Accessing GPDs through meson production

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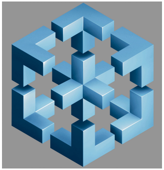
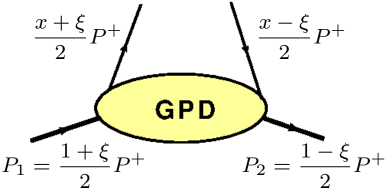


(Escher 3D, Al Borge)

For2926, Regensburg, Feb 17, 2023

Outline

Generalized Parton Distributions



$P = P_1 + P_2 \qquad \Delta = P_2 - P_1$

$\Delta^2 = t$ momentum transfer

$\xi = -\frac{\Delta^+}{P^+}$ longitudinal momentum transfer (skewness)

$$F^a(x,\xi,t;\mu) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\mathcal{O}^a(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

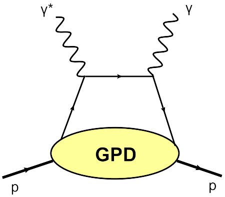
$a \in \{q, g\}$, $\mu \ldots$ factorization scale

- vector (H^a, E^a) and axial-vector GPDs ($\widetilde{H}^a$, $\widetilde{E}^a$)
  $\to$ chiral-even ($\mathcal{O}^q = \bar{q}(z)\Gamma q(-z)$, $\Gamma = \gamma^+, \gamma^+\gamma_5$)
- transversity GPDs (H_T^a, E_T^a, $\widetilde{H}_T^a$, $\widetilde{E}_T^a$)
  $\to$ chiral-odd ($\Gamma = i\sigma^{+i}$)

H^a, $\widetilde{H}^a$, $H_T^q \xrightarrow{\xi=0,t=0}$ PDFs

Selected exclusive processes

Deeply virtual Compton scattering (DVCS)

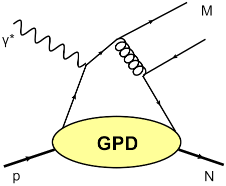


$$\gamma^* p \to \gamma p$$

factorization:

[Collins, Freund '99]

Deeply virtual production of mesons (DVMP)



$$\gamma^* N \to MN'$$

factorization:

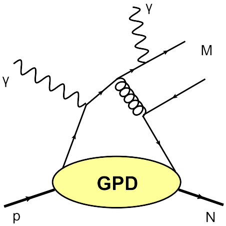
[Collins, Frankfurt, Strikman '97]

$(\gamma_L^* N \to MN')$

[Collins, Diehl '99]

~~$(\gamma^* N \to V_T N')$~~

Deeply virtual production of photon-meson pair



$$\gamma N \to \gamma M N'$$

factorization:
[Qiu, Yu '22]

$(MN \to \gamma\gamma N'$ and similar $2 \to 3$ processes)

M		J^{PC}	DA	GPDs
S, V$_L$	$(q_i\bar{q}_j)$	0^{++} , 1^{--}	$\phi_{\text{asym}}, \phi_{\text{sym}}$	(H, E)
S	(gg)	0^{++}	ϕ_{sym}	(H_g, E_g)
PS, PV$_L$	$(q_i\bar{q}_j)$	0^{-+}, 1^{+-}	$\phi_{\text{sym}}, \phi_{\text{asym}}$	$(\widetilde{H}, \widetilde{E})$
PS	(gg)	0^{-+}	ϕ_{asym}	$(\widetilde{H}_g, \widetilde{E}_g)$
V$_T$	$(q_i\bar{q}_j)$	1^{--}	ϕ_{sym}	(H_T, E_T)
PV$_T$	$(q_i\bar{q}_j)$	1^{+-}	ϕ_{asym}	$(\widetilde{H}_T, \widetilde{E}_T)$
T	(gg)	2^{++}	ϕ_{asym}	$(H_{Tg}, E_{Tg}, \ldots)$

$(q_i\bar{q}_j)$: $P = (-1)^{l+1}$, $C = (-1)^{l+s}$ $(i = j)$, (gg): $P = (-1)^l$, $C = 1$

- $\gamma^*\gamma \to PS(S,T) \Rightarrow$ DVCS
- $\gamma_L^* M^\pm \to M^\pm$,
  $\gamma_L^* S(V_L) \to V_L(S)$, $\gamma_L^* PV_L(PS) \to PS(PV_L) \Rightarrow$ DVMP
- $\gamma\gamma \to M^\pm\, M^\pm$,
  $\gamma\gamma \to PS(S)\; PS(S)$, $\gamma\gamma \to S\; PS$,
  $\gamma\gamma \to V(PV)\; V(PV)$, $\gamma\gamma \to PV\; V$,
  $\gamma\gamma \to T\; PS$, $\gamma\gamma \to T\; S$ $\Rightarrow \gamma p \to (\gamma M)N$

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  $\gamma^*_L S$ $(V_L) \to V_L$ (S), $\gamma^*_L PV_L(PS) \to PS(PV_L) \Rightarrow$ DV(V_L)P
- $\gamma\gamma \to M^\pm\ M^\pm$,
  $\gamma\gamma \to PS(S)\ PS(S)$, $\gamma\gamma \to S\ PS$,
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- $\gamma\gamma \to M^\pm \, M^\pm$,
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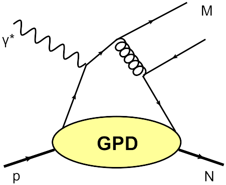
Meson Production: handbag factorization



$\gamma^{(*)} N \to MN'$

	DEEPLY VIRTUAL	WIDE ANGLE
	$Q^2 >>, -t <<$	$-t, -u, s >>$
	DVMP	WAMP
	[Collins, Frankfurt, Strikman '97]	[Huang, Kroll '00]
	• factorization $\mathcal{H}^a \otimes GPD$	• arguments for factorization $\mathcal{H}^a(1/x \otimes GPD(\xi = 0))$
	• GPDs at small $(-t)$	• GPDs at large $(-t)$

$\mathcal{H}^a$ ... parton subprocess helicity amplitudes

$\Rightarrow \mathcal{M}$ ... hadron helicity amplitudes

$\Rightarrow$ observables (cross sections, asymmetries)

Meson Production status

- DV (V_L) P:
  - tw-2 predictions $(\underline{\gamma_L^* N \to V_L N'})$ can describe the data
  - tw-3 calculations $(\underline{\gamma_T^* N \to V_{L,T} N'})$ [Anikin, Teryaev '02], [Golosk., Kroll '13]
- DV (PS) P:
  - tw-2 predictions $(\underline{\gamma_L^* N \to \pi N'})$ bellow the data [HERMES '09] [JLab '12,'16, '20] [COMPAS '19] $\Rightarrow$ importance of $\gamma_T^* N \to \pi N'$

  $\Rightarrow$ tw-3 calculations $(\underline{\gamma_T^* N \to \pi N'})$ with transversity (chiral-odd) GPDs $(H_T^q ...)$ [Goloskokov, Kroll '10] (2-body, i.e., WW approximation), [Ahmad, Goldstein Liuti '09, Goldstein, Hernandez, Liuti '13]
- WA (PS) P:
  - tw-2 results [Huang, Kroll '00] bellow the data [SLAC '76], [JLab '05, '18] for photoproduction $(Q^2 = 0)$
  - tw-3 2-body π photoproduction vanishes [Huang, Jakob, Kroll, P-K '03]

  $\Rightarrow$ tw-3 (2- and 3-body) prediction to π_0 photoproduction [Kroll, P-K '18] fitted to CLAS data [CLAS '18]; photoproduction of η, η' mesons [Kroll, P-K. '22] [preliminary GlueX '20]

  $\Rightarrow$ tw-3 prediction for $\pi^\pm, \pi^0$ photo- and electroproduction $(Q^2 < -t)$ [Kroll, P-K. '21]; extension to DV (PS) P

DVMP

Transition form factors

$$ {}^a\mathcal{T}(\xi, t, Q^2) = \int \mathrm{d}x \int \mathrm{d}u\, T^a(x, \xi, u, \mu_\varphi, \mu_F)\, F^a(x, \xi, t, \mu_\varphi)\, \phi(u, \mu_F) $$

$a = q, g$ or NS,S(Σ, g)

hard-scattering amplitude (known up to NLO)

$$ T^a(x, \xi, u, \mu_\varphi, \mu_F) = \frac{\alpha_s(\mu_R)}{4\pi} T^{a(1)}(x, \xi, u) + \frac{\alpha_s^2(\mu_R)}{(4\pi)^2} T^{a(2)}(x, \xi, u, \mu_R, \mu_\varphi, \mu_F) + \cdots $$

distribution amplitude (DA) evolution, similary GPD (F^a) evolution (known up to NNLO)

$$ \phi(x; \mu_F, \mu_0) = \phi^{(0)}(u, \mu_F, \mu_0) + \frac{\alpha_s(\mu_F)}{4\pi} \phi^{(1)}(u, \mu_F, \mu_0) + \frac{\alpha_s^2(\mu_F)}{(4\pi)^2} \phi^{(2)}(u, \mu_F, \mu_0) + \cdots $$

$\rightarrow$ evolution simpler to implement in conformal momentum representation [Müller '98]

From x space to conformal momentum space

$$ {}^a\mathcal{T}(\xi, t, Q^2) = \int \mathrm{d}x \int \mathrm{d}u \; T^a(x, \xi, y, \mu^2)) \; F^a(x, \xi, t, \mu^2) \; \phi(u, \mu^2) $$

F...GPDs,a=q,g or NS,S(Σ,g)

conformal moments (analogous to Mellin moments in DIS $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$)

[Müller, Lautenschläger, P-K., Schäfer 2014] [Duplančić, Müller, P-K. 2017]

$$ {}^a\mathcal{T}(\xi, t, \mathcal{Q}^2) = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i \pm \left\{ \begin{matrix} \tan \\ \cot \end{matrix} \right\} \left(\frac{\pi j}{2} \right) \right] \xi^{-j-1} $$

$$ \times \left[\mathrm{T}_{jk}(\mathcal{Q}^2/\mu^2) \overset{k}{\otimes} \phi_{\mathrm{M},k}(\mu^2) \right] F_j^{\mathsf{a}}(\xi, t, \mu^2) $$

all channels calculated to NLO :

$\mathcal{H}_M^{q(+)}, \mathcal{E}_M^{q(+)}, \mathcal{H}_M^{\mathrm{g}}, \mathcal{E}_M^{\mathrm{g}}$ $\widetilde{\mathcal{H}}_M^{q(-)}, \widetilde{\mathcal{E}}_M^{q(-)}$	1_L^{--}=V$_L$ 0^{-+}=PS	$\mathcal{H}_M^{q(-)}, \mathcal{E}_M^{q(-)}$ $\widetilde{\mathcal{H}}_M^{q(+)}, \widetilde{\mathcal{E}}_M^{q(+)}, \widetilde{\mathcal{H}}_M^{\mathrm{g}}, \widetilde{\mathcal{E}}_M^{\mathrm{g}}$	0^{++}=S 1_L^{+-}=PV$_L$

(x-space, conformal mom. space, imaginary parts for disp. relations)

NLO predictions

[Müller, Lautenschlager, P-K., Schäfer '14], [Duplančić, Müller, P-K., '17]

- large NLO corrections and model dependence
- results sensitive to the choice of DA
- LO evolution important
- NLO calculations should include NLO evolution
- evolution effects can be called moderate, except for H/E at small ξ
- NLO global DIS+DVCS+DVMP fits needed

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NLO for DV V_L production

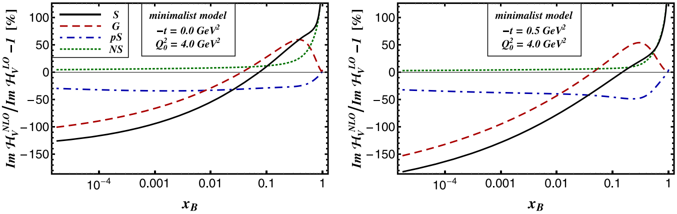


Fig. 6. Relative NLO corrections to the imaginary part of the flavor singlet TFF $\mathcal{F}_V^S$ (solid) broken down to the gluon (dashed), pure singlet quark (dash-dotted) and 'non-singlet' quark (dotted) at $t = 0$ GeV^2 (left panel) and $t = -0.5$ GeV^2 (right panel) at the initial scale $\mathcal{Q}_0^2 = 4$ GeV^2.

[Müller, Lautenschlager, P-K., Schäfer '14]

- big $\ln(1/\xi)$ terms for $\xi <<$, i.e, $j = 0$ pole,
  in gluon evolution and gluon coefficient function

NLO for DV PS/PV production

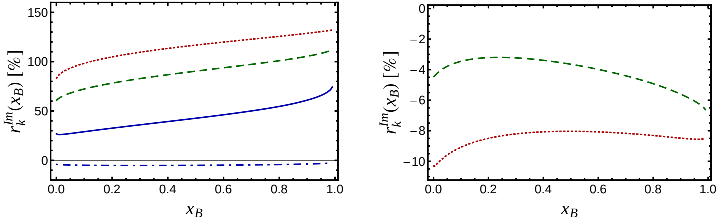


Figure 2: Relative NLO corrections (36) to the imaginary part of the TFF (21) versus x_{B} for the $k = 0$ (solid), $k = 2$ (dashed), $k = 4$ (dotted) partial waves arising from the quark-quark channel (left panel) and quark-gluon channel (right panel). The pure singlet quark contribution for $k = 0$ is shown as dash-dotted line in the left panel.

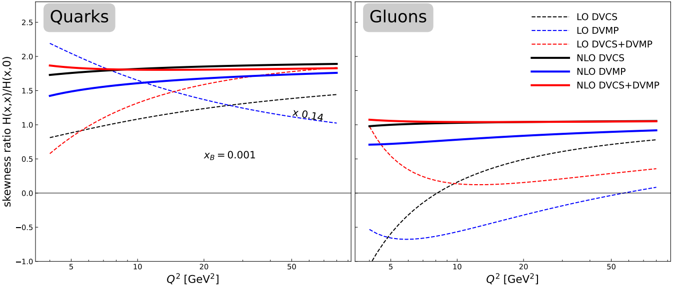
[Duplančić, Müller, P-K., '17]

- NLO corrections higher for higher DA conformal moments ⇒ important for non-asymptotic DAs
- the role of gluons (PV production) smaller since LO vanishes

Global NLO fits (DIS+DVCS+DVV$_L$P)

small-x global fits to HERA collider data (ρ_0 and ϕ)

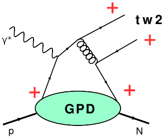
- LO: [Meskauskas, Müller '11] ($\chi^2/n_{\mathrm{d.o.f}} \approx 2$)
- NLO: [Lautenschlager, Müller, Schäfer '13]
- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit using GEPARD software: $\chi^2/n_{\mathrm{d.o.f}} = 254.3/231$



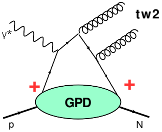
talk K. Kumerički

PS meson production to twist-3

$\mathcal{H}^P_{0\lambda,\mu\lambda}$ . . . non-flip subprocess amplitudes (twist-2)

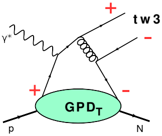


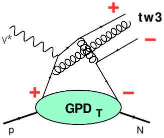
and



for η_1

$\mathcal{H}^P_{0-\lambda,\mu\lambda}$ . . . flip subprocess amplitudes (twist-3)





Note: just meson DA tw-3 contributions ($\mu_\pi = 2$ GeV)

distribution amplitudes (DAs):

twist-2 ($q\bar{q}$) : ϕ_P

2-body ($q\bar{q}$) twist-3 ϕ_{Pp}, $\phi_{P\sigma}$ 3-body ($q\bar{q}g$) twist-3 ϕ_{3P}

$\to$ connected by equations of motion (EOMs)

Subprocess amplitudes: twist-3

General structure:

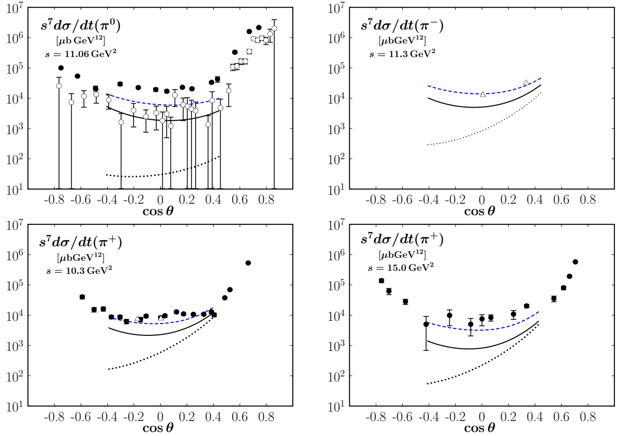
$$\begin{aligned} \mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\ &= \big(\mathcal{H}^{P,\phi_{Pp}} + \underbrace{\mathcal{H}^{P,\phi_{P2}^{EOM}}\big) + \big(\mathcal{H}^{P,q\bar{q}g,C_F}}_{} + \mathcal{H}^{P,q\bar{q}g,C_G}\big) \\ &= \mathcal{H}^{P,\phi_{Pp}} + \mathcal{H}^{P,\phi_{3P},C_F} + \mathcal{H}^{P,\phi_{3P},C_G} \end{aligned}$$

- 2- and 3-body contributions necessary for gauge invariance
- photoproduction ($Q \to 0$): $\mathcal{H}^{P,\phi_{Pp}} = 0$ [Kroll, P-K '18]
- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{P,\phi_{Pp}}$ $\int_0^1 \frac{d\tau}{\bar{\tau}} \phi_{Pp}(\tau)$

    $\Rightarrow$ modified hard-scattering picture (with $k_\perp$) [Golosk., Kroll, '10]
  - complete twist-3 contribution [Kroll, P-K '21]
  - work in progress in modified and collinear picture (effect. m_g^2)

Photoproduction (π)

[Kroll, P-K '21]



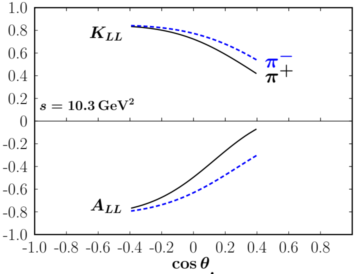
solid curves: complete twist-3
dotted curves: twist-2

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well below the data [Huang, Kroll '00]

Spin effects - photoproduction

[Kroll, P-K '21]: $\pi^\pm$



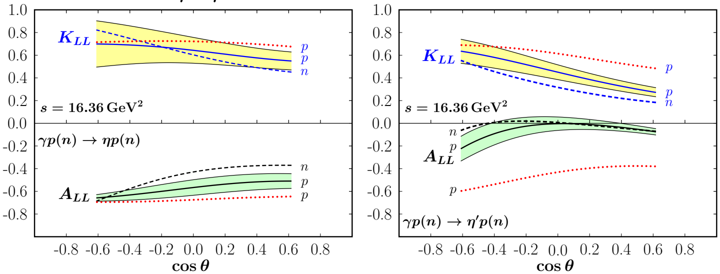
$A_{LL}(K_{LL})\ldots$ correlation of the helicities of the photon and incoming (outgoing) nucleon

$$A_{LL}^{P,tw2} = K_{LL}^{P,tw2}$$

$$A_{LL}^{P,tw3} = -K_{LL}^{P,tw3}$$

$\rightarrow$ characteristic signature for dominance of twist-3 (like $\sigma_T \gg \sigma_L$ in DVMP)

[Kroll, P-K '22]: η, η'



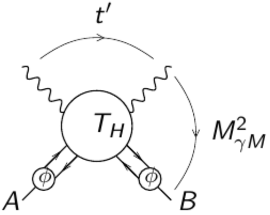
$\rightarrow$ in contrast to π and η, for η' dominance of twist-2 and sensitivity to gluons

Summary

- WA (PS) P:
  - meson's twist-3 contributions (γ^*_T) dominate for πs and η
  - different combinations of form factors ⇒possibility of extraction ⇒ large $-t$ behaviour of transversity GPDs (F^q_T)
- DV (PS) P
  - twist-3 dominates (γ^*_T)
  - complete (2- and 3-body) analysis underway
  - twist-2 (γ^*_L) NLO contributions available and should be tested
- DV (V$_L$) P
  - twist-2 (γ^*_L) contributions can describe the data
  - NLO tw2 contributions available for implementation; included in GeParD ⇒ global DIS+DVCS+DVMP fits performed
- Experimental goals
  - clear L/T separation (eg., for DVπP JLab, Hall C)

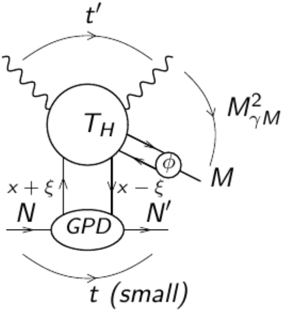
Photon meson photoproduction

$$\gamma + N \to \gamma + M + N'$$



large angle factorisation
à la Brodsky Lepage

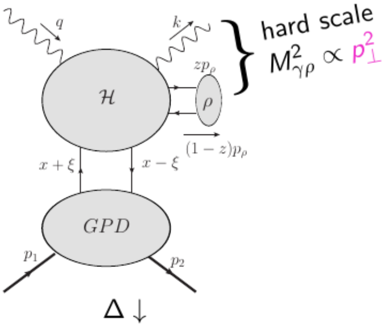
$\longrightarrow$



(á la "time-like" DVCS)

Kinematics

$$\gamma(q) + N(p_1) \to \gamma(k) + \rho(p_\rho, \varepsilon_\rho) + N'(p_2)$$



$$u' = (p_\rho - q)^2$$
$$t' = (k - q)^2$$
$$s' = M^2_{\gamma\rho} = (k + p_\rho)^2$$

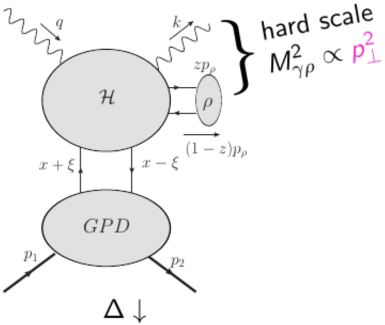
$$t = (p_2 - p_1)^2$$
$$s = S^2_{\gamma N} = (q + p_1)^2$$

$$\xi = \frac{\tau}{2-\tau}, \; \tau = \frac{M^2_{\gamma\rho}}{S^2_{\gamma N} - M^2}$$

- factorization requires:
  $-u', -t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$

Kinematics

$$\gamma(q) + N(p_1) \to \gamma(k) + \rho(p_\rho, \varepsilon_\rho) + N'(p_2)$$



$$u' = (p_\rho - q)^2 \gg$$
$$t' = (k - q)^2 \gg$$
$$s' = M^2_{\gamma\rho} = (k + p_\rho)^2 \gg$$

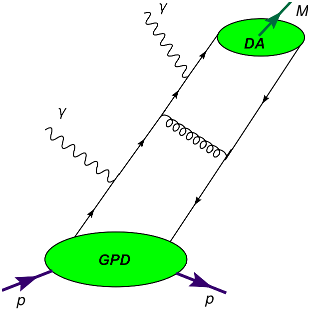
$$t = (p_2 - p_1)^2 \ll$$
$$s = S^2_{\gamma N} = (q + p_1)^2$$

$$\xi = \frac{\tau}{2-\tau},\ \tau = \frac{M^2_{\gamma\rho}}{S^2_{\gamma N} - M^2}$$

- factorization requires: $-u', -t' > 1\ \mathrm{GeV}^2$ and $(-t)_{\min} \leqslant -t \leqslant .5\ \mathrm{GeV}^2$

Photon-meson photoproduction

$$\gamma^* q \to \gamma q (q\bar{q})$$



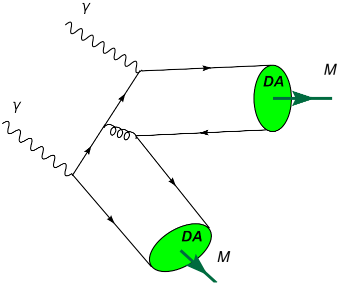
LO ρ mesons: [Boussarie, Pire, Szymanowsky, Wallon '16]

LO $\pi^\pm$ mesons: [Duplančić, P-K, Pire, Szymanowski, Wallon '18]

$$\pi^\pm : H, E, \widetilde{H}, \widetilde{E}$$
$$\rho^0_L : H, E, \widetilde{H}, \widetilde{E}$$
$$\rho^0_T : H_T, E_T, \widetilde{H}_T, \widetilde{E}_T$$

Meson pair production

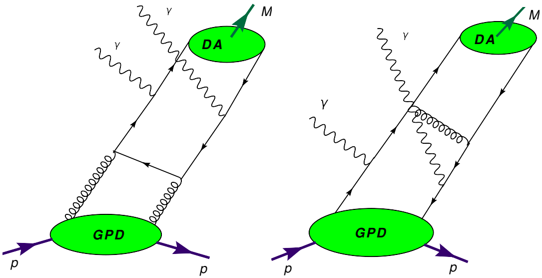
$$\gamma^* \gamma \to (q\bar{q})(q\bar{q})$$



NLO: [Nižić '87, Duplančić, Nižić '06]

Photon-π^0 photoproduction

$$\gamma q \to \gamma(q\bar{q})q,\ \gamma g \to \gamma(q\bar{q})g$$



$(M)\pi^0$ photoproduction

$$\gamma\gamma \to (q\bar{q})(q\bar{q})$$
$$\gamma\gamma \to (gg)(q\bar{q})$$

$\gamma\gamma \to (PS)\pi^0 \quad \to \widetilde{H}, \widetilde{E}$

$\gamma\gamma \to (S)\pi^0 \quad \to H, E$

$\gamma\gamma \to (PS)_g\pi^0 \quad \to \widetilde{H}_g, \widetilde{E}_g$

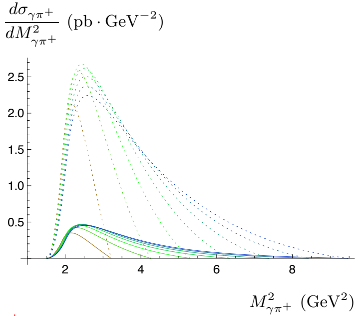
$\gamma\gamma \to (S)_g\pi^0 \quad \to H_g, E_g$

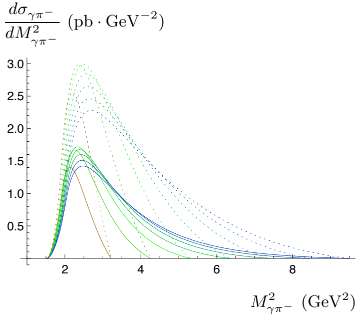
$\gamma\gamma \to (T)_g\pi^0$

$\to H_{Tg}, E_{Tg}, \widetilde{H}_{Tg}, \widetilde{E}_{Tg}$

LO: [Bayer, Grozin '85]

$\pi^\pm$





π^+ photoproduction (proton target) π^- photoproduction (neutron target)

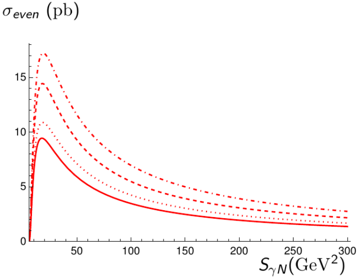
$S_{\gamma N}$ vary in the set 8, 10, 12, 14, 16, 18, 20 GeV^2 (from left to right)

solid: "valence" model

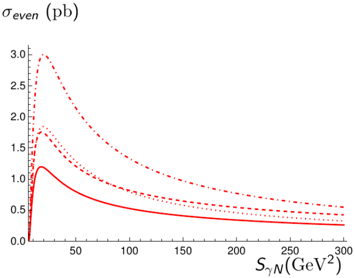
dotted: "standard" model

[Duplančić, P-K, Pire, Szymanowski, Wallon '18]

ρ_L^0



(proton target)

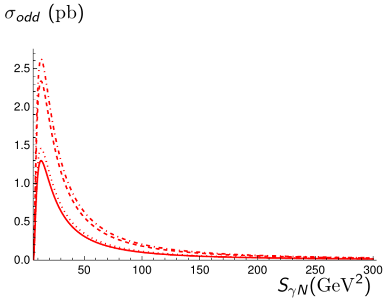


(neutron target)

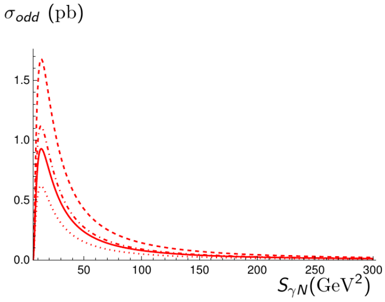
[Duplančić, P-K, Nabeebaccus, Pire, Szymanowski, Wallon '22]

	$\phi_{\rm as}(z) = 6z(1-z)$	$\phi_{\rm hol}(z) = \frac{8}{\pi}\sqrt{z(1-z)}$
"valence"	solid	dashed
"standard"	dotted	dash-dotted

ρ_T^0



(proton target)



(neutron target)

[Duplančić, P-K, Nabeebaccus, Pire, Szymanowski, Wallon '22]

	$\phi_{\mathrm{as}}(z) = 6z(1-z)$	$\phi_{\mathrm{hol}}(z) = \frac{8}{\pi}\sqrt{z(1-z)}$
"valence"	solid	dashed
"standard"	dotted	dash-dotted

Summary

$\gamma N \to (\gamma M) N'$

- provides additional channel for extracting GPDs
- it can probe chiral-odd GPDs at the leading twist
- proof of factorisation for this family of processes
- good statistics in various experiments, particularly at JLab
- small ξ limit of GPDs can be investigated by exploiting high energies available at EIC

Conclusions

- Meson production processes promissing in accessing additional information about GPDs.
- Meson distribution amplitudes additional nontrivial nonperturbative input.

Conclusions

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- Meson distribution amplitudes additional nontrivial nonperturbative input.

Thank you.

- elementary hard-scattering amplitudes for twist-2 collinear approximation (t=0):
  - DVCS ($\gamma^* q \to \gamma^{(*)} q$)
    $\Leftrightarrow$ meson transition form factor ($\gamma^* \gamma^{(*)} \to (q\bar{q})$)
  - DVMP ($\gamma^* q \to (q\bar{q}) q$)
    $\Leftrightarrow$ meson electromagnetic form factor, i.e., meson-to-meson ff ($\gamma^* (q\bar{q}) \to (q\bar{q})$)
- bookkeeping of momentum fractions

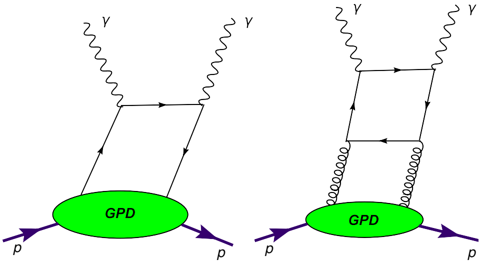
$$\frac{\xi + x}{2\xi} = u \qquad (\frac{\xi - x}{2\xi} = 1 - u)$$

but u real so care with $i\epsilon$ in propagators, or a posteriori analytical continuation of energy, i.e., ξ and not u:

$$u \to \frac{\xi - i\epsilon + x}{2(\xi - i\epsilon)} = \frac{\xi + x}{2\xi} + i\epsilon \text{sign} x$$

(D)DVCS

$$\gamma^* q \to \gamma^{(*)} q,\ \gamma^* g \to \gamma^{(*)} g$$



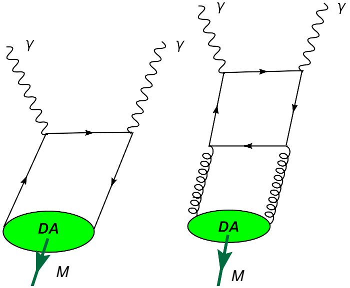
NLO: [Ji, Belitsky et al, Mankiewicz et al, '97]
[Pire, Szymanowski, Wagner '11]

β_0 proportional NNLO: [Belitsky, Schäfer '98]

NNLO from conf. sym: [Müller '05, Kumerički, Müller, P-K. '07]

Meson transition form factor

$$\gamma^* \gamma^{(*)} \to (q\bar{q}),\ \gamma^* \gamma^{(*)} \to (gg)$$



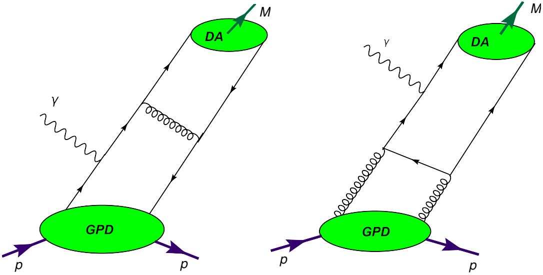
NLO: [..., Kroll, P-K '02] [Kroll, P-K '19]

β_0 proportional NNLO: [Melić, Nižić, Passek '01]

NNLO from conf. sym: [Melić, Müller, Passek '02]

DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$



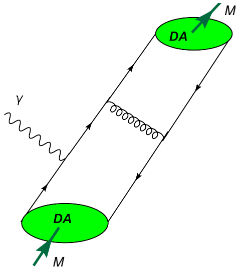
NLO DV PS$^+$ prod.: [Belitsky and Müller '01]

NLO DV V$_L$ prod.: [Ivanov et al '04,]

NLO DV V$_L$ (corr.), PS, (S, PV$_L$) prod.: [Duplančić, Müller, P-K. '17]

Meson em form factor

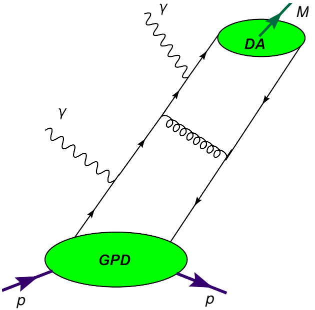
$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



NLO: [..., Melić et al '99]

Photon-meson photoproduction

$$\gamma^* q \to \gamma q (q\bar{q})$$

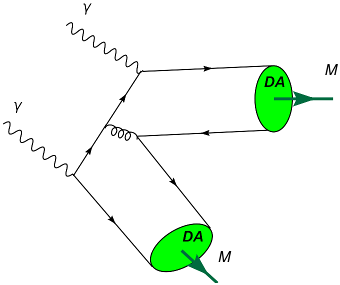


LO V mesons: [Boussarie, Pire, Szymanowsky, Wallon '16]

LO PS mesons: [Duplančić, P-K, Pire, Szymanowski, Wallon '18]

Meson pair production

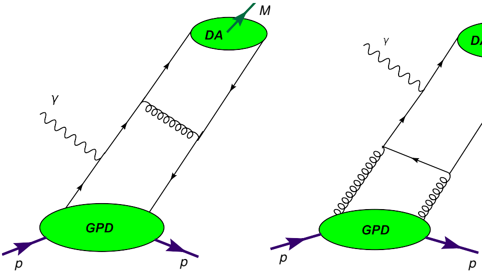
$$\gamma^* \gamma \to (q\bar{q})(q\bar{q})$$



NLO: [Nižić '87, Duplančić, Nižić '06]

DVMP

$$\gamma^* q \to (q\bar{q})q,\ \gamma^* g \to (q\bar{q})g$$



NLO DV PS$^+$ prod.: [Belitsky and Müller '01]

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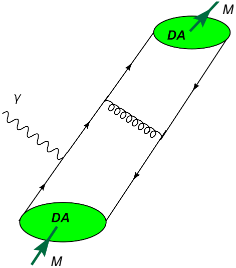
NLO DV V$_L$ (corr.), PS, (S, PV$_L$) prod.: [Duplančić, Müller, P-K. '17]

Meson em form factor

$$\gamma^*(q\bar{q}) \to (q\bar{q})$$



NLO: [..., Melić et al '99]

$$\gamma_L^*(M^\pm) \to M^\pm,$$
$$\gamma_L^*(S) \to V_L,\ \gamma_L^*(V_L) \to S$$
$$\gamma_L^*(PV_L) \to PS,\ \gamma_L^*(PS) \to PV_L$$

$\Rightarrow$ DVMP

About "$\otimes$": DVCS

- factorization formula for singlet DVCS CFFs:

$$ {}^{S}\mathcal{H}(\xi, t, \mathcal{Q}^2) = \int \mathrm{d}x \, \boldsymbol{C}(x, \xi, \mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \boldsymbol{H}(x, \xi, t, \mu^2) $$

- ... in terms of conformal moments

(analogous to Mellin moments in DIS: $x^n \to C_n^{3/2}(x), C_n^{5/2}(x)$):

$$ = 2 \sum_{j=0}^{\infty} \xi^{-j-1} \boldsymbol{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \boldsymbol{H}_j(\xi = \eta, t, \mu^2) $$

$$ H_j^q(\eta, \dots) = \frac{\Gamma(3/2)\Gamma(j+1)}{2^{j+1}\Gamma(j+3/2)} \int_{-1}^{1} \mathrm{d}x \, \eta^{j-1} C_j^{3/2}(x/\eta) H^q(x, \eta, \dots) $$

...

H_j^a even polynomials in η with maximal power η^{j+1}

- series summed using Mellin-Barnes integral over complex j:

$$ = \frac{1}{2i} \int_{c-i\infty}^{c+i\infty} dj \left[i + \tan\left(\frac{\pi j}{2} \right) \right] \xi^{-j-1} \boldsymbol{C}_j(\mathcal{Q}^2/\mu^2, \alpha_s(\mu)) \, \boldsymbol{H}_j(\xi, t, \mu^2) $$

[Müller 2006, Kumerički, Müller, P-K., Schäfer 2006, 2007]

Modelling conformal moments

$$\boldsymbol{H}_j(\eta, t) = \underbrace{\begin{pmatrix} N'_\Sigma F_\Sigma(t)\,\mathrm{B}\big(1+j-\alpha_\Sigma(0), 8\big) \\ N'_\mathsf{G} F_\mathsf{G}(t)\,\mathrm{B}\big(1+j-\alpha_\mathsf{G}(0), 6\big) \end{pmatrix}}_{\text{Leading partial wave}} + \begin{pmatrix} s_\Sigma \\ s_\mathsf{G} \end{pmatrix} \begin{pmatrix} \text{subleading partial waves, } \eta\text{-dependence!} \end{pmatrix}$$

- Leading wave – simplest case:
  (at NLO data can be fitted with leading wave only)
  - Regge-inspired ansatz

$$\alpha_a(t) = \alpha_a(0) + 0.15t \qquad F_a(t) = \frac{j+1-\alpha(0)}{j+1-\alpha(t)} \left(1 - \frac{t}{M_0^{a\,2}}\right)^{-p_a}$$

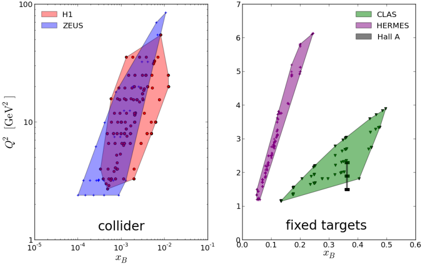
  - for $t = 0$ corresponds to x-space PDFs of the form

$$\Sigma(x) = N'_\Sigma\, x^{-\alpha_\Sigma(0)}\, (1-x)^7 \; ; \qquad G(x) = N'_\mathsf{G}\, x^{-\alpha_\mathsf{G}(0)}\, (1-x)^5$$

  - fit parameters: N_Σ, $\alpha_\Sigma(0)$, $\alpha_G(0)$ (DIS) and M_0^Σ (DVCS)
    ($M_0^G = \sqrt{0.7}$ GeV from J/Ψ prod.)

Experimental status

DVCS



[from Kumericki et al. 2015]

DVMP

- in the last decade: vector meson (ρ, J/Ψ, ϕ) production at H1 and ZEUS (HERA, DESY), COMPASS (CERN), pseudoscalar mesons (π, η) at CLAS (JLab) . . .

$\rightarrow$ new results from JLab@12 (2018)

COMPASS@LHC

EIC (Electron Ion Collider at Brokhaven, 2030)
LHeC proposed

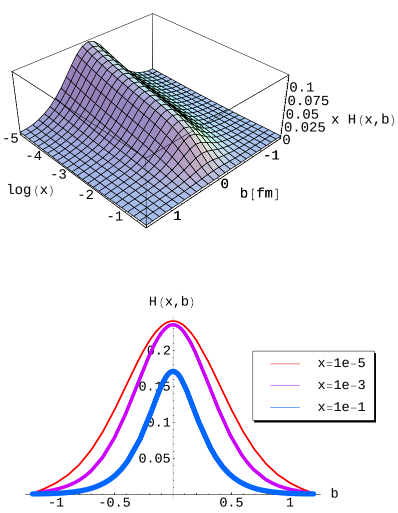
Parton probability density

- Fourier transform of GPD for $\eta = 0$ can be interpreted as probability density depending on x and transversal distance b
  [Burkardt '00, '02]

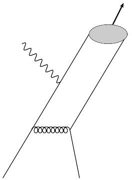
$$H(x,\vec{b}) = \int \frac{d^2\vec{\Delta}}{(2\pi)^2}\, e^{-i\vec{b}\cdot\vec{\Delta}} H(x,\eta=0,\Delta^2=-\vec{\Delta}^2)$$

Three-dimensional image of a proton

Quarks: Gluons:



Subprocess amplitudes $\mathcal{H}$

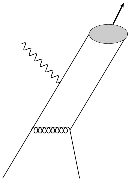


$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\begin{aligned}\mathcal{P}_2^\pi \quad \sim \quad & f_\pi \Big\{ \gamma_5 \not{q}' \phi_\pi(\tau, \mu_F) \\ & + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) \\ & - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, \frac{q'^\mu n^\nu}{q' \cdot n} \, \phi'_{\pi\sigma}(\tau, \mu_F) \\ & + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}\end{aligned}$$

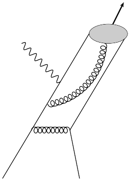
Subprocess amplitudes $\mathcal{H}$



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\mathcal{P}_2^\pi \sim f_\pi \Big\{ \gamma_5 \, q\!\!\!/' \phi_\pi(\tau, \mu_F) + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \frac{q'^\mu n^\nu}{q' \cdot n} \phi'_{\pi\sigma}(\tau, \mu_F) + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}$$

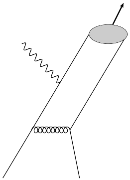


$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \frac{i}{g} \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}$$

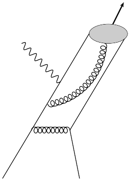
Subprocess amplitudes $\mathcal{H}$



$q\bar{q} \to \pi$ projector [Beneke, Feldmann '00]

$$(\tau q' + k_\perp) + (\bar{\tau} q' - k_\perp) = q'$$

$$\mathcal{P}_2^\pi \sim f_\pi \Big\{ \gamma_5 \, \not{q}' \phi_\pi(\tau, \mu_F) + \mu_\pi(\mu_F) \Big[\gamma_5 \, \phi_{\pi p}(\tau, \mu_F) - \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \frac{q'^\mu n^\nu}{q' \cdot n} \phi'_{\pi\sigma}(\tau, \mu_F) + \frac{i}{6} \gamma_5 \, \sigma_{\mu\nu} \, q'^\mu \phi_{\pi\sigma}(\tau, \mu_F) \frac{\partial}{\partial k_{\perp\nu}} \Big] \Big\}_{k_\perp \to 0}$$



$q\bar{q}g \to \pi$ projector [Kroll, P-K '18]

$$\tau_a q' + \tau_b q' + \tau_g q' = q'$$

$$\mathcal{P}_3^\pi \sim f_{3\pi}(\mu_F) \frac{i}{g} \gamma_5 \, \sigma_{\mu\nu} q'^\mu g_\perp^{\nu\rho} \frac{\phi_{3\pi}(\tau_a, \tau_b, \tau_g, \mu_F)}{\tau_g}$$

$\mu_\pi = m_\pi^2/(m_u + m_d) \cong 2$ GeV, $f_{3\pi} \sim \mu_\pi$

distribution amplitudes (DAs):

twist-2 $(q\bar{q})$: ϕ_π

2-body $(q\bar{q})$ twist-3 $\phi_{\pi p}$, $\phi_{\pi\sigma}$ 3-body $(q\bar{q}g)$ twist-3 $\phi_{3\pi}$

$\to$ connected by equations of motion (EOMs)

Helicity amplitudes $\mathcal{M}$ for WAMP

$$\mathcal{M}^P_{0+,\mu+} = \frac{e_0}{2}\sum_\lambda \Big[\mathcal{H}^P_{0\lambda,\mu\lambda}\left(R^P_V(t) + 2\lambda\, R^P_A(t)\right) \quad \to \text{twist-2}$$

$$-2\lambda \frac{\sqrt{-t}}{2m}\mathcal{H}^P_{0-\lambda,\mu\lambda}\, \bar{S}^P_T(t)\Big] \quad \to \text{twist-3}$$

$$\mathcal{M}^P_{0-,\mu+} = \frac{e_0}{2}\sum_\lambda \Big[\frac{\sqrt{-t}}{2m}\mathcal{H}^P_{0\lambda,\mu\lambda}\, R^P_T(t) \quad \to \text{twist-2}$$

$$-2\lambda \frac{t}{2m^2}\mathcal{H}^P_{0-\lambda,\mu\lambda}\, S^P_S(t)\Big] + e_0 \mathcal{H}^P_{0-,\mu+}\, S^P_T(t) \quad \to \text{twist-3}$$

μ photon helicity, $\lambda\ldots$ quark helicities, $P \in \{\pi^\pm, \pi^0, \eta_8, \eta_1, \eta, \eta'\}$,

$$R^a_V(t) = \int \frac{dx}{x} H^a(x, \xi = 0, t)$$ ...form factors

$$a \in \{u, d\} \Rightarrow R^{\pi^\pm}_V = R^u_V - R^d_V,\; R^{\pi^0}_V = \frac{1}{\sqrt{2}}\left(e_u R^u_V - e_d R^d_V\right)$$

$$R^{\eta_8}_V \approx \frac{1}{\sqrt{2}} R^{\eta_1}_V \approx \frac{1}{\sqrt{6}}\left(e_u R^u_V + e_d R^d_V\right)$$

$$(H, \tilde{H}, E) \to (R_V, R_A, R_T)$$

$$(H_T, \tilde{H}_T, \bar{E}_T) \to (S_T, S_S, \bar{S}_T)$$ transversity GPDs

DAs and EOMs

$$\tau\phi_{\pi p}(\tau)+\frac{\tau}{6}\,\phi'_{\pi\sigma}(\tau)-\frac{1}{3}\,\phi_{\pi\sigma}(\tau) \;=\; \phi^{EOM}_{\pi 2}(\bar\tau)$$

$$\bar\tau\phi_{\pi p}(\tau)-\frac{\bar\tau}{6}\phi'_{\pi\sigma}(\tau)-\frac{1}{3}\phi_{\pi\sigma}(\tau) \;=\; \phi^{EOM}_{\pi 2}(\tau)$$

$$\phi^{EOM}_{\pi 2}(\tau)=2\frac{f_{3\pi}}{f_\pi\mu_\pi}\int_0^{\bar\tau}\frac{d\tau_g}{\tau_g}\,\phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$

- EOMs and symmetry properties
  $\Rightarrow$ the subprocess amplitudes in terms of two twist-3 DAs and 2- and 3-body contributions combined
- combined EOMs $\rightarrow$ first order differential equation $\Rightarrow$ from known form of $\phi_{3\pi}$ [Braun, Filyanov '90] one determines $\phi_{\pi p}$ (and $\phi_{\pi\sigma}$)

Note: $q\bar q g$ projector and EOMs were derived using light-cone gauge for constituent gluon

Subprocess amplitudes: twist-2

Transverse photon polarization ($\mu = \pm 1$) T

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,\mu\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \frac{\sqrt{-\hat{t}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \Big[(2\lambda\mu+1)\left(\frac{(\hat{s}\tau+Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{s}\bar{\tau}(Q^2\bar{\tau}-\hat{t}\tau)}\, e_a + \frac{(\hat{s}\tau-Q^2)(\hat{s}+Q^2)-\hat{u}Q^2\bar{\tau}}{\hat{u}\tau(Q^2\tau-\hat{t}\bar{\tau})}\, e_b\right) + (2\lambda\mu-1)\left(\frac{\hat{u}\, e_a}{(Q^2\bar{\tau}-\hat{t}\tau)} + \frac{\hat{s}\bar{\tau}\, e_b}{\tau(Q^2\tau-\hat{t}\bar{\tau})}\right)\Big]$$

Longitudinal photon polarization L

$$\mathcal{H}^{\pi,tw2}_{0\lambda,\,0\lambda} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \lambda\, \frac{Q\sqrt{-\hat{u}\hat{s}}}{\hat{s}+Q^2} \int_0^1 d\tau\, \phi_\pi(\tau) \left(\frac{\hat{u}\, e_a}{\hat{s}(Q^2\bar{\tau}-\hat{t}\tau)} - \frac{(\hat{t}+\tau\hat{u})\, e_b}{\tau\hat{u}(Q^2\tau-\hat{t}\bar{\tau})}\right)$$

→ photoproduction ($Q \to 0$): $\mathcal{H}^{\pi,tw2}_{L}\Big|_{Q\to 0} = 0$

$$\mathcal{H}^{\pi,tw2}_{T}\Big|_{Q\to 0} \sim f_\pi\, C_F\, \alpha_s(\mu_R)\, \frac{1}{\sqrt{-\hat{t}}} \int_0^1 \frac{d\tau}{\tau}\, \phi_\pi(\tau)\, \big((1+2\lambda\mu)\,\hat{s} - (1-2\lambda\mu)\,\hat{u}\big) \left(\frac{e_a}{\hat{s}} + \frac{e_b}{\hat{u}}\right)$$

→ DVMP ($\hat{t} \to 0$): $\mathcal{H}^{\pi,tw2}_{T}\Big|_{\hat{t}\to 0} = 0$

$\mathcal{H}^{\pi,tw2}_{L}\Big|_{\hat{t}\to 0}$: $\quad \hat{s} = -\frac{\xi - x}{2\xi}\, Q^2\,,\ \hat{u} = -\frac{\xi + x}{2\xi}\, Q^2$ ⇒ well known LO result for DVMP

Subprocess amplitudes: twist-3

General structure:

$$\begin{aligned}\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\ &= \Big(\mathcal{H}^{P,\phi_{\pi p}} + \underbrace{\mathcal{H}^{P,\phi_{\pi 2}^{EOM}}\Big) + \Big(\mathcal{H}^{P,q\bar{q}g,C_F}} + \mathcal{H}^{P,q\bar{q}g,C_G}\Big) \\ &= \mathcal{H}^{P,\phi_{\pi p}} + \mathcal{H}^{P,\phi_{3\pi},C_F} + \mathcal{H}^{P,\phi_{3\pi},C_G}\end{aligned}$$

- 2-body twist-3 $\sim C_F$; 3-body C_F and C_G proportional parts
- C_G part is separately gauge invariant
- the sum of 2- and 3-body C_F parts is gauge invariant (QED and QCD)
- no end-point singularities for $\hat{t} \neq 0$!

Subprocess amplitudes: twist-3 at $Q <<$ or $\hat{t} <<$

General structure:

$$\begin{aligned}
\mathcal{H}^{P,tw3} &= \mathcal{H}^{P,tw3,q\bar{q}} + \mathcal{H}^{P,tw3,q\bar{q}g} \\
&= \left(\mathcal{H}^{P,\phi_{\pi p}} + \underbrace{\mathcal{H}^{P,\phi_{\pi 2}^{EOM}}\right) + \left(\mathcal{H}^{P,q\bar{q}g,C_F}}_{} + \mathcal{H}^{P,q\bar{q}g,C_G}\right) \\
&= \mathcal{H}^{P,\phi_{\pi p}} + \mathcal{H}^{P,\phi_{3\pi},C_F} + \mathcal{H}^{P,\phi_{3\pi},C_G}
\end{aligned}$$

- $\mathcal{H}_L^{P,tw3} \sim Q\sqrt{-t} \to 0$ both for $Q \to 0$ and $\hat{t} \to 0$
- photoproduction ($Q \to 0$):
  - $\mathcal{H}^{P,\phi_{\pi p}} = 0$ [Kroll, P-K '18]
- DVMP ($\hat{t} \to 0$):
  - end-point singularities in $\mathcal{H}^{P,\phi_{\pi p}}$ [Goloskokov, Kroll '10]
  - $\mathcal{H}^{P,\phi_{\pi 2}^{EOM}} = 0$

Subprocess amplitudes: twist-3 at $Q \to 0$, $t \to 0$

photoproduction

$$\mathcal{H}^{P,tw3}_{0-\lambda,\,\mu\lambda}|_{Q^2\to 0} \sim (2\lambda-\mu)\, f_{3\pi}\, \alpha_S(\mu_R)\, \sqrt{-\hat{u}\hat{s}} \int_0^1 d\tau \int_0^{\bar\tau} \frac{d\tau_g}{\tau_g}\, \phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$
$$\times \Bigg[C_F \left(\frac{1}{\bar\tau^2} - \frac{1}{\bar\tau(\bar\tau-\tau_g)}\right)\left(\frac{e_a}{\hat{s}^2}+\frac{e_b}{\hat{u}^2}\right) +$$
$$-C_G\, \frac{2}{\tau\tau_g}\, \frac{\hat{t}}{\hat{s}\hat{u}} \left(\frac{e_a}{\hat{s}}+\frac{e_b}{\hat{u}}\right)\Bigg]$$

DVMP

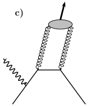
$$\mathcal{H}^{P,\phi_{\pi p}}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim (2\lambda+\mu)\; f_\pi \mu_\pi C_F \alpha_S(\mu_R) \sqrt{-\frac{\hat{u}}{\hat{s}}}\left[\frac{e_a}{\hat{s}}+\frac{\hat{s}}{\hat{u}}\frac{e_b}{\hat{u}}\right] \int_0^1 \frac{d\tau}{\bar\tau}\phi_{\pi p}(\tau)$$

$$\mathcal{H}^{P,C_F,\phi_{3\pi}}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim -(2\lambda+\mu)\, f_{3\pi}\, C_F \alpha_S(\mu_R) \sqrt{-\frac{\hat{u}}{\hat{s}}}\left(\frac{e_a}{\hat{s}}+\frac{\hat{s}}{\hat{u}}\frac{e_b}{\hat{u}}\right)$$
$$\times \int_0^1 \frac{d\tau}{\bar\tau^2} \int_0^{\bar\tau} \frac{d\tau_g}{\tau_g(\bar\tau-\tau_g)}\, \phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$

$$\mathcal{H}^{P,qqg,C_G}_{0-\lambda,\mu\lambda}|_{\hat{t}\to 0} \sim (2\lambda+\mu)\; f_{3\pi}\, C_G \alpha_S(\mu_R) \frac{Q^2}{\sqrt{-\hat{s}\hat{u}}}\left(\frac{e_a}{\hat{s}}+\frac{e_b}{\hat{u}}\right)$$
$$\times \int_0^1 \frac{d\tau}{\bar\tau} \int_0^{\bar\tau} \frac{d\tau_g}{\tau_g(\bar\tau-\tau_g)}\, \phi_{3\pi}(\tau,\bar\tau-\tau_g,\tau_g)$$

Subprocess amplitudes $\mathcal{H}^{\eta_8,\eta_1} \to \mathcal{H}^{\eta,\eta'}$

Novel features:

1. 

$|gg\rangle$ states contribute to twist-2

- $\mathcal{H}^{\pi,tw2} \Rightarrow \mathcal{H}^{\eta_8,tw2}, \mathcal{H}^{\eta_{1,q},tw2}$ $\qquad (\phi_\pi, f_\pi) \to (\phi_{\eta_8}, f_{\eta_8}), (\phi^q_{\eta_1}, f_{\eta_1})$

$\mathcal{H}^{\eta_1} = \mathcal{H}^{\eta_{1q},tw2} + \mathcal{H}^{\eta_{1g},tw2}$ $\qquad \phi^q_{\eta_1}$ and $\phi^g_{\eta_1}$ mix under evolution

- $\mathcal{H}^{\pi,tw3} \Rightarrow \mathcal{H}^{P,tw3}$ $\qquad (\phi_{3\pi}, f_\pi, f_{3\pi}) \to (\phi_{3P}, f_P, f_{3P})$

2. flavour-mixing:
   - simplest: flavour-mixing embedded in the decay constants

$$f^8_\eta = f_8 \cos\theta_8 \qquad f^1_\eta = -f_1 \sin\theta_1$$
$$f^8_{\eta'} = f_8 \sin\theta_8 \qquad f^1_{\eta'} = f_1 \cos\theta_1$$

[review Feldmann '00]

Pion distribution amplitudes

Twist-2 DA: $\phi_\pi(\tau,\mu_F) = 6\tau\bar{\tau}\left[1 + a_2(\mu_F)\, C_2^{3/2}(2\tau-1)\right]$

Twist-3 DAs:

$$\begin{aligned}\phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F) &= 360\tau_a\tau_b\tau_g^2\Big[1+\omega_{1,0}(\mu_F)\,\frac{1}{2}(7\tau_g-3)\\ &+ \omega_{2,0}(\mu_F)\,(2-4\tau_a\tau_b-8\tau_g+8\tau_g^2)\\ &+ \omega_{1,1}(\mu_F)\,(3\tau_a\tau_b-2\tau_g+3\tau_g^2)\Big]\end{aligned}$$

[Braun, Filyanov '90]

using EOMs [Kroll, P-K '18]:

$$\begin{aligned}\phi_{\pi p}(\tau,\mu_F) &= 1+\frac{1}{7}\frac{f_{3\pi}(\mu_F)}{f_\pi\mu_\pi(\mu_F)}\Big(7\,\omega_{1,0}(\mu_F)-2\,\omega_{2,0}(\mu_F)-\omega_{1,1}(\mu_F)\Big)\\ &\times\Big(10\,C_2^{1/2}(2\tau-1)-3\,C_4^{1/2}(2\tau-1)\Big)\,,\quad \phi_{\pi\sigma}(\tau)=\ldots\end{aligned}$$

Parameters:

- $a_2(\mu_0) = 0.1364 \pm 0.0213$ at $\mu_0 = 2$ GeV [Braun et al '15] (lattice)
- $\omega_{10}(\mu_0) = -2.55\,, \omega_{10}(\mu_0) = 0.0$ and $f_{3\pi}(\mu_0) = 0.004\ \mathrm{GeV}^2$. [Ball '99]
- $\omega_{20}(\mu_0) = 8.0$ [Kroll, P-K '18] fit to π^0 photoproduction data [CLAS '17]

Evolution of the decay constants and DA parameters taken into account.
Choice of scales: ${\mu_R}^2 = {\mu_F}^2 = \hat{t}\hat{u}/\hat{s}$

η, η' distribution amplitudes

Twist-2 DA:

$$\phi_8(\tau,\mu_F) = 6\tau\bar{\tau}\left[1 + a_2^8(\mu_F)\, C_2^{3/2}(2\tau-1)\right]$$

$$\begin{aligned} \phi_{1,q}(\tau,\mu_F) &= 6\tau\bar{\tau}\left[1 + a_2^1(\mu_F)\, C_2^{3/2}(2\tau-1)\right] \\ \phi_{1,g}(\tau,\mu_F) &= 30\tau^2\bar{\tau}^2\left[1 + a_2^g(\mu_F)\, C_1^{5/2}(2\tau-1)\right] \end{aligned}$$

Twist-3 DAs:

assumption

$$\phi_{38}(\tau_a,\tau_b,\tau_g,\mu_F) = \phi_{31}(\tau_a,\tau_b,\tau_g,\mu_F) \approx \phi_{3\pi}(\tau_a,\tau_b,\tau_g,\mu_F)$$

Parameters:

- $a_2^8(\mu_0) = -0.039$, $a_2^1(\mu_0) = -0.057$, $a_2^g(\mu_0) = 0.038$ [Kroll, KPK '13], and other choices tested
- $f_{38}(\mu_0) = 0.86 f_{3\pi}(\mu_0) \Leftarrow$ [Ball '99; Braun, Filyanov '90]
- $f_{31}(\mu_0) = 0.86 f_{3\pi}(\mu_0) \Leftarrow \eta$ exp: [GlueX preliminary '20]
- mixing parameters from [Feldmann, Kroll, Stech '98]

Form factors and GPDs

$R_i \ldots 1/x$ moment of $\xi = 0$ GPD (K_i)

- $R_V(\leftarrow H)$, $R_T(\leftarrow E)$ from nucleon form factor analysis [Diehl, Kroll '13]
- $R_A(\leftarrow \tilde{H})$ form factor analysis and WACS KLL asymmetry [Kroll '17]
- $S_T(\leftarrow H_T)$, $\bar{S}_T(\leftarrow \bar{E}_T)$ low $-t$ from DVMP analysis [Goloskokov, Kroll '11]
- $S_S(\leftarrow \tilde{H}_T) \cong \bar{S}_T/2$ $(\bar{E}_T = 2\tilde{H}_T + E_T)$

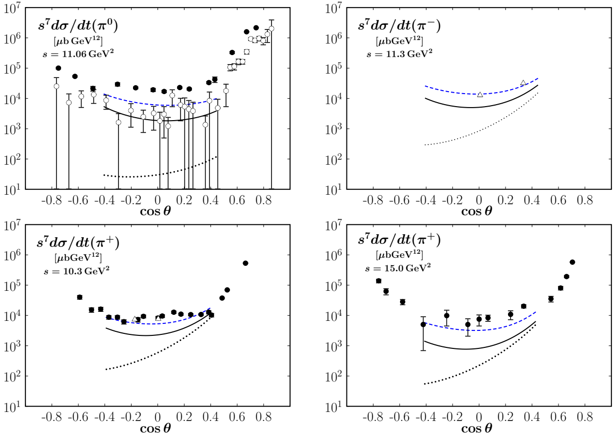
GPD parameterization [Diehl, Feldmann, Jakob, Kroll '04, Diehl, Kroll '13]

$$K_i^a = k_i^a(x) \exp\left[t f_i^a(x)\right], \; f_i^a(x) = \left(B_i^a - \alpha_i'^a \ln x\right)(1-x)^3 + A_i^a x(1-x)^2$$

- strong $x - t$ correlation
- power behaviour for large $(-t)$
- choice for transversity GPDs $A = 0.5$ GeV^{-2}

Photoproduction (π)

[Kroll, P-K '21]



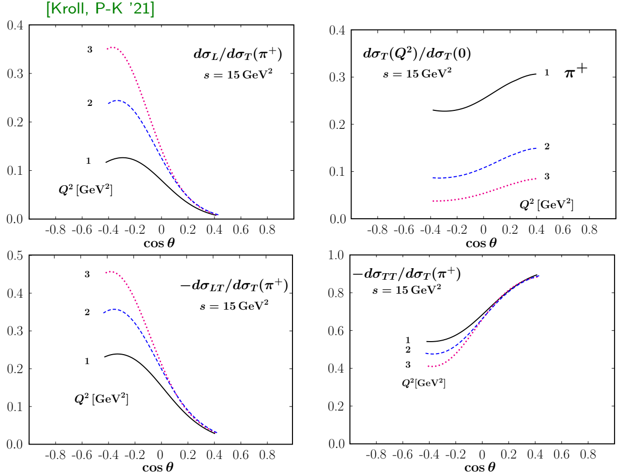
theoretical predictions with parameters from [Kroll, P-K '18] (fit of π^0 twist-3 prediction to [CLAS '17] data)

solid curves: complete twist-3
dotted curves: twist-2
dashed curves: $\omega_{20} = 10.3$
$\mu_R = \mu_F = 1$ GeV

exp data:
full circles [SLAC '76]
open circles [CLAS '17]
triangles [JLab, Hall A '05]

- twist-2 prediction well beyond the data [Huang, Kroll '00]
- scaling: s^{-7} (s^{-8}) twist-2 (twist-3) $\rightarrow$ effective s^{-9} $\rightarrow$ too strong

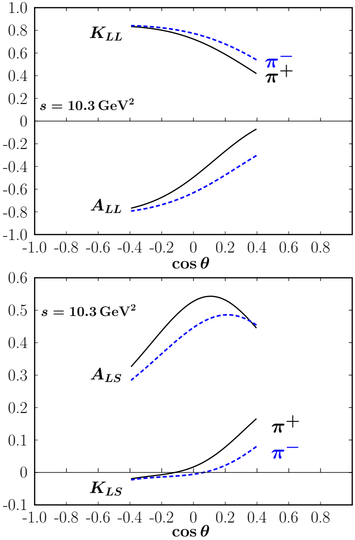
Electroproduction (π)



- both for σ_L and σ_{LT} no twist-2 and twist-3 interference
  $\Rightarrow$ information on S_T (H_T)
- information on S_S ($\tilde{H}_T$) from σ_{TT} (suppressed for DVMP)

Spin effects - photoproduction

[Kroll, P-K '21]



$A_{LL}(K_{LL})\ldots$ correlation of the helicities of the photon and incoming (outgoing) nucleon

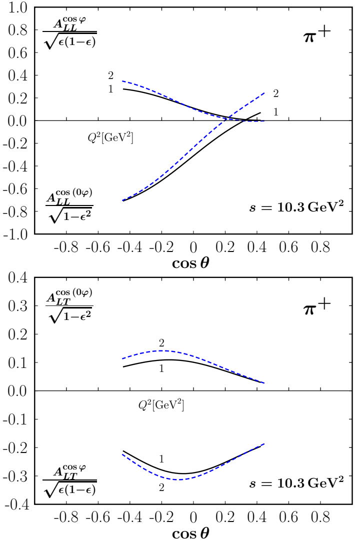
$$A_{LL}^{P,tw2} = K_{LL}^{P,tw2}$$
$$A_{LL}^{P,tw3} = -K_{LL}^{P,tw3}$$

$\rightarrow$ characteristic signature for dominance of twist-3 (like $\sigma_T \gg \sigma_L$ in DVMP)

$A_{LS}(K_{LS})\ldots$ correlation of the helicities of the photon and sideway polarization of the incoming (outgoing) nucleon

Spin effects - electroproduction

[Kroll, P-K '21]



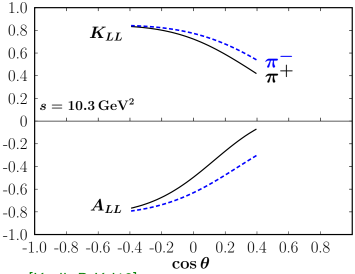
$A_{LL}(K_{LL})$ have two modulations for electroproduction

($\rightarrow$ measured for DVMP [CLAS '15])

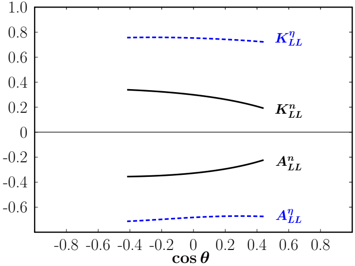
$A_{LT}(K_{LT})$ ... correlation between the lepton helicity and transversal target polarization

Spin effects - photoproduction

[Kroll, P-K '21]



[Kroll, P-K '18]



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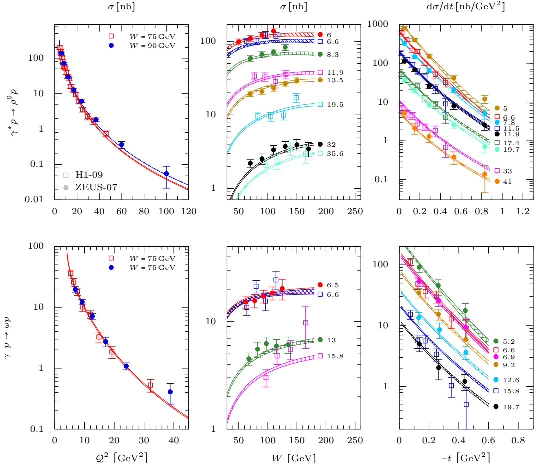
$A_{LL}(K_{LL})$ for π^0 photoproduction on neutron and η photoproduction

Global NLO fits (DIS+DVCS+DVV$_L$P)

small-x global fits to HERA collider data

- LO: [Meskauskas, Müller '11] $(\chi^2/n_{\rm d.o.f} \approx 2)$
- NLO: [Lautenschlager, Müller, Schäfer '13] (normalization of experimental DVMP datasets treated as fitting parameters)



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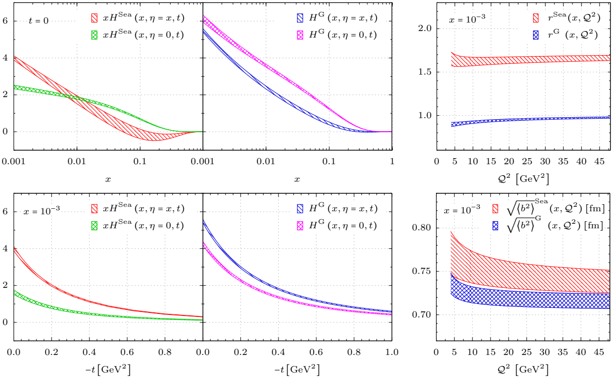
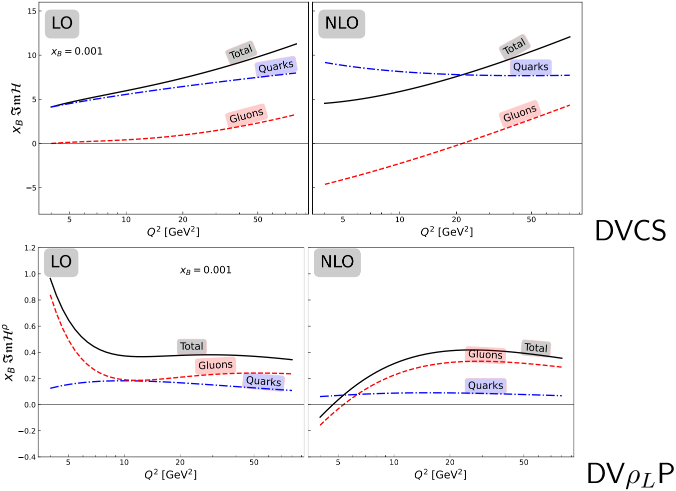


Global NLO fits (DIS+DVCS+DVV_LP)

- hard scattering amplitude corrected [Duplančić, Müller, P-K. '17]
- new NLO fit using GEPARD software: $\chi^2/n_{\rm d.o.f} = 254.3/231$



[preliminary K. Kumerički at Transversity 2022]

Global NLO fits (DIS+DVCS+DVV$_L$P)

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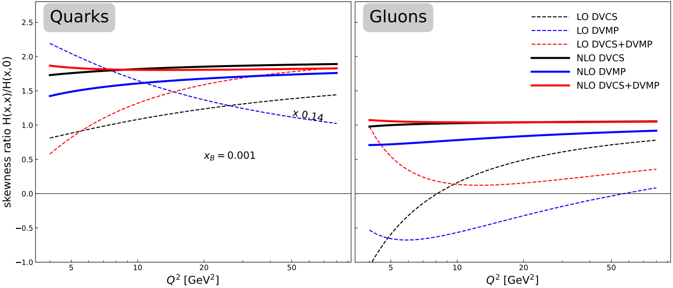

[preliminary K. Kumerički at Transversity 2022]

Global NLO fits

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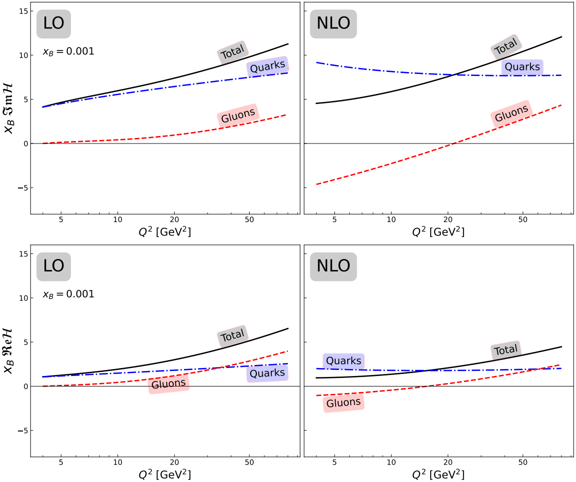

[K. Kumerički at Transversity 2022]

Global NLO fits

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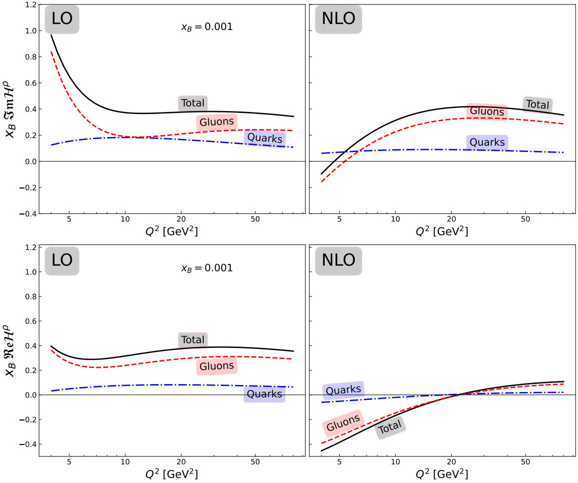


[K. Kumerički at Transversity 2022]

Prospects at experiments

Counting rates: JLab

Good statistics: For example, at JLab Hall B:

- untagged incoming $\gamma \Rightarrow$ Weizsäcker-Williams distribution
- with an expected luminosity of $\mathcal{L} = 100\ \mathrm{nb}^{-1}s^{-1}$, for 100 days of run:
  - ρ_L^0 (on p) : $\approx 2.4 \times 10^5$
  - ρ_T^0 (on p) : $\approx 4.2 \times 10^4$ (Chiral-odd)
  - ρ_L^+ : $\approx 1.4 \times 10^5$
  - ρ_T^+ : $\approx 6.7 \times 10^4$ (Chiral-odd)
  - π^+ : $\approx 1.8 \times 10^5$

Prospects at experiments
Counting rates: COMPASS

At COMPASS:

- Taking a luminosity of $\mathcal{L} = 0.1\ \mathrm{nb}^{-1} s^{-1}$, and 300 days of run,
  - ρ_L^0 (on p) : $\approx 1.2 \times 10^3$
  - ρ_T^0 (on p) : $\approx 1.5 \times 10^2$ (Chiral-odd)
  - ρ_L^+ : $\approx 7.4 \times 10^2$
  - ρ_T^+ : $\approx 2.6 \times 10^2$ (Chiral-odd)
  - π^+ : $\approx 4.5 \times 10^2$
- Lower numbers due to low luminosity (factor of 10^3 less than JLab!)

Prospects at experiments

Counting rates: EIC

- At the future EIC, with an expected integrated luminosity of $10\,\mathrm{fb}^{-1}$ (about 100 times smaller than JLab):
  - ρ_L^0 (on p) : $\approx 2.4 \times 10^4$
  - ρ_T^0 (on p) : $\approx 2.4 \times 10^3$ (Chiral-odd)
  - ρ_L^+ : $\approx 1.5 \times 10^4$
  - ρ_T^+ : $\approx 4.2 \times 10^3$ (Chiral-odd)
  - π^+ : $\approx 1.3 \times 10^4$
- Small ξ study: $160 < S_{\gamma N} < 20000$ ($5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$):
  - ρ_L^0 (on p) : $\approx 2.3 \times 10^3$
  - ρ_T^0 (on p) : ≈ 6.5 (Chiral-odd) (tiny)
  - ρ_L^+ : $\approx 1.8 \times 10^3$
  - π^+ : $\approx 1.0 \times 10^3$

Prospects at experiments

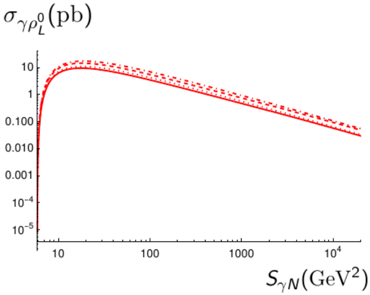
LHC at UPC

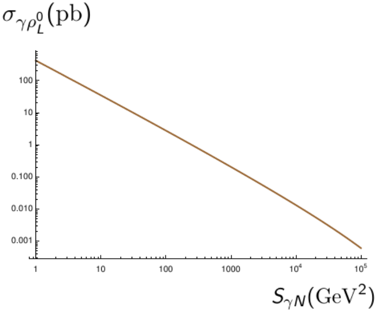
For p-Pb UPCs at LHC (integrated luminosity of 1200 nb^{-1}):

- With future data from runs 3 and 4,
  - $\rho^0_L : \approx 1.6 \times 10^4$
  - $\rho^0_T : \approx 1.7 \times 10^3$ (Chiral-odd)
  - $\rho^+_L : \approx 1.0 \times 10^4$
  - $\rho^+_T : \approx 2.9 \times 10^3$ (Chiral-odd)
  - $\pi^+ : \approx 9.0 \times 10^3$
- With $160 < S_{\gamma N} < 20000$, probing $5 \cdot 10^{-5} < \xi < 5 \cdot 10^{-3}$:
  - $\rho^0_L : \approx 1.6 \times 10^3$
  - $\rho^+_L : \approx 1.2 \times 10^3$
  - $\pi^+ : \approx 6.5 \times 10^2$

Prospects at experiments

Why counting rates lower UPCs at LHC?





- Photon flux enhanced by a factor of Z^2, but drops rapidly with $S_{\gamma N} \Longrightarrow$ *Low luminosity not compensated by larger photon flux.*
- LHC great for high energy, but JLab better in terms of luminosity.
- Still, LHC gives us access to the small ξ region of GPDs!