x_P, \vec{z}_{2\perp}; \xi, \overrightarrow{\Delta}_\perp; \mu^2, \zeta_2) = w_{N'N}^{[\Gamma_2]}(x_P, \vec{z}_{2\perp}; \xi, \overrightarrow{\Delta}_\perp; \mu^2, \delta^+)\sqrt{S\left(\vec{z}_{2\perp}; \mu^2, (\delta^+)^2 \frac{\zeta_2}{p^+_{N,q} p^+_{N',\bar{q}}}\right)}$$

$$\Phi_\pi^{[\Gamma_2']}(x_\pi, \vec{z}_{2\perp}; \mu^2, \bar{\zeta}_2) = \phi_\pi^{[\Gamma_2']}(x_\pi, \vec{z}_{2\perp}; \mu^2, \delta^-)\sqrt{S\left(\vec{z}_{2\perp}; \mu^2, (\delta^-)^2 \frac{\bar{\zeta}_2}{p^-_{\pi,q} p^-_{\pi,\bar{q}}}\right)}$$

$\rightarrow$ $$\frac{d}{d\ln\mu} \ln W_{N'N}^{[\Gamma]}(x_P, z_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu, \zeta) = \gamma_W\left(a_s, \mu, \zeta\right), \qquad \frac{d}{d\ln\mu} \ln \Phi_\pi^{[\Gamma]}(x, \vec{z}_\perp; \mu, \zeta) = \gamma_\Phi\left(a_s, \mu, \zeta\right)$$

$\rightarrow$ $$\frac{d}{d\ln\zeta} \ln W_{N'N}^{[\Gamma]}(x_P, \vec{z}_\perp; \xi, \overrightarrow{\Delta}_\perp; \mu, \zeta) = -D(\vec{z}_\perp; \mu), \qquad \frac{d}{d\ln\zeta} \ln \Phi_\pi^{[\Gamma]}(x, \vec{z}_\perp; \mu, \zeta) = -D(\vec{z}_\perp; \mu)$$

# Renormalization:comment

In Rodini-Vladimirov paper the AD for operators like one appearing in GTMD is calculated ($k_N$, $k_{N'}$ are quark momenta)

$$\gamma_W(a_s, \mu^2, \zeta) = \gamma(a_s, \mu^2, \zeta) + 2a_s \ln \frac{(q^+)^2}{k^+_{N,q} k^+_{N',\bar{q}}}, \qquad \gamma_\Phi(a_s, \mu^2, \zeta) = \gamma(a_s, \mu^2, \zeta) + 2a_s \ln \frac{(q^-)^2}{k^-_{\pi,q} k^-_{\pi,\bar{q}}},$$

$$S(\delta^+\delta^-, z_\perp^2) = R(\delta^+/q^+, \zeta, z_\perp^2) R(\delta^-/q^-, \zeta, z_\perp^2)$$

In the present setting $(q^+)^2 = q_1^+ q_2^+ = p^+_{N,q} p^+_{N',\bar{q}} = k^+_{N,q} k^+_{N',\bar{q}}$ and $(q^-)^2 = q_1^- q_2^- = p^-_{\pi,q} p^-_{\pi,\bar{q}} = k^-_{\pi,q} k^-_{\pi,\bar{q}}$

and no imaginary part in the AD are generated

# Cancellation of the sum of AD

The sum of all the AD, including the one of the hard factor, must cancel. This implies

$$H = |C(Q_1^2, \mu^2)|^2 |C(Q_2^2, \mu^2)|^2$$

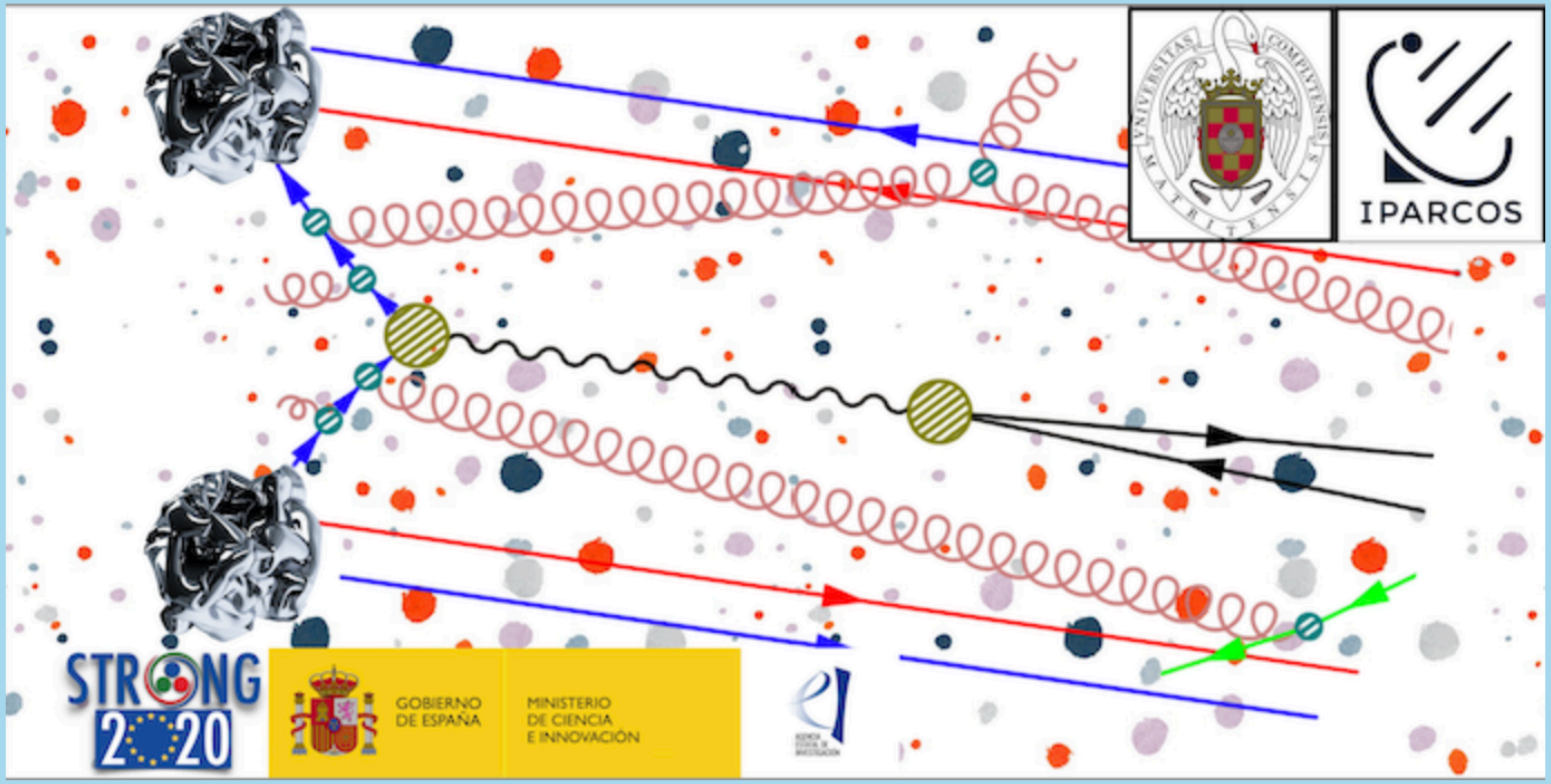
Where $C$ are the quark form factor coefficients as in DY. Checked at one loop.

# Conclusions

The GTMD were identified time ago and some of their properties like evolution have been studied.

In order to make a phenomenological analysis we need

- TPE exclusive processes
- $$\frac{|q_{i\perp}|}{\sqrt{q_i^2}} \ll 1$$
- They can appear together with other functions: more and better processes should be considered
- A better calculation of hard factors, matching coefficients, etc.




# Resummation, Evolution, Factorization 2023 (REF2023)

23-27 octubre 2023
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