Resummation of threshold logarithms in DVCS

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DVCS

$$\gamma^*(q)\ N(p) \longrightarrow \gamma(q')\ N(p')$$

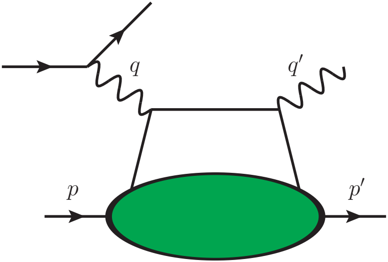
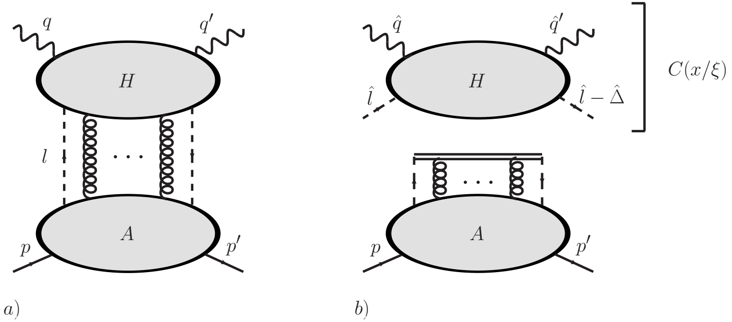


Figure: Handbag approximation

Kinematical parameters

$$P = \frac{p+p'}{2}, \quad t = (p'-p)^2, \quad Q^2 = -q^2, \quad M^2 = p^2 = p'^2, \quad x_B = \frac{Q^2}{2p\cdot q},$$
$$\xi = \frac{p^+ - p'^+}{p^+ + p'^+} \approx \frac{x_B}{2-x_B}$$



- a) Leading region to DVCS amplitude, b) Factorized form after performing expansion. Dashed lines can be quarks or transversely polarized gluons.
- Vector contribution to the amplitude

$$V(\xi,t) = \sum_q \int_{-1}^{1} \frac{dx}{\xi} C_q(x/\xi) F_q(x,\xi,t) + \int_{-1}^{1} \frac{dx}{\xi^2} C_g(x/\xi) F_g(x,\xi,t),$$

where F_q, F_g are GPDs.

- A subtlety: Parton lines connecting H and A are assumed collinear to target. This order of size assumption fails (naively) at "breakpoints" $x = \pm\xi$, where one of the two lines becomes soft.

- Example: hard propagator in handbag diagram ($l^+ = (x+\xi)P^+$)

$$(l+q)^2 + i0 = 2(x-\xi)(l^- + q^-)P^+ + l_\perp^2 + O(-t, m^2) + i0.$$

Canonical p.c.: $|x-\xi| \gg \max(|t|, m^2, |l_\perp^2|)/Q^2$.
Collinear approximation: Set $l_\perp^2, t, m^2, l^-$ to zero.

- Point $x = \xi$ is inside the integration region. **But:** Deformation of the integration contour is possible [Collins & Freund, 98], restoring the validity of the collinear approximation.

- A caveat: GPD has discontinuous derivative at $x = \xi$. Contour deformation fails!
- **But:** Can write GPD for $x > 0$ as

$$F(x,\xi) = \Theta(x-\xi)\,\underbrace{F_{x>\xi}(x,\xi)}_{\text{analytic}} + \Theta(\xi - x)\,\underbrace{F_{x<\xi}(x,\xi)}_{\text{analytic}}$$

and then

$$\int_0^1 dx\ C(x/(\xi - i0))F(x,\xi)$$
$$= \underbrace{\int_0^1 dx\ C(x/(\xi - i0))F_{x<\xi}(x,\xi)}_{\text{deform contour here}} + \int_\xi^1 dx\ C(x/\xi)\underbrace{\left(F_{x>\xi}(x,\xi) - F_{\xi<x}(x,\xi)\right)}_{\text{gives additional suppression at } x=\xi}$$

- Conclusion: $|x - \xi|$ does not get very small under the convolution integral.

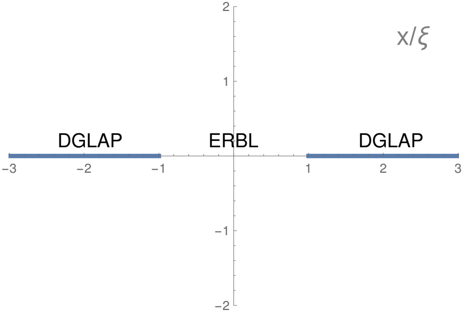
- Leading terms of the quark CF for $x \to \xi$ [Braun, Manashov, Moch, Schoenleber, 20]

$$C_q = \frac{1}{1 - x/\xi}\left[1 + \frac{\alpha_s C_F}{4\pi}\log^2\left(\frac{\xi - x}{2\xi}\right) + \frac{1}{2}\left(\frac{\alpha_s C_F}{4\pi}\log^2\left(\frac{\xi - x}{2\xi}\right)\right)^2 + ...\right],$$

- Question: Does this exponentiate to all orders? Is it possible to resum terms of the form $\frac{\alpha_s^n}{1 \pm x/\xi}\log^k(x \pm \xi)$ to all orders? $\to$ Yes.

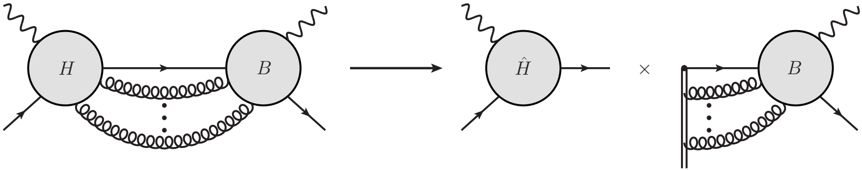
- Although these logs are not large, one can
  $\to$ predict higher order terms in the CF
  $\to$ get contributions that might be relevant for precision physics

- We consider leading contributions to the CF near the partonic thresholds $\hat{s} = \frac{x-\xi}{2\xi} Q^2 = 0$ (i.e. $x = \xi$) and $\hat{u} = -\frac{x+\xi}{2\xi} Q^2 = 0$ (i.e. $x = -\xi$) $\Rightarrow$ CF has poles with protruding branch cuts at $\hat{s} = 0$ and $\hat{u} = 0$.



- CF is (anti-)symmetric w.r.t. to $x \to -x$, so it is enough to only consider $x \to \xi$. Can focus on region $\hat{s} \to 0$.

- Key observation: quark CF itself factorizes near $x = \xi$. The outgoing parton leg becomes soft and the intermediate quark propagator (previously hard) becomes collinear to outgoing photon.



- Get factorization theorem

$$C_q(x/\xi, \mu = Q) = \frac{h(Q^2, \nu^2) f(-\hat{s}, \nu^2)}{1 - x/\xi} + O((x - \xi)^0).$$

- Use independence on new factorization scale ν to resum logs.

- Resummed result:

$$C_q(x/\xi, Q, \mu = Q)$$

$$\sim \frac{\bar{h}(\alpha_s(Q))\bar{f}(\alpha_s(\sqrt{-\hat{s}}))}{1 - x/\xi} \exp\left\{\frac{1}{2}\int_{-\hat{s}}^{Q^2} \frac{d\mu^2}{\mu^2}\Big[-\Gamma_{\mathsf{cusp}}(\alpha_s(\mu))\log\Big(\frac{-\hat{s}}{\mu^2}\Big) + \bar{\gamma}_f(\alpha_s(\mu))\Big]\right\}$$

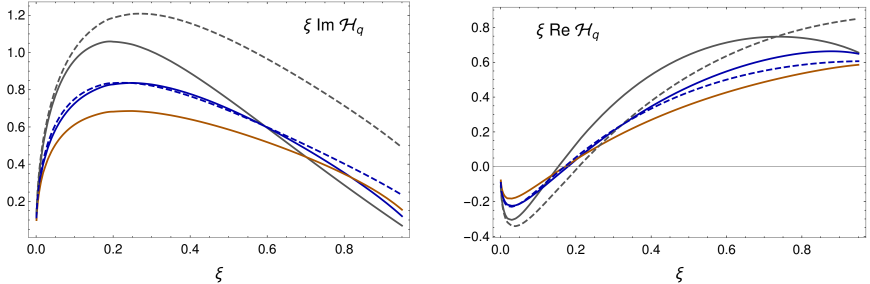
$$\overset{\mathsf{LL}}{\sim} \frac{1}{1 - x/\xi}\exp\left\{\frac{8\pi C_F}{\alpha_s(Q)\beta_0^2}(1 - r + r\log r)\right\} \sim \frac{1}{1 - x/\xi}\exp\left\{\frac{\alpha_s(Q)}{\pi}C_F\log^2\Big(\frac{\xi - x}{2\xi}\Big)\right\},$$

where $r = 1 + \frac{\alpha_s(Q)}{4\pi}\beta_0\log(\frac{\xi - x}{2\xi})$.

- All ingredients for NNLL can be obtained from the NNLO expression for C_q

- Need to integrate over running coupling $\alpha_s(\sqrt{-\hat{s}})$. **But:** recall that the convolution integral is *defined* on a deformed contour $\Rightarrow$ The Landau pole is naturally avoided.

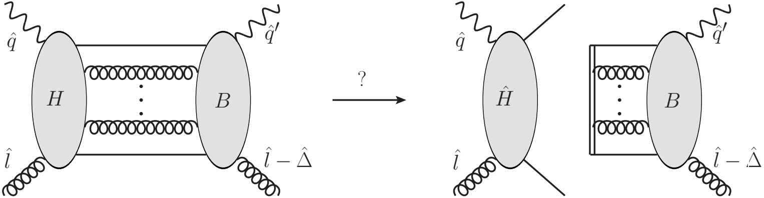
- Preliminary numerical study (simple model for quark GPD)



Gray: LO/NLL, Blue: NLO/NNLL, Brown: NNLO
Straight lines: Fixed order, Dashed lines: with resummation.

- Small corrections at small ξ. Sizable corrections at large $\xi \to 1$. This is because for $\xi \to 1$, the point $x = \xi$ approaches the endpoint of the contour.

- What about the gluon CF? Expect something like



- **But:** Decoupling of the collinear gluons from the hard subgraph H is not clear, since H has an external gluon. Modifications needed? Is this a known problem?

- The same resummation formula applies for the quark axial-vector case, i.e. also to the CF of the pion-photon transition form factor.
- Numerical analysis was performed for the most simple GPD model. Consider impact for more realistic models.
- For gluon CF a similar factorization theorem probably holds. Corrections small, since gluon contribution small at large ξ?