Quark mass effects in double parton distributions.

a consistent treatment

February 17, 2023

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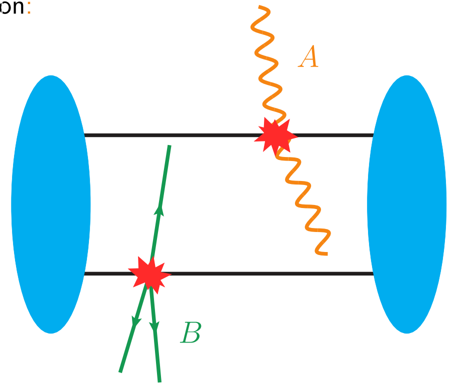
Part I

What is double parton scattering?

What is double parton scattering?

Double parton scattering (DPS) describes two individual hard interactions in a single hadron-hadron collision:

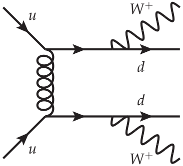


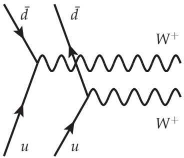
- Already observed at previous colliders at CERN and at the Tevatron.
- More data available from the LHC and more to come from HL-LHC.

DPS is naturally associated with the situation where the final state can be separated into two subsets with individual hard scales.

When is DPS relevant and why is it interesting?

- Whilst generally suppressed compared to single parton scattering (SPS), DPS may be enhanced for final states with small transverse momenta or large separation in rapidity.
- When production of final states via SPS involves small coupling constants or higher orders, DPS may give leading contributions (like-sign W production):





$\longrightarrow$ background to the search for new physics with like-sign lepton pairs.

- Relative importance of DPS increases with collision energy ($\sigma_{\text{DPS}} \sim \text{PDF}^4$ vs. $\sigma_{\text{SPS}} \sim \text{PDF}^2$).
- DPS gives access to information about hadron structure not accessible in other processes: spatial, spin, and colour correlations between two partons.

Factorization for DPS.

Pioneering work already in the 80's:

LO factorisation formula based on a parton model picture [Politzer, 1980; Paver and Treleani, 1982; Mekhfi, 1985]

$$\sigma_{pp\to A,B} = \hat{\sigma}_{ik\to A}(x_1\bar{x}_1 s)\,\hat{\sigma}_{jl\to B}(x_2\bar{x}_2 s) \times \int d^2\boldsymbol{y}\, F_{ij}(x_1, x_2, \boldsymbol{y}; Q_1^2, Q_2^2)\, F_{kl}(\bar{x}_1, \bar{x}_2, \boldsymbol{y}; Q_1^2, Q_2^2)$$

Increasing interest in DPS in the LHC era:

- Many experimental data already from previous colliders at CERN and Tevatron, new measurements from LHC with more to come in the HL phase.
- Progress also from theory:
  - Systematic QCD description. [Blok et al., 2011; Diehl et al., 2011; Manohar and Waalewijn, 2012; Ryskin and Snigirev, 2012]
  - Factorization proof for double DY. [Diehl, Gaunt, PP, Schäfer, 2015; Diehl and Nagar, 2019]
  - Disentangling SPS and DPS. [Gaunt and Stirling, 2011; Diehl, Gaunt and Schönwald, 2017]

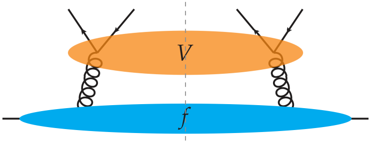
Part II

Small distance DPDs and quark mass effects.

Perturbative splitting in DPDs.

In the limit of small distance $\boldsymbol{y}$ the leading contribution to a DPD is due to the perturbative splitting of one parton into two and can be calculated in perturbation theory:

$$F_{a_1 a_2}(x_i, y, \mu) \overset{\boldsymbol{y}\to 0}{=} \frac{1}{\pi y^2} \left[V_{a_1 a_2, a_0}(y, \mu) \underset{12}{\otimes} f_{a_0}(\mu) \right] (x_i)$$



At LO the convolution reduces to a simple product:

$$F^{(1)}_{a_1 a_2}(x_i, y, \mu) \overset{\boldsymbol{y}\to 0}{=} \frac{a_s}{\pi y^2} V^{(1)}_{a_1 a_2, a_0} \left(\frac{x_1}{x_1 + x_2} \right) \frac{f_{a_0}(x_1 + x_2 \mu)}{x_1 + x_2}$$

with

$$V^{(1)}_{gg,g}(z) = 2\, C_A \left(\frac{\bar{z}}{z} + \frac{z}{\bar{z}} + z\bar{z} \right)$$

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formally OPE of
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Gaussian damping for numerical studies

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with

$$V^{(1)}_{q\bar{q},g}(z) = T_F\left(z^2 + \bar{z}^2\right)$$

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Gaussian damping for numerical studies

with

$$V^{(1)}_{qg,q}(z) = C_F \frac{1+z}{\bar{z}}$$

The “splitting scale”.

At which scale μ_{split} should the splitting be evaluated?

The natural scale of the splitting is set by the interparton distance y of the observed partons:

$$\mu_{\text{split}}(y) \sim \frac{1}{y}$$

In order to avoid evaluation of the splitting at non-perturbative scales for large y define:

$$\mu_{\text{split}}(y) = \frac{b_0}{y^*(y)}$$

with

$$y^*(y) = \frac{y}{\sqrt[4]{1 + y^4/y_{\max}^4}}, \qquad y_{\max} = \frac{b_0}{\mu_{\min}}$$

where y^* is adapted from b^* in TMD studies.

Small y splitting and massive quarks.

What happens when the scale at which the splitting is evaluated is similar to the mass of a heavy quark?

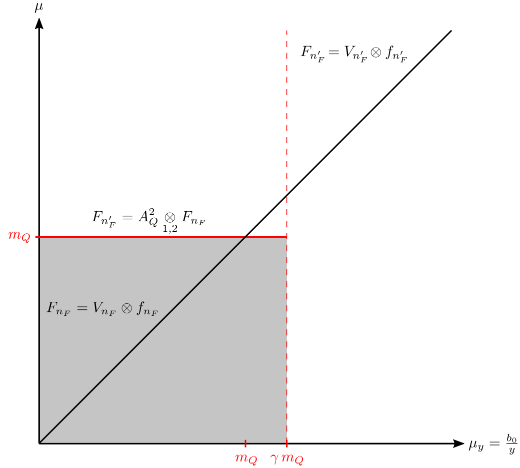
Should the heavy quark be treated as massless, massive, or absent in the evaluation of the splitting?

Consider and compare in the following two different schemes:

- purely massless scheme:
  - heavy quarks treated as decoupling for $\mu_{\text{split}} \lesssim m_Q$,
  - heavy quarks treated as massless for $\mu_{\text{split}} \gtrsim m_Q$.
- "massive" scheme:
  - heavy quarks treated as decoupling for $\mu_{\text{split}} \ll m_Q$,
  - heavy quarks treated as massive for $\mu_{\text{split}} \sim m_Q$,
  - heavy quarks treated as massless for $\mu_{\text{split}} \gg m_Q$.

Purely massless quarks.

The simplest scheme to handle massive quarks is to treat them as absent below a certain scale and as massless above a certain scale.



- Below $\mu_y = \gamma\, m_Q$ the DPD is initialized for n_F massless flavours with a n_F flavour PDF.

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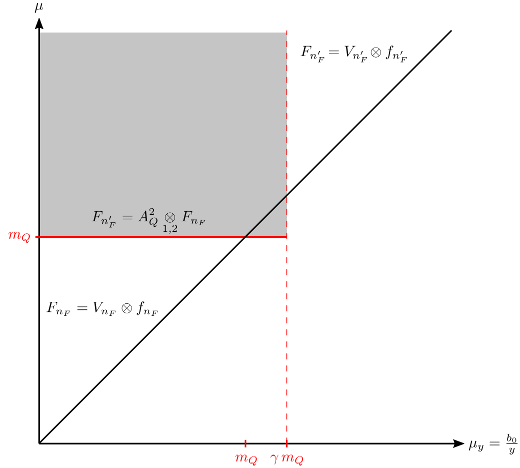

- Below $\mu_y = \gamma\, m_Q$ the $n_F + 1$ DPD is obtained by flavour matching.

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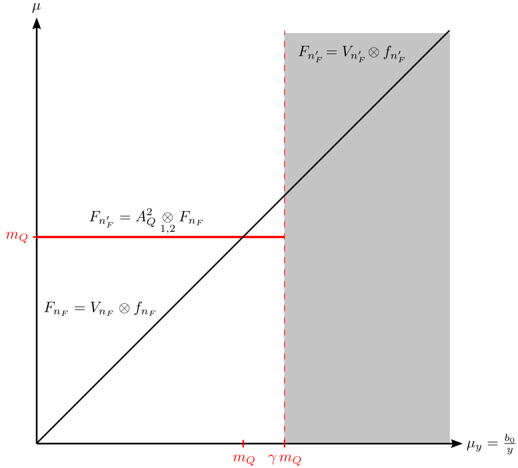
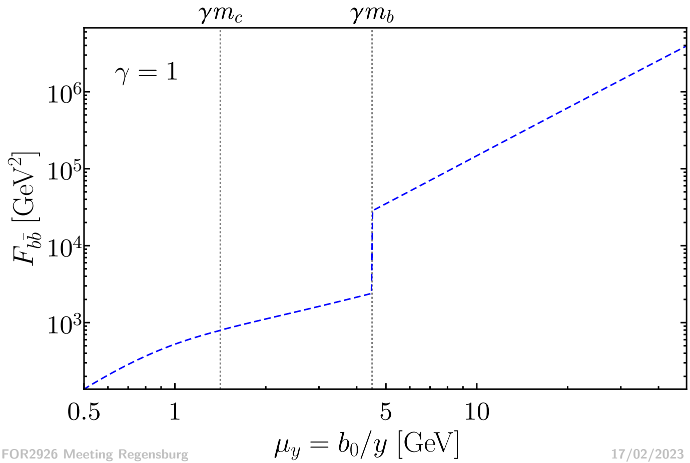


- Above $\mu_y = \gamma\, m_Q$ the DPD is initialized for $n_F + 1$ massless flavours with a $n_F + 1$ flavour PDF.

Purely massless quarks.

Consider $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ initialized with the scheme shown in the previous slide:

$$F_{b\bar{b}}(x_1 = x_2 = m_{\text{dijet}}/\sqrt{s},\ y,\ m_{\text{dijet}})$$

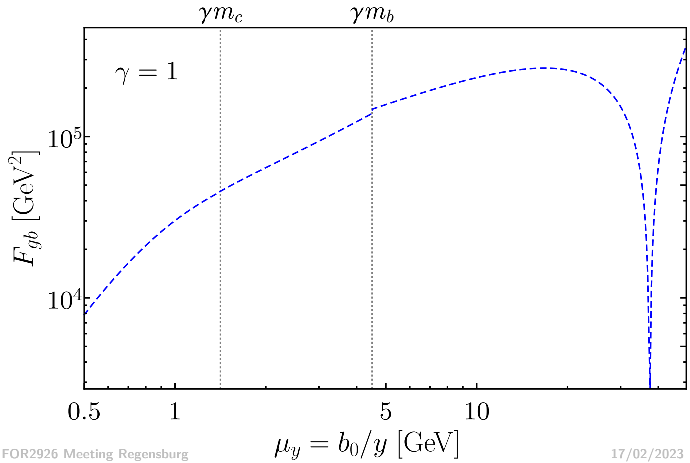


- Below $\mu_y = m_b$ the $b\bar{b}$ DPD is produced only by flavour matching and evolution.
- Above $\mu_y = m_b$ the $b\bar{b}$ DPD is produced by a direct (massless) $g \to q\bar{q}$ splitting.

Purely massless quarks.

Consider $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ initialized with the scheme shown in the previous slide:

$$F_{gb}(x_1 = x_2 = m_{\text{dijet}}/\sqrt{s},\ y,\ m_{\text{dijet}})$$

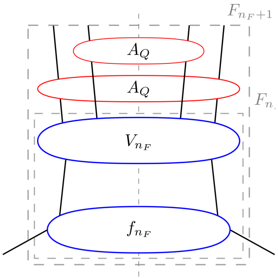


- At LO the gb DPD is produced by a direct splitting only for $\mu_y > \gamma m_b$.
- Heavy quark effects in the splitting seem to be unimportant.

A more realistic treatment of quark mass effects.

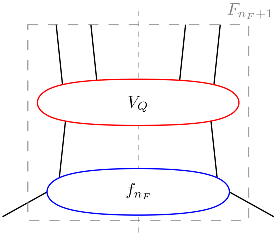
In the splitting DPDs one can distinguish three regions of μ_{split}:

$\mu_{\text{split}} \ll m_Q$:



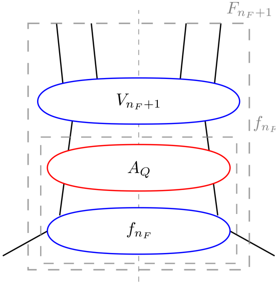
- In the splitting the heavy quarks decouple.
- $n_F + 1$ DPDs obtained by flavour matching.

$\mu_{\text{split}} \sim m_Q$:



- Heavy quarks treated as massive in the splitting kernel V_Q.

$\mu_{\text{split}} \gg m_Q$:



- Heavy quarks can be treated as massless in the splitting.

Massive DPD splitting kernels.

Just like the massless V_{n_F} kernels the massive V_Q kernels can be computed in perturbation theory!

At leading order the only splitting with massive quarks is $g \to Q\bar{Q}$, where the kernel reads:

$$V^{(1)}_{Q\bar{Q},g}(z_1, z_2, m_Q, y) = T_f\,(m_Q\,y)^2\left[(z_1^2+z_2^2)\,K_1^2(m_Q\,y) + K_0^2(m_Q\,y)\right]\,\delta(1-z_1-z_2)$$

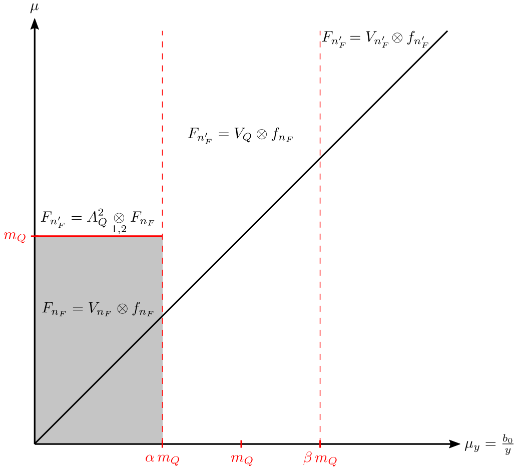
with the following limiting behaviour for small and large μ_{split} (corresponding to large and small $m_Q\,y$, respectively):

$$\begin{aligned}
\mu_{\text{split}} \ll m_Q: &\qquad V^{(1)}_{Q\bar{Q},g}(z, m_Q, y) \longrightarrow 0 \\
\mu_{\text{split}} \gg m_Q: &\qquad V^{(1)}_{Q\bar{Q},g}(z_1, z_2, m_Q, y) \longrightarrow T_f(z_1^2+z_2^2)\,\delta(1-z_1-z_2) = V^{(1)}_{q\bar{q},g}(z_1, z_2)
\end{aligned}$$

$\longrightarrow$ The massive kernel interpolates between the regions where the heavy quark decouples and where it can be treated as massless!

One heavy flavour.

Consider now the initialization of a splitting DPD with one heavy flavour (where $\alpha \ll 1$ and $\beta \gg 1$):



- Below $\mu_y = \alpha\, m_Q$ the DPD is initialized for n_F massless flavours with a n_F flavour PDF.

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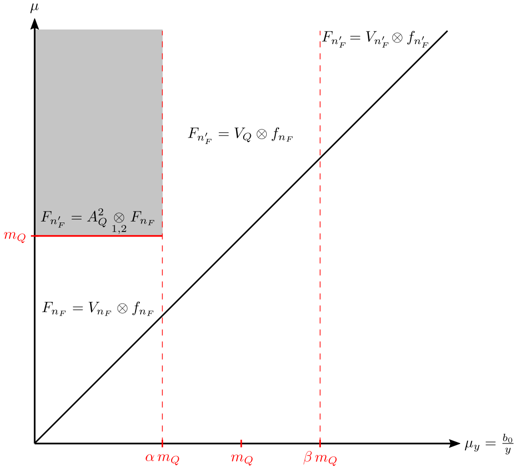

- Below $\mu_y = \alpha\, m_Q$ the $n_F + 1$ DPD is obtained by flavour matching.

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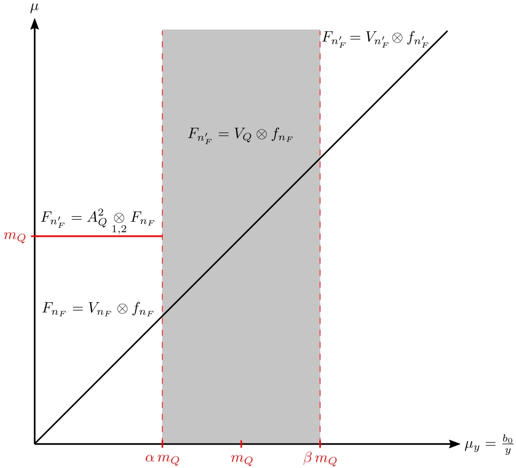

- For $\alpha\, m_Q < \mu_y < \beta\, m_Q$ the DPD is initialized for n_F massless and one massive flavours with a n_F flavour PDF.

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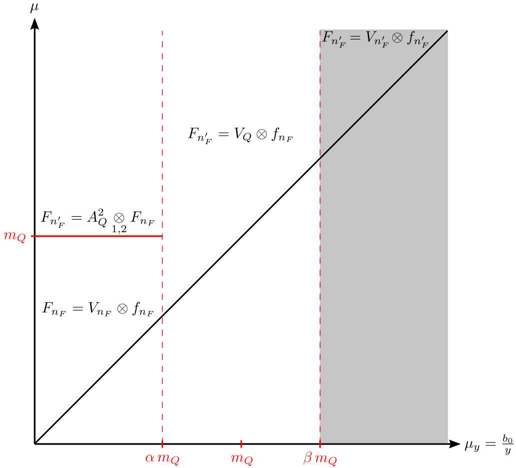

- Above $\mu_y = \beta\, m_Q$ the DPD is initialized for $n_F + 1$ massless flavours with a $n_F + 1$ flavour PDF.

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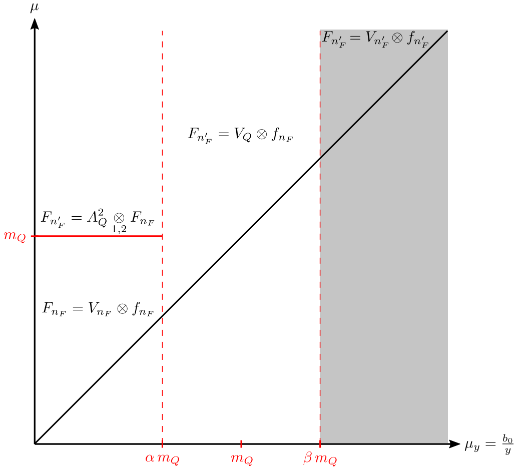


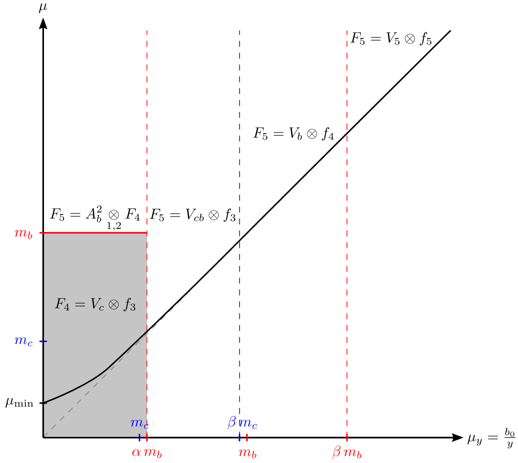
- Above $\mu_y = \beta\, m_Q$ the DPD is initialized for $n_F + 1$ massless flavours with a $n_F + 1$ flavour PDF.

What happens for charm and bottom which have to be treated as massive simultaneously?

Quark mass effects in the $1 \to 2$ splitting.

Two heavy flavours: charm and bottom.

Consider now the initialization of a splitting DPD with massive c and b quarks:



- Below $\mu_y = \alpha\, m_b$ the DPD is initialized for 3 massless and one heavy flavours with a 3 flavour PDF.

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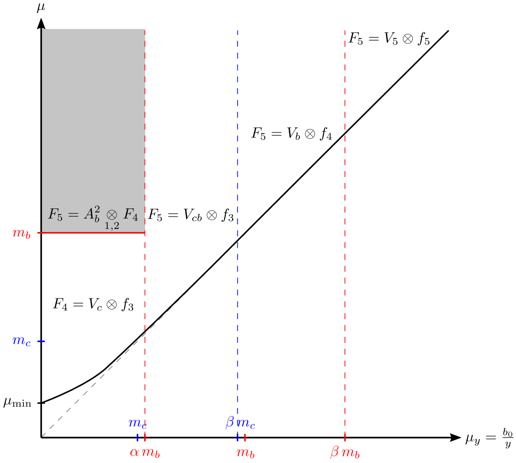

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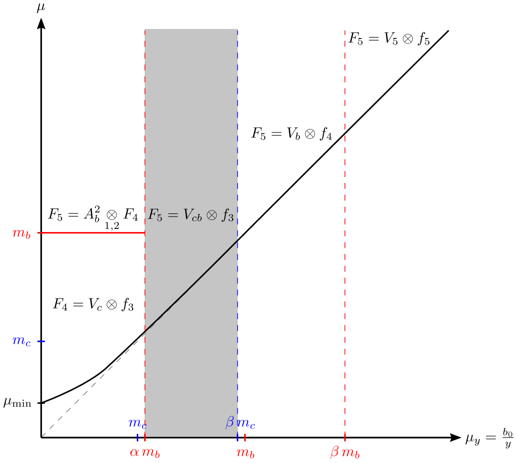

- For $\alpha\, m_b < \mu_y < \beta\, m_c$ the DPD is initialized for 3 massless and two massive flavours with a 3 flavour PDF.

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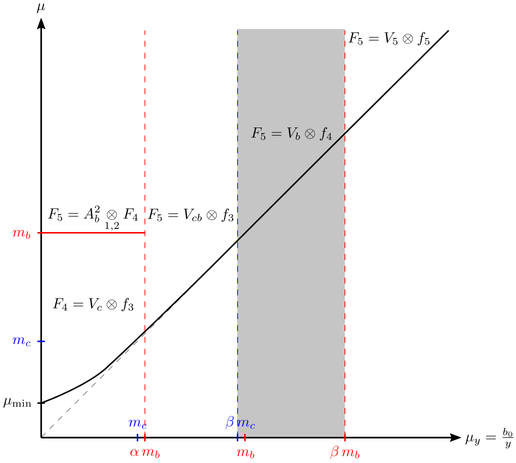

- For $\beta\, m_c < \mu_y < \beta\, m_b$ the DPD is initialized for 4 massless and one massive flavours with a 4 flavour PDF.

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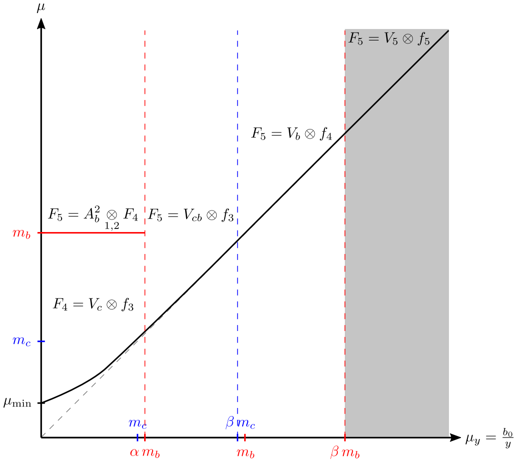

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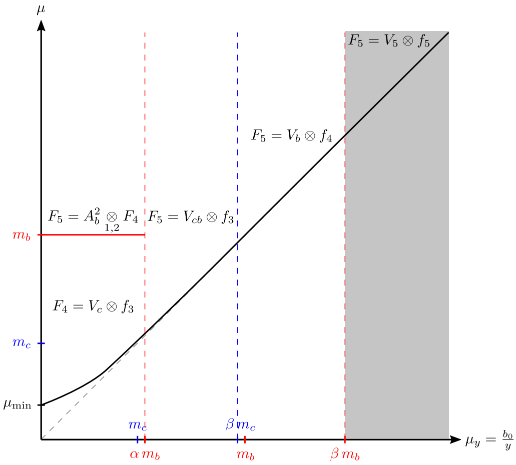

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Let's see how the DPDs look like in this scheme!

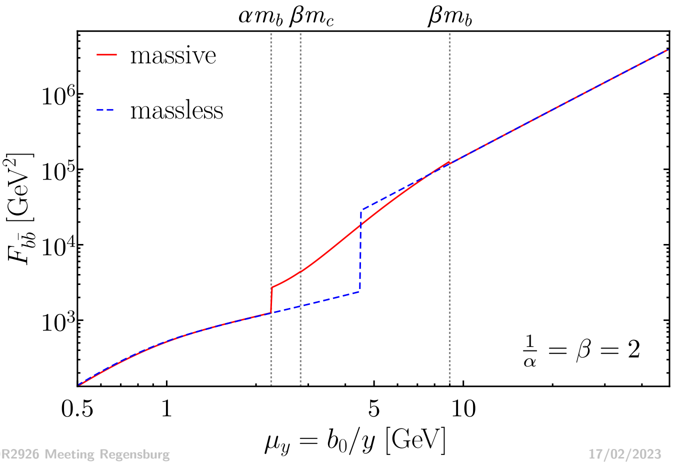
Part III

Numerical studies.

DPDs in the massive scheme.

Consider now $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ for dijet production, initialized with the scheme shown in the previous slide (for different α and β):

$$F_{b\bar{b}}(x_1 = x_2 = m_{\text{dijet}}/\sqrt{s},\ y,\ m_{\text{dijet}})$$



- DPDs still discontinuous, but greatly improved compared to the massless scheme!

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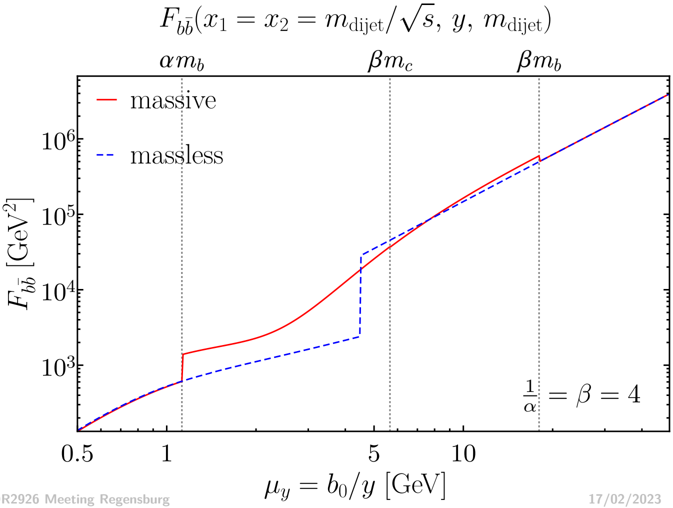

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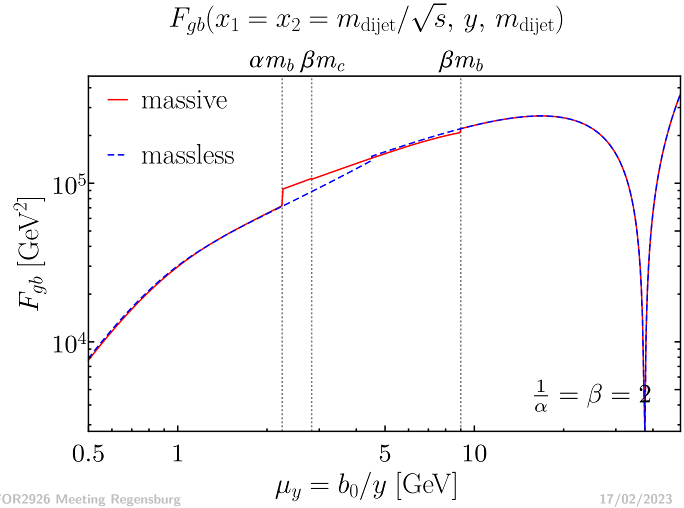

- Increased discontinuity for gb at $\mu_y = \alpha m_b$ due to direct production of $\bar{b}b$ DPD!
- Increased discontinuity for gb at $\mu_y = \beta m_b$ due to more production modes in the massless case!

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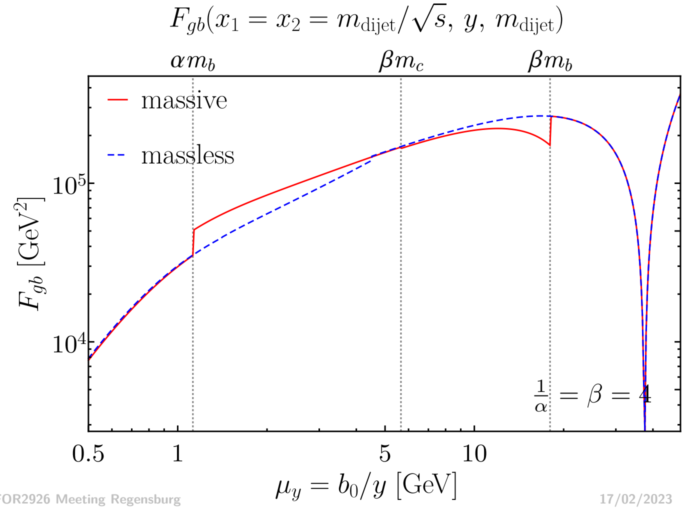

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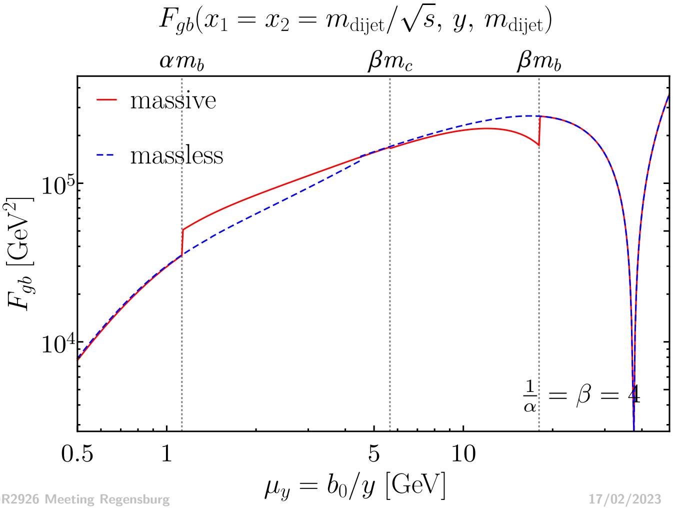
DPDs in the massive scheme.

Consider now $n_F = 5$ LO splitting DPDs at $\mu_1 = \mu_2 = m_{\text{dijet}} = 25\,\text{GeV}$ for dijet production, initialized with the scheme shown in the previous slide (for different α and β):



- Smallest discontinuities for $\beta = 2$ and $\alpha = \frac{1}{4}$!
- Seen also in other DPDs and at different scales.

DPD luminosities.

In order to study the effect of heavy quarks on DPS cross sections, consider DPD luminosities, i.e. products of DPDs integrated over y:

$$\mathcal{L}_{a_1 a_2 b_1 b_2}(x_{1a}, x_{2a}, x_{1b}, x_{2b}; \nu, \mu_1, \mu_2) = \int\limits_{b_0/\nu} \mathrm{d}^2\boldsymbol{y}\, F_{a_1 a_2}(x_{1a}, x_{2a}, y; \mu_1, \mu_2) F_{b_1 b_2}(x_{1b}, x_{2b}, y; \mu_1, \mu_2)$$

where the lower cut-off regulates the y^{-4} splitting singularity.

Here we include also “intrinsic” non-splitting contributions to the DPDs, modelled as:

$$F^{\text{int}}_{a_1 a_2}(x_1, x_2, y; \mu_1, \mu_2) = \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^2 (1 - x_2)^2} \frac{\exp\left(-\frac{y^2}{4h_{a_1 a_2}}\right)}{4\pi h_{a_1 a_2}} f_{a_1}(x_1, \mu_1)\, f_{a_2}(x_2, \mu_2)$$

In the following all possible combinations containing splitting DPDs are considered:

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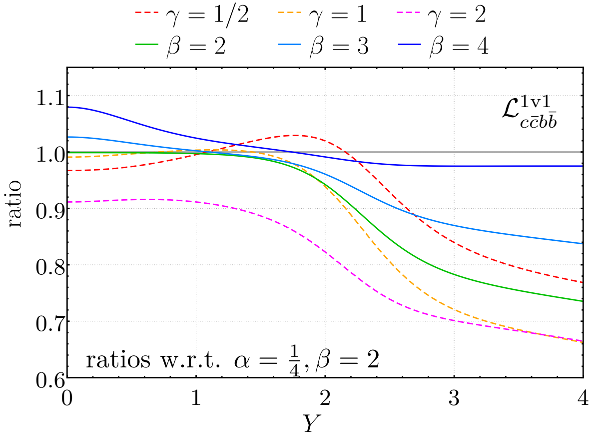
split x split (1v1), split x int (1v2), int x split (2v1).

DPD luminosities in the massive scheme.

Consider now ratios of LO DPD luminosities for dijet production with different scheme parameters:



Jets at rapidities Y and $-Y$:

$$x_{1a} = \frac{m_{\text{dijet}}}{\sqrt{s}} \exp(Y)$$

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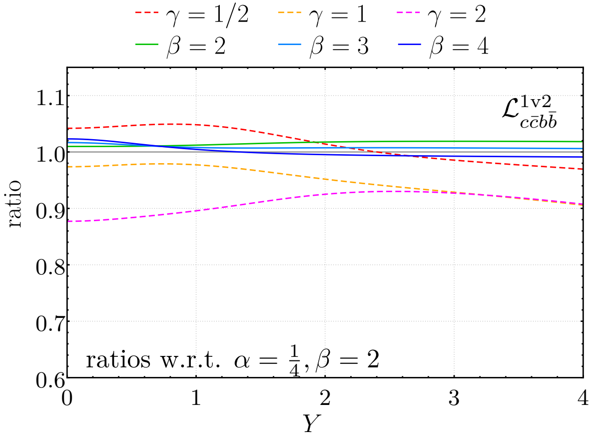

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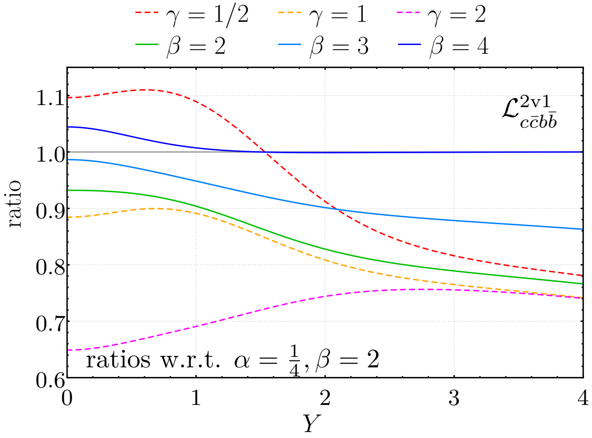

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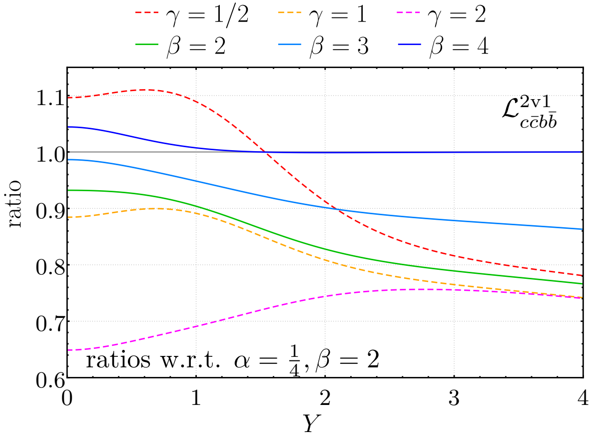

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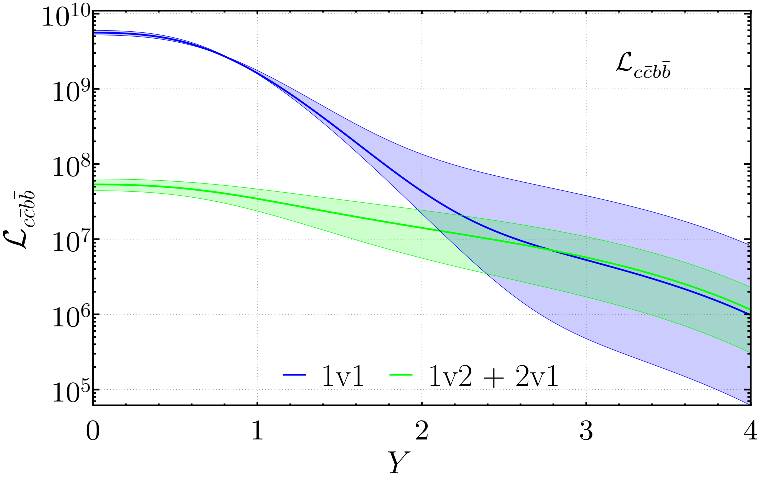
$\longrightarrow$ Smaller dependence of luminosities on α and β compared to γ!

DPD luminosities in the massive scheme: Scale dependence.

Finally consider the dependence of DPD luminosities involving LO splitting DPDs on the scale μ_{split} (varied by a factor of 2):



- Note that the 1v1 luminosities contain the squared uncertainties of the splitting DPDs!

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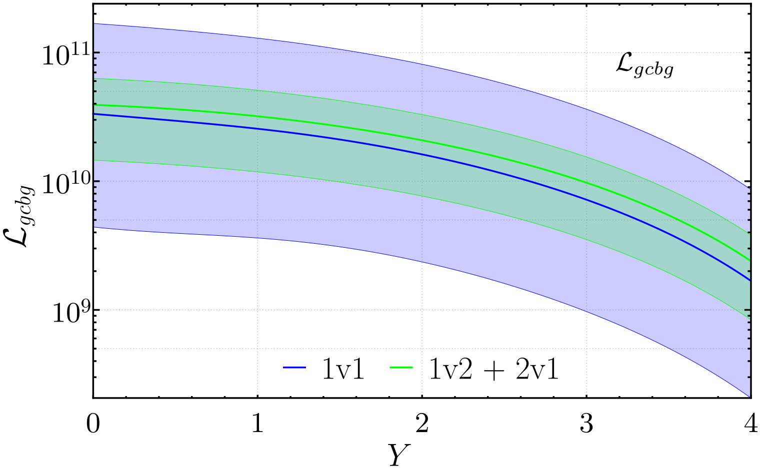

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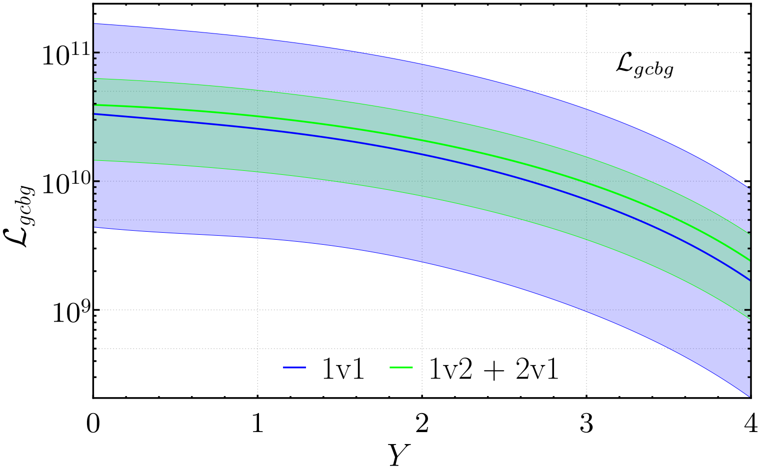

- Large scale uncertainties hint at importance of higher order splitting!

DPD luminosities in the massive scheme: Scale dependence.

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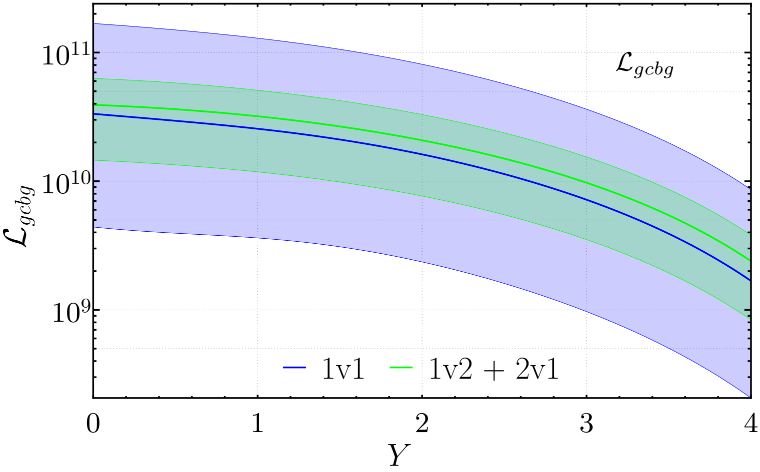

- Massless NLO kernels already calculated!
  [Diehl, Gaunt, PP, Schäfer, 2019; Diehl, Gaunt, PP, 2021]

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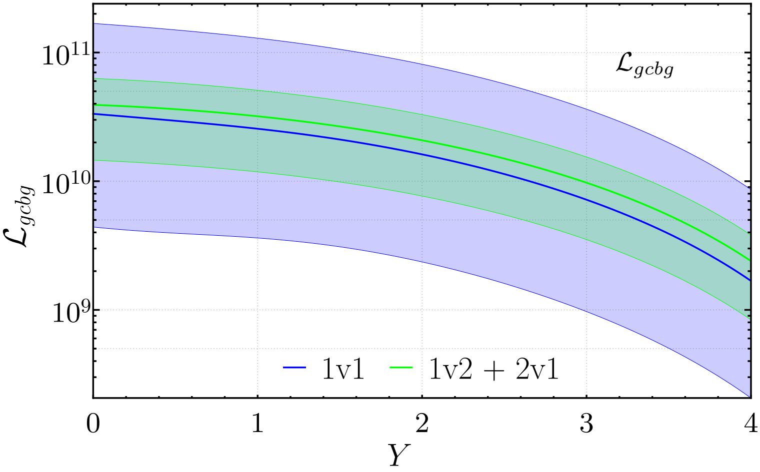

- Massive NLO kernels still unknown!

Part IV

Massive NLO kernels.

Constraints for the massive NLO kernels.

For now a full calculation of the massive NLO kernels is out of reach for us (involves massive two-loop diagrams).

$\longrightarrow$ construct approximate solutions!

To this end make use of the following constraints:

- RGE dependence of the massive kernels.
- Small and large distance limits of the massive kernels.
- DPD number and momentum sum rules.

The limiting behaviour and RGE dependence are uniquely fixed by these constraints, while the DPD sum rules constrain also intermediate inter parton distances!

RGE dependence of the massive NLO kernels.

The RGE dependence of the massive NLO kernels is completely fixed by LO perturbative ingredients:

Scale dependence of the massive NLO kernels:

$$\begin{aligned}\frac{\mathrm{d}}{\mathrm{d}\log\mu^2} V^{Q,n_F(2)}_{a_1a_2,a_0} &= \sum_{b_1} P^{n_F+1(0)}_{a_1b_1} \underset{1}{\otimes} V^{Q(1)}_{b_1a_2,a_0} + \sum_{b_2} P^{n_F+1(0)}_{a_2b_2} \underset{2}{\otimes} V^{Q(1)}_{a_1b_2,a_0} \\ &\quad - \sum_{b_0} V^{Q(1)}_{a_1a_2,b_0} \underset{12}{\otimes} P^{n_F(0)}_{b_0a_0} + \frac{\beta_0^{n_F+1}}{2} V^{Q(1)}_{a_1a_2,a_0} \\ &= v^{n_F,\,\mathrm{RGE}}_{a_1a_2,a_0}\end{aligned}$$

where the $V^{Q(1)}$ are the massive LO kernels and the $P^{n_F(0)}_{ab}$ are the LO DGLAP kernels.

Limiting behaviour of the massive NLO kernels.

For small and large interparton distances the massive kernels can be expressed in terms of convolutions of massless kernels and flavour matching kernels:

Small distance limit:

$$V^{Q,n_F(2)}_{a_1a_2,a_0} \xrightarrow{y\to 0} \delta^{n_F}_{a_0 l}\, V^{n_F+1(2)}_{a_1a_2,a_0} + \sum_{b_0} V^{n_F+1(1)}_{a_1a_2,b_0} \underset{12}{\otimes} A^{Q(1)}_{b_0a_0}\,,$$

Large distance limit:

$$V^{Q,n_F(2)}_{a_1a_2,a_0} \xrightarrow{y\to\infty} V^{n_F(2)}_{a_1a_2,a_0} + \sum_{b_1} A^{Q(1)}_{a_1b_1} \underset{1}{\otimes} V^{(1)}_{b_1a_2,a_0} + \sum_{b_2} A^{Q(1)}_{a_2b_2} \underset{2}{\otimes} V^{(1)}_{a_1b_2,a_0} + A^{Q(1)}_{\alpha}\, V^{(1)}_{a_1a_2,a_0}\,.$$

Sum rules for the massive NLO kernels.

The Gaunt-Stirling DPD sum rules can be used to derive sum rules for the massive kernels:

Momentum sum rule:

$$\sum_{a_2} \int_2 X_2 \int_{y_\beta}^{y_\alpha} \mathrm{d}^2 y \, V^{Q,n_F(2)}_{a_1 a_2, a_0} = (1 - X) \, A^{Q(2)}_{a_1 a_0}$$

$$+ \sum_{a_2} \int_2 X_2 \left[U^{n_F(2)}_{a_1 a_2, a_0}(r_\alpha) - U^{n_F+1(2)}_{a_1 a_2, a_0}(r_\beta) \right] + A^{(1)}_\alpha \sum_{a_2} \int_2 X_2 \, U^{(1)}_{a_1 a_2, a_0}(r_\alpha)$$

$$+ \sum_{b_1, a_2} A^{Q(1)}_{a_1 b_1} \underset{1}{\otimes} \left(\int_2 X_2 \, U^{(1)}_{b_1 a_2, a_0}(r_\alpha) \right) - \sum_{a_2, b_0} \left(\int_2 X_2 \, U^{(1)}_{a_1 a_2, b_0}(r_\beta) \right) \otimes \left(X A^{Q(1)}_{b_0 a_0} \right)$$

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The Gaunt-Stirling DPD sum rules can be used to derive sum rules for the massive kernels:

Number sum rule:

$$\int_2 \int_{y_\beta}^{y_\alpha} \mathrm{d}^2 y \frac{1}{\pi y^2} V^{Q,n_F(2)}_{a_1 a_{2_v}, a_0} = \left(\delta_{a_1 \bar{a}_2} - \delta_{a_1 a_2} - \delta_{a_2 \bar{a}_0} + \delta_{a_2 a_0}\right) A^{Q(2)}_{a_1 a_0}$$
$$+ \int_2 \left[U^{n_F(2)}_{a_1 a_{2_v}, a_0}(r_\alpha) - U^{n_F+1(2)}_{a_1 a_{2_v}, a_0}(r_\beta) \right] + A^{(1)}_\alpha \int_2 U^{(1)}_{a_1 a_{2_v}, a_0}(r_\alpha)$$
$$+ \sum_{b_1} A^{Q(1)}_{a_1 b_1} \otimes \left(\int_2 U^{(1)}_{b_1 a_{2_v}, a_0}(r_\alpha) \right) - \sum_{b_2} \left(\int_2 U^{(1)}_{a_1 a_{2_v}, b_0}(r_\beta) \right) \otimes A^{Q(1)}_{b_0 a_0}$$

Ansatz for the massive NLO kernels.

The following ansatz fulfils the RGE and limiting behaviour constraints:

$$
\begin{aligned}
V^{Q,n_F(2)}_{a_1a_2,a_0} = {} & V^{n_F[2,0]}_{a_1a_2,a_0} + V^{n_F[2,1]}_{a_1a_2,a_0} \log\frac{m_Q^2}{\mu_y^2} + k_{00}(y\,m_Q)\, v^{n_F,I}_{a_1a_2,a_0}(z_1,z_2) \\
& + k_{11}(ym_Q)\left(V^{n_F+1[2,0]}_{a_1a_2,a_0} - V^{n_F[2,0]}_{a_1a_2,a_0}\right) - k_{02}(ym_Q)\left(V^{n_F+1[2,1]}_{a_1a_2,a_0} - V^{n_F[2,1]}_{a_1a_2,a_0}\right) \\
& + \log\frac{\mu^2}{m_Q^2}\, v^{n_F,\mathrm{RGE}}_{a_1a_2,a_0}(z_1,z_2)\,,
\end{aligned}
$$

where

$$
k_{ij}(w) = w^2 K_i(w) K_j(w)\,.
$$

$\longrightarrow$ Sum rules can be used to constrain $v^{n_F,I}_{a_1a_2,a_0}$!

Part V

Summary.

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At small interparton distances y DPDs can be matched onto PDFs with perturbative $1 \to 2$ splitting kernels:

- Splitting evaluated at $\mu_{\text{split}} \sim 1/y$.
- For $\mu_{\text{split}} \sim m_Q$ quark mass effects have to be taken into account!

Consistent treatment of quark mass effects:

- Heavy quark decouples for $\mu_{\text{split}} \ll m_Q$.
- Heavy quark treated as massive for $\mu_{\text{split}} \sim m_Q$.
- Heavy quark treated as massless for $\mu_{\text{split}} \gg m_Q$.

Including quark mass effects leads to DPDs with smaller discontinuities and stabilizes DPD luminosities compared to the purely massless case!

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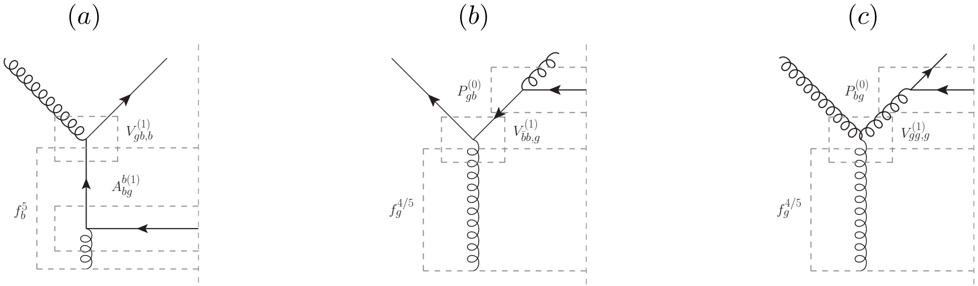
Thank you for your attention!

Part VI

Backup.

F_{gb}: massless vs. massive scheme



(a)
- Only contributes in the massless scheme.
- DPD produced by direct splitting, no evolution necessary.

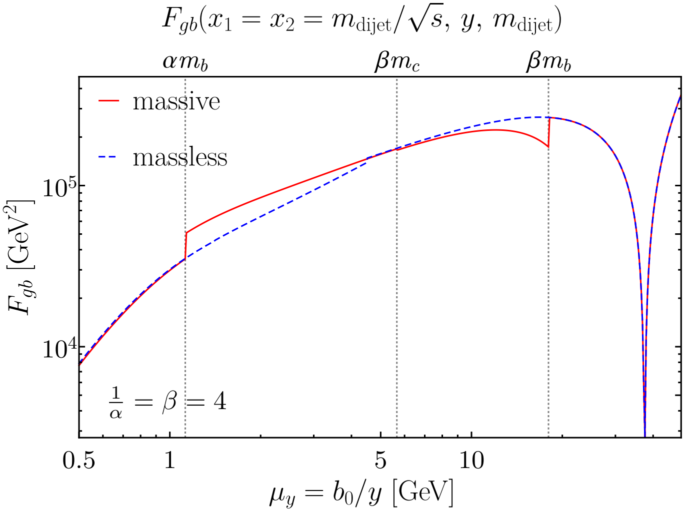
(b)
- Contributes in the massive and massless schemes.
- DPD only produced by evolution.

(c)
- Contributes in the massive and massless schemes.
- DPD only produced by evolution.

- Contributions (b) and (c) vanish when the splitting scale is identical to the target scale!

Quark mass effects in the $1 \to 2$ splitting.

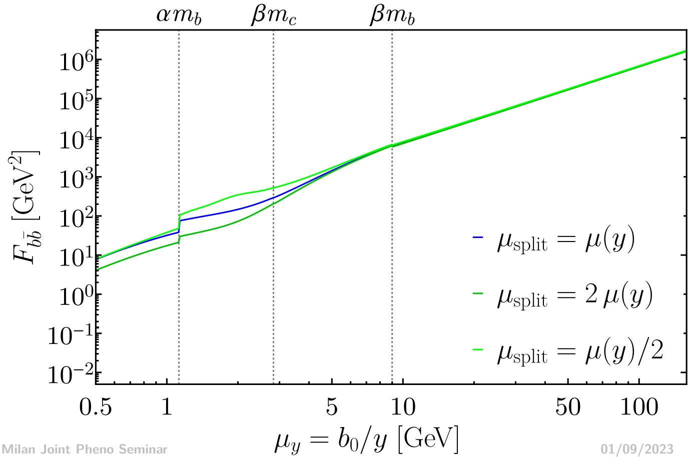
F_{gb}: massless vs. massive scheme



Scale dependence of splitting DPDs: in depth.

In order to understand the μ_{split} dependence of LO DPD luminosities involving $q\bar{q}$ DPDs consider the scale variation of the involved DPDs ($x_1 = \frac{m_W}{\sqrt{s}} \exp Y$, $x_2 = \frac{m_W}{\sqrt{s}} \exp -Y$):

Central rapidity ($Y = 0$):



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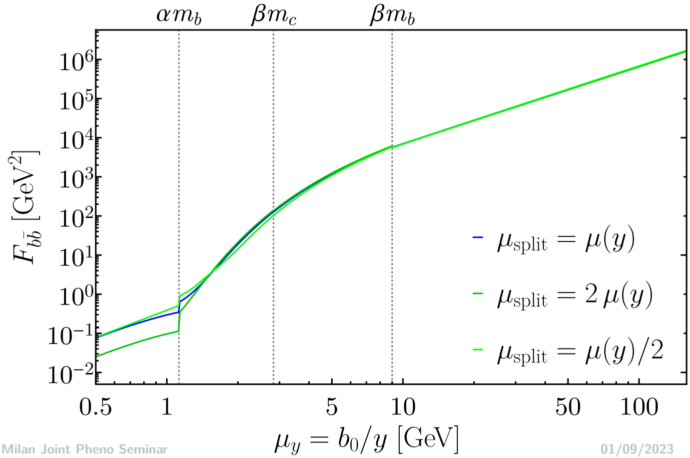
Central rapidity ($Y = 0$), only $g \to q\bar{q}$ splitting:



- Contribution from $g \to gg$ and $q \to qg, gq$ splitting and evolution negligible for central rapidity ($x_1 = x_2$).
- Only scale variation from initial gluon PDF.

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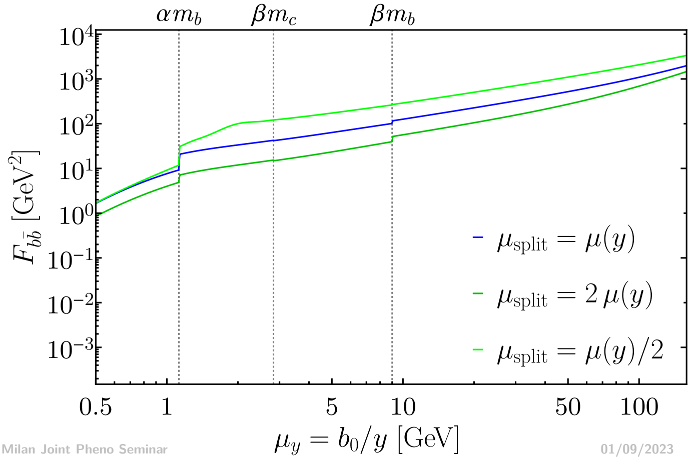
Non-central rapidity ($Y = 3$):



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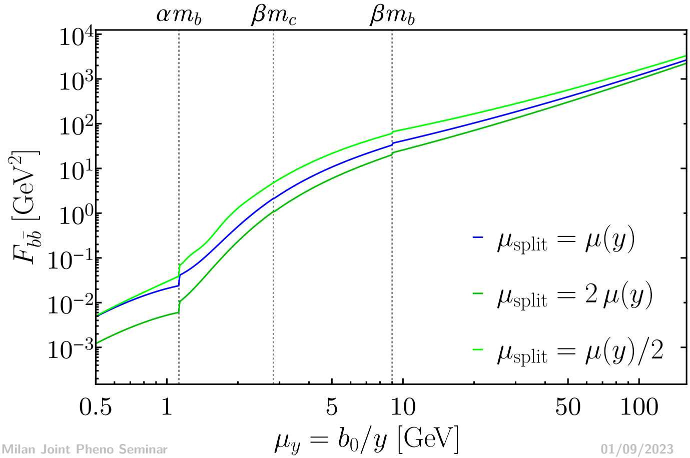
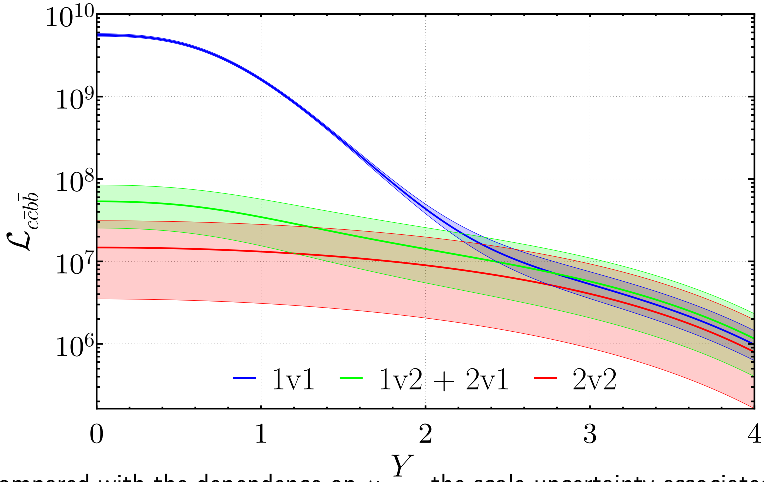
Non-central rapidity ($Y = 3$), only $g \to q\bar{q}$ splitting:



- Sizeable contribution from $g \to gg$ and $q \to qg, gq$ splitting and evolution for non-central rapidity ($x_1 \ll x_2$).
- In addition to scale variation from initial gluon PDF also uncertainties from evolution.

DPD luminosities in the massive scheme: Matching scale dependence.

Finally consider the dependence of LO DPD luminosities for dijet production on the flavour matching scales (at LO, varied by a factor of 2):



$$x_{1a} = \frac{m_W}{\sqrt{s}} \exp(Y)$$

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Compared with the dependence on μ_{split} the scale uncertainty associated with flavour matching is small!

Quark mass effects in the $1 \to 2$ splitting.

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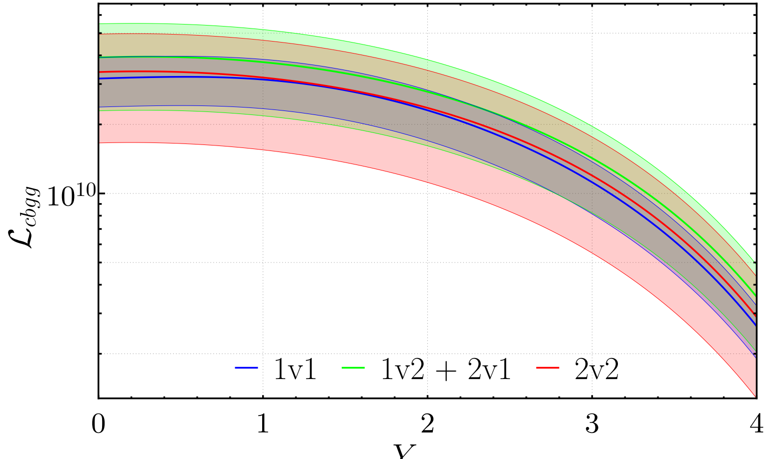

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