Chiral and trace anomalies in Compton scattering

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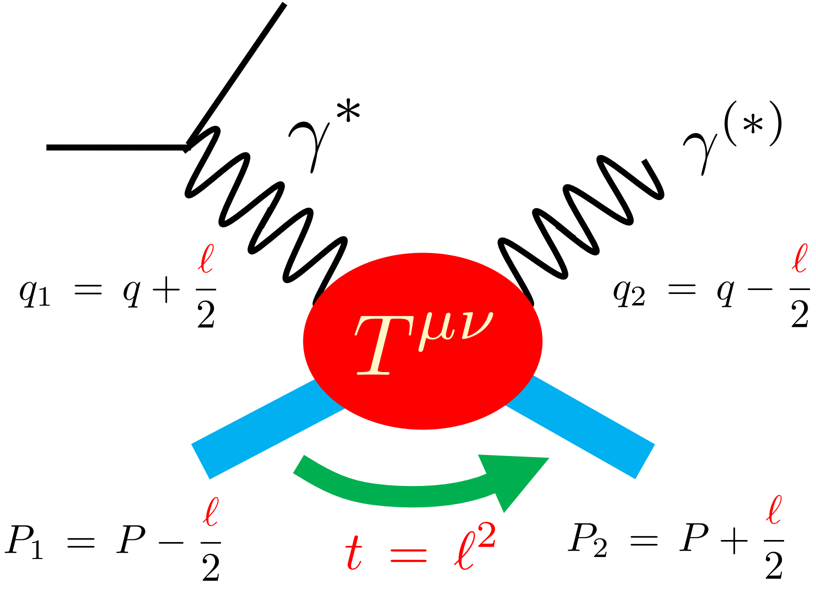
Outline:

- The main observation
- Implications & possible remedy
- Relation to axial anomaly
- Symmetric case and trace anomaly

The main observation



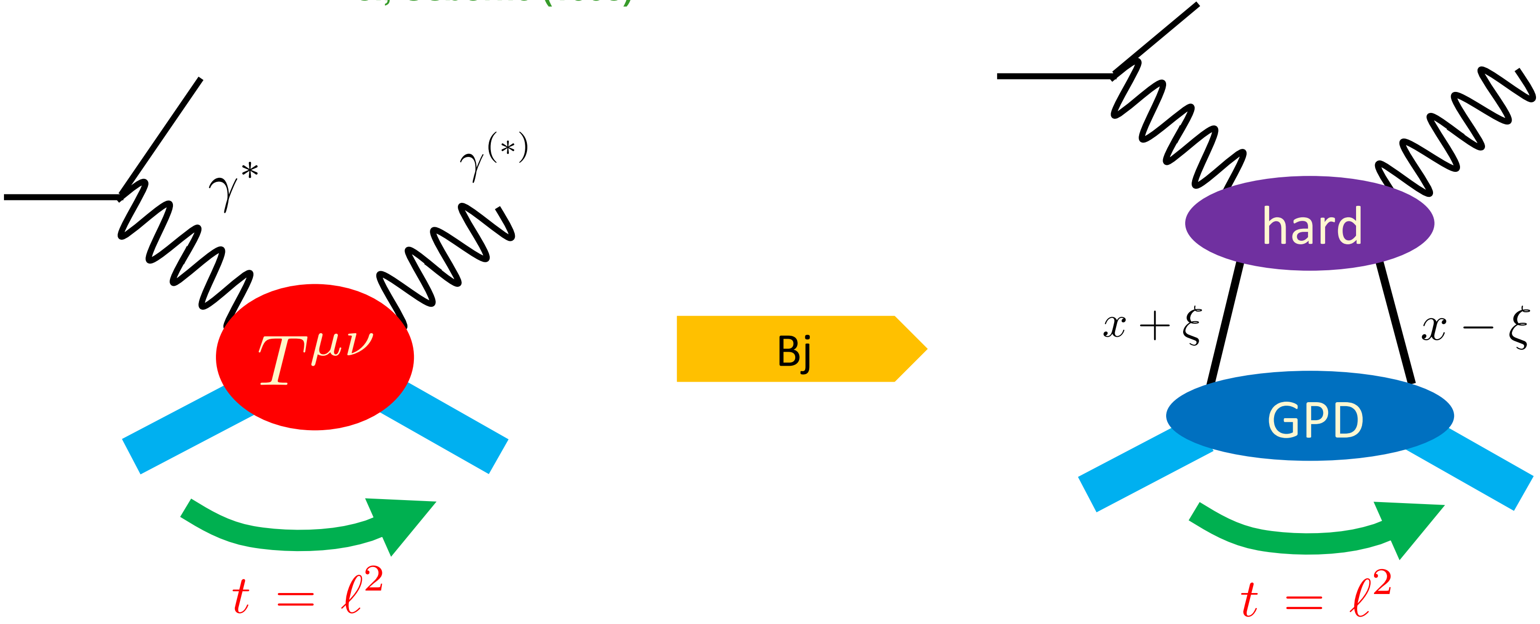
$$x_B \equiv \frac{-q^2}{2P \cdot q} \equiv \frac{Q^2}{2P \cdot q}$$

$$\xi \equiv \frac{q_2^2 - q_1^2}{4P \cdot q} = \frac{-q \cdot \ell}{2P \cdot q}$$

$$T^{\mu\nu} = T^{\mu\nu}(x_B, \xi, t, Q^2)$$

Factorization

Collins, Freund;
Ji, Osborne (1998)



$$T^{\mu\nu}(x_B, \xi, t, Q^2) = \sum_{a=q,\bar{q},g} \int \frac{dx}{x} \mathcal{C}_a^{\mu\nu}\left(\frac{x_B}{x}, \frac{\xi}{x}, \alpha_s(Q^2)\right) f_a(x, \xi, t, Q^2) + \text{P.C.}$$

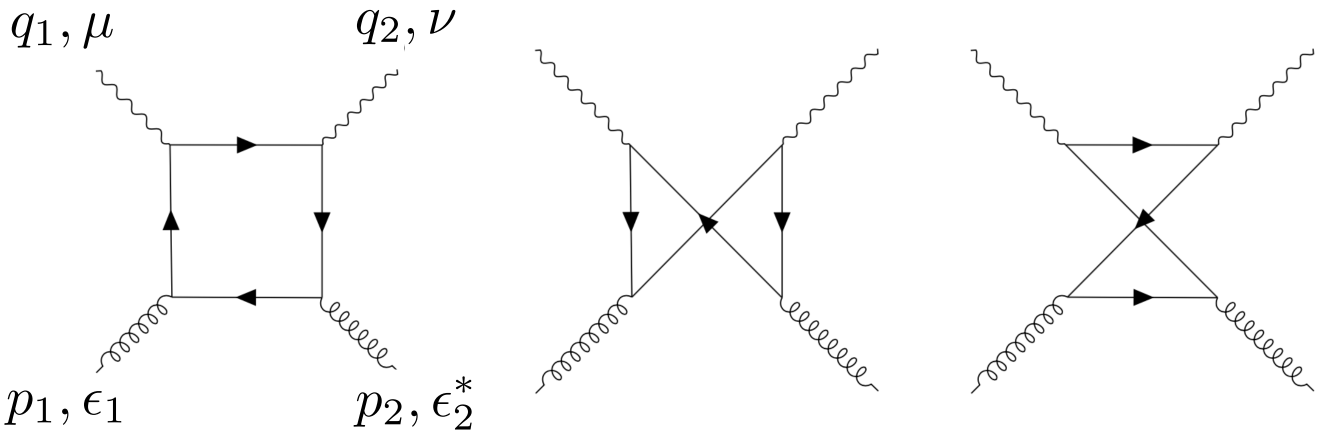
e.g. $\left(\frac{\sqrt{-t}}{Q}\right)^k, \left(\frac{m_p}{Q}\right)^k$

Braun, Manashov, Pirnay

→ extraction of GPDs

$$T^{\mu\nu}(x_B, \xi, t, Q^2) = \sum_{a=q,\bar{q},g} \int \frac{dx}{x} \underbrace{\mathcal{C}_a^{\mu\nu}\left(\frac{x_B}{x}, \frac{\xi}{x}, \alpha_s(Q^2)\right)}_{\text{pQCD}} f_a(x, \xi, t, Q^2) + \text{P.C.}$$

now look at NLO corrections associated with incoming gluons:



let's consider imaginary part of antisymmetric part of $T^{\mu\nu}$

First, entirely forward kinematics: $\xi = 0,\ \ell^2 = 0$

→ imaginary part of antisymmetric tensor gives contribution to g_1

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \,\mathrm{Im} T_{\mu\nu}\big|_{\mathrm{box}} = \frac{\alpha_s}{4\pi}\left(\sum_f e_f^2\right) \bar{u}(P_2)\left[\left(-\frac{1}{\varepsilon}\,\Delta P_{qg} + \delta C_g^{\overline{\mathrm{MS}}}\right)\otimes \Delta G(x_B)\,\gamma^\alpha\gamma_5\right] u(P_1)$$

standard coll. log

DIS coefficient function

Carlitz, Collins, Mueller
Ratcliffe; WV (~1990)

$$\delta C_g^{\overline{\mathrm{MS}}}(\hat{x}) = (2\hat{x}-1)\left[\ln\frac{1-\hat{x}}{\hat{x}} - 1\right] + 2(1-\hat{x})$$

Now do same with $\xi = 0$, $\ell^2 \neq 0$ (but $\ell^2 \to 0$ where possible):

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \, \mathrm{Im} T_{\mu\nu}\big|_{\rm box} = \frac{\alpha_s}{4\pi}\Big(\sum_f e_f^2\Big)\, \bar{u}(P_2)\left[\left(\Delta P_{qg} \ln\frac{Q^2}{-\ell^2} + \delta C_g^{\rm off}\right)\otimes \Delta G(x_B)\,\gamma^\alpha\gamma_5 + \frac{\ell^\alpha}{\ell^2}\,\delta C_g^{\rm anom}\otimes \tilde{\mathcal{F}}(x_B)\,\gamma_5\right] u(P_1)$$

Tarasov, Venugopalan '20

$$\delta C_g^{\rm off}(\hat{x}) = (2\hat{x}-1)\left[\ln\frac{1}{\hat{x}(1-\hat{x})} - 1\right] \qquad \delta C_g^{\rm anom}(\hat{x}) = 2(1-\hat{x})$$

$$\tilde{\mathcal{F}}(x,\ell^2) \equiv \frac{iP^+}{\bar{u}(P_2)\gamma_5 u(P_1)}\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\langle P_2|F_a^{\mu\nu}(-z^-/2)\tilde{F}^a_{\mu\nu}(z^-/2)|P_1\rangle$$

Tarasov, Venugopalan
Hatta
Radyushkin, Zhao

Why not seen before?

$$T^{\mu\nu}(x_B,\xi,t,Q^2) = \sum_{a=q,\bar{q},g} \int \frac{dx}{x}\, \mathcal{C}_a^{\mu\nu}\left(\frac{x_B}{x},\frac{\xi}{x},\alpha_s(Q^2)\right) f_a(x,\xi,t,Q^2) + \text{P.C.}$$

$\ell^2 = 0$ from start: $\quad \xi \approx \dfrac{-\ell^+}{2P^+} \rightarrow \ell = -2\xi P \qquad \varepsilon_1^{\rho}\varepsilon_2^{*\sigma} \rightarrow \epsilon^{\rho\sigma+-}$

→ known off-forward gluon NLO coefficient function

$$\begin{aligned}\text{Im}[\ldots] = &-\frac{1}{\epsilon}\,\frac{(2\hat{x}-1-\hat{\xi}^2)+2(\hat{\xi}-\hat{x})\,\Theta(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}\\ &+\frac{1}{1-\hat{\xi}^2}\left[(2\hat{x}-1-\hat{\xi}^2)\,\ln\frac{1-\hat{x}}{\hat{x}}+3-4\hat{x}+\hat{\xi}^2\right]\\ &+\frac{2\,(\hat{\xi}-\hat{x})\,\Theta(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}\left[\ln\frac{\hat{\xi}-\hat{x}}{\hat{x}}-2\right]\end{aligned}$$

Belitsky, Müller
Ji, Osborne
Pire, Szymanowski, Wagner

Implications and possible remedy

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \, \mathrm{Im} T_{\mu\nu}\big|_{\mathrm{box}} = \frac{\alpha_s}{4\pi} \Big(\sum_f e_f^2 \Big) \, \bar{u}(P_2) \Bigg[\Bigg(\Delta P_{qg} \ln \frac{Q^2}{-\ell^2} + \delta C_g^{\mathrm{off}} \Bigg) \otimes \Delta G(x_B) \, \gamma^\alpha \gamma_5 + \frac{\ell^\alpha}{\ell^2} \, \delta C_g^{\mathrm{anom}} \otimes \tilde{\mathcal{F}}(x_B) \, \gamma_5 \Bigg] u(P_1)$$

according to factorization of Compton amplitude:

$$-\epsilon^{\alpha\beta\mu\nu} P_\beta \, \mathrm{Im} T_{\mu\nu}\big|_{\mathrm{box}} = \frac{1}{2} \sum_f e_f^2 \, \bar{u}(P_2) \Big[\gamma^\alpha \gamma_5 \big(\tilde{H}_f(x_B, \xi, \ell^2) + \tilde{H}_f(-x_B, \xi, \ell^2) \big) + \frac{\ell^\alpha \gamma_5}{2M} \big(\tilde{E}_f(x_B, \xi, \ell^2) + \tilde{E}_f(-x_B, \xi, \ell^2) \big) \Big] u(P_1) + \mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

correction $\propto \dfrac{\langle F\tilde{F} \rangle}{\ell^2}$ to $\tilde{E}_f$ at one loop! (twist-4, but unsuppressed)

Compton amplitude must have finite forward limit!

$$-\epsilon^{\alpha\beta\mu\nu}P_\beta\,\mathrm{Im}T_{\mu\nu}\big|_{\mathrm{box}} = \frac{1}{2}\sum_f e_f^2\,\bar{u}(P_2)\Big[\gamma^\alpha\gamma_5\big(\tilde{H}_f(x_B,\xi,\ell^2)+\tilde{H}_f(-x_B,\xi,\ell^2)\big) + \frac{\ell^\alpha\gamma_5}{2M}\big(\tilde{E}_f^{\mathrm{bare}}(x_B,\xi,\ell^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\xi,\ell^2)\big)\Big]u(P_1)$$

tree

one-loop

$$\tilde{E}_f(x_B,\ell^2)+\tilde{E}_f(-x_B,\ell^2) = \tilde{E}_f^{\mathrm{bare}}(x_B,\ell^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\ell^2)+\frac{\alpha_s}{2\pi}\frac{2M}{\ell^2}\,\delta C_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B,\ell^2)$$

where $\tilde{E}_f^{\mathrm{bare}}(x_B,\ell^2)+\tilde{E}_f^{\mathrm{bare}}(-x_B,\ell^2) \approx -\frac{\alpha_s}{2\pi}\frac{2M}{\ell^2}\,\delta C_g^{\mathrm{anom}}\otimes\tilde{\mathcal{F}}(x_B,\ell^2=0)$

Does this make any sense?

GPDs related to singlet axial and pseudoscalar form factors:

$$\sum_f \int_0^1 dx\,\left(\tilde{H}_f(x,\xi,\ell^2) + \tilde{H}_f(-x,\xi,\ell^2)\right) \;=\; g_A(\ell^2)$$

$$\sum_f \int_0^1 dx\left(\tilde{E}_f(x,\xi,\ell^2) + \tilde{E}_f(-x,\xi,\ell^2)\right) \;=\; g_P(\ell^2)$$

$$= \tilde{E}_f^{\text{bare}}(x_B,\ell^2) + \tilde{E}_f^{\text{bare}}(-x_B,\ell^2) + \frac{\alpha_s}{2\pi}\frac{2M}{\ell^2}\,\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B,\ell^2)$$

$$\approx -\frac{\alpha_s}{2\pi}\frac{2M}{\ell^2}\,\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B,\ell^2=0) + \frac{\alpha_s}{2\pi}\frac{2M}{\ell^2}\,\delta C_g^{\text{anom}} \otimes \tilde{\mathcal{F}}(x_B,\ell^2)$$

obtain

$$\frac{g_P(\ell^2)}{2M} = -\frac{i}{\ell^2}\left(\left.\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right|_{\ell^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right)$$

finite at $\ell^2 = 0$

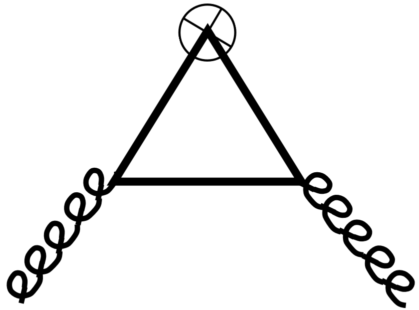
Relation to axial anomaly

g_A, g_P appear in matrix element of $J_5^\alpha = \sum_f \bar{\psi}_f \gamma^\alpha \gamma_5 \psi_f$:

$$\langle P_2 | J_5^\alpha | P_1 \rangle = \bar{u}(P_2) \left[\gamma^\alpha \gamma_5 \, g_A(\ell^2) + \frac{\ell^\alpha \gamma_5}{2M} \, g_P(\ell^2) \right] u(P_1) \qquad \ell = P_2 - P_1$$

anomaly: $\partial_\alpha J_5^\alpha = -\frac{n_f \, \alpha_s}{4\pi} F^{\mu\nu} \tilde{F}_{\mu\nu}$

triangle diagram:



$$\langle p_2 | J_5^\alpha | p_1 \rangle = \frac{n_f \alpha_s}{4\pi} \frac{i \ell^\alpha}{\ell^2} \langle p_2 | F \tilde{F} | p_1 \rangle$$

massless pole in $g_P(\ell^2)$ should cancel

$$\langle P_2|J_5^\alpha|P_1\rangle = \bar{u}(P_2)\left[\gamma^\alpha\gamma_5\, g_A(\ell^2) + \frac{\ell^\alpha\gamma_5}{2M}\, g_P(\ell^2)\right] u(P_1)$$

obtain

$$2M g_A(\ell^2) + \frac{\ell^2 g_P(\ell^2)}{2M} = i\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}$$

$$\frac{g_P(\ell^2)}{2M} = -\frac{i}{\ell^2}\left(\left.\frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right|_{\ell^2=0} - \frac{\langle P_2|\frac{n_f\alpha_s}{4\pi}F\tilde{F}|P_1\rangle}{\bar{u}(P_2)\gamma_5 u(P_1)}\right)$$

consistent as long as $g_A(\ell^2) \approx g_A(0) = \Delta\Sigma$

massless pole in g_P is cancelled (instead pole at $m^2_{\eta'}$)

Jaffe, Manohar
Tarasov, Venugopalan

Symmetric case and trace anomaly

$$\mathrm{Im}T^{\mu\nu}_{\mathrm{sym}}\Big|_{\mathrm{box}} \approx \left(-g^{\mu\nu}+\frac{q^\mu q^\nu}{q^2}\right)\bar{F}_1^{\mathrm{off}}(x_B,\xi,\ell^2)+\left(P^\mu-\frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu-\frac{P\cdot q}{q^2}q^\nu\right)\frac{2x_B\bar{F}_2^{\mathrm{off}}(x_B,\xi,\ell^2)}{Q^2}$$

where, e.g.

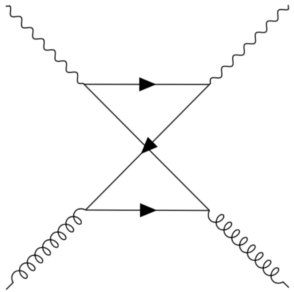
$$\bar{F}_2^{\mathrm{off}}(x_B,\xi,\ell^2) \approx x_B\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\left[\left(P_{qg}\ln\frac{Q^2}{-\ell^2}+C_{2g}^{\mathrm{off}}\right)\otimes g(x_B)+\frac{1}{\ell^2}C^{\mathrm{anom}}\otimes' \mathcal{F}(x_B,\xi,\ell^2)\,\frac{\bar{u}(P_2)u(P_1)}{2M}\right]$$

$$C^{\mathrm{anom}}\otimes' \mathcal{F}(x_B,\xi,\ell^2) \equiv \int_{x_B}^1\frac{dx}{x}\,\frac{\hat{x}(1-\hat{x})}{1-\hat{\xi}^2}\,\mathcal{F}(x,\xi,\ell^2)-\frac{\theta(\xi-x_B)}{2}\int_{-1}^1\frac{dx}{x}\,\frac{\hat{x}(\hat{\xi}-\hat{x})}{1-\hat{\xi}^2}\,\mathcal{F}(x,\xi,\ell^2)$$

twist-4 scalar GPD

Hatta, Zhao; Radyushkin, Zhao

$$\mathcal{F}(x,\xi,\ell^2) = -4xP^+M\int\frac{dz^-}{2\pi}e^{ixP^+z^-}\frac{\langle P_2|F^{\mu\nu}(-z^-/2)F_{\mu\nu}(z^-/2)|P_1\rangle}{\bar{u}(P_2)u(P_1)}$$



According to factorization:

$$\mathrm{Im}T^{\mu\nu}_{\mathrm{sym}} = \sum_f \frac{e_f^2}{4P^+}\left[\frac{4x_B^2}{Q^2}\left(P^\mu - \frac{P\cdot q}{q^2}q^\mu\right)\left(P^\nu - \frac{P\cdot q}{q^2}q^\nu\right) - g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right]$$

$$\times\,\bar{u}(P_2)\left[\big(H_f(x_B,\xi,\ell^2) - H_f(-x_B,\xi,\ell^2)\big)\gamma^+ + \frac{i\sigma^{+\lambda}\ell_\lambda}{2M}\big(E_f(x_B,\xi,\ell^2) - E_f(-x_B,\xi,\ell^2)\big)\right]u(p_1)$$

$$+\,\mathcal{O}(\alpha_s) + \mathcal{O}(1/Q^2)$$

$$\sum_f e_f^2\, x_B\big(H_f(x_B,\xi,\ell^2) - H_f(-x_B,\xi,\ell^2)\big) = \sum_f e_f^2\, x_B\big(H_f^{\mathrm{bare}}(x_B,\xi,\ell^2) - H_f^{\mathrm{bare}}(-x_B,\xi,\ell^2)\big)$$

$$+\,\frac{\alpha_s}{2\pi}\Big(\sum_f e_f^2\Big)\frac{x_B}{\ell^2}\,C^{\mathrm{anom}}\otimes'\mathcal{F}(x_B,\xi,\ell^2)$$

(likewise for E_f)

GPDs related to gravitational form factors:

$$\int_0^1 dx_B\, x_B\, \left(H_f(x_B,\xi,\ell^2) - H_f(-x_B,\xi,\ell^2)\right) \;=\; A_f(\ell^2) + \xi^2 D_f(\ell^2)$$

$$\int_0^1 dx_B\, x_B\, \left(E_f(x_B,\xi,\ell^2) - E_f(-x_B,\xi,\ell^2)\right) \;=\; B_f(\ell^2) - \xi^2 D_f(\ell^2)$$

$$= \sum_f e_f^2\, x_B \left(H_f^{\text{bare}}(x_B,\xi,\ell^2) - H_f^{\text{bare}}(-x_B,\xi,\ell^2)\right) \,+\, \frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\frac{x_B}{\ell^2}\, C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,\ell^2)$$

$$\approx\, -\frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\frac{x_B}{\ell^2}\, C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,\ell^2=0) \,+\, \frac{\alpha_s}{2\pi}\left(\sum_f e_f^2\right)\frac{x_B}{\ell^2}\, C^{\text{anom}} \otimes' \mathcal{F}(x_B,\xi,\ell^2)$$

(again, likewise for E_f)

obtain

$$\sum_f e_f^2\left(A_f(\ell^2)+\xi^2 D_f(\ell^2)\right) \approx \frac{T_R\alpha_s}{12\pi\ell^2}\left(\sum_f e_f^2\right)\left[\left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2F_{\alpha\beta}|P\rangle}{(P^+)^2}+\xi^2\langle P|F^2|P\rangle\right)-\left(\ldots\right)_{\ell^2=0}\right]$$

$$\sum_f e_f^2\left(B_f(\ell^2)-\xi^2 D_f(\ell^2)\right) \approx -\frac{T_R\alpha_s}{12\pi\ell^2}\left(\sum_f e_f^2\right)\left[\left(\frac{\langle P|F^{\alpha\beta}(i\overleftrightarrow{D}^+)^2F_{\alpha\beta}|P\rangle}{(P^+)^2}+\xi^2\langle P|F^2|P\rangle\right)-\left(\ldots\right)_{\ell^2=0}\right]$$

finite at $\ell^2=0$

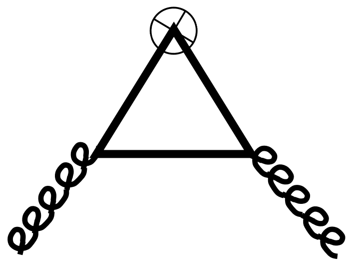
Form factors appear in matrix element of energy-momentum tensor:

$$\langle P_2|\Theta_f^{\mu\nu}|P_1\rangle = \frac{1}{M}\,\bar{u}(P_2)\left[P^\mu P^\nu A_f + (A_f+B_f)\frac{P^{(\mu}i\sigma^{\nu)\rho}\ell_\rho}{2} + \frac{D_f}{4}(\ell^\mu\ell^\nu - g^{\mu\nu}\ell^2) + M^2\bar{C}_f g^{\mu\nu}\right]u(P_1)$$

trace anomaly $\quad \partial\mu\left(\Theta^{\mu\nu}\,x_\nu\right) = \Theta^\mu_\mu = \dfrac{\beta(g)}{2g}\,F^{\mu\nu}F_{\mu\nu}$

triangle diagram: Giannotti, Mottola

$$\langle p_2|\Theta^{\mu\nu}_{\rm QED}|p_1\rangle = -\frac{e^2}{24\pi^2\ell^2}\left(p^\mu p^\nu + \frac{\ell^\mu\ell^\nu - \ell^2 g^{\mu\nu}}{4}\right)\langle p_2|F^{\alpha\beta}F_{\alpha\beta}|p_1\rangle + \ldots$$



our result consistent with cancellation of massless pole, e.g.

$$\sum_f D_f(\ell^2) \approx M\,\frac{T_R n_f \alpha_s}{6\pi\ell^2}\left(\left.\frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\right|_{\ell^2=0} - \frac{\langle P_2|F^2|P_1\rangle}{\bar{u}(P_2)u(P_1)}\right)$$

Summary:

- QCD factorization for Compton amplitude:
  cornerstone of GPD programs

- have found unexpected contributions at $\mathcal{O}(\alpha_s)$
  when keeping $\ell^2 \neq 0$

- associated with chiral and trace anomalies:

$$T^{\mu\nu} \sim \frac{\langle F^{\alpha\beta}\tilde{F}_{\alpha\beta}\rangle}{\ell^2}, \quad \frac{\langle F^{\alpha\beta}F_{\alpha\beta}\rangle}{\ell^2}$$

- cancellation of poles?