Status and prospects of GPD determinations

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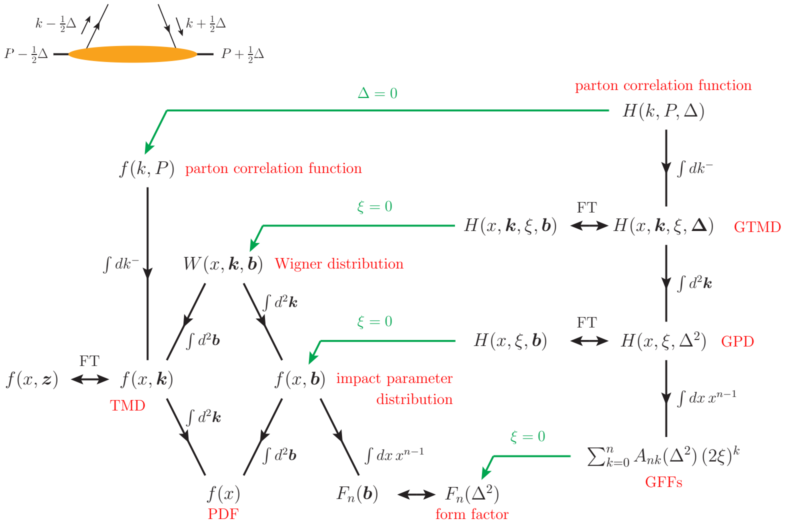
Outline

1. Introduction
2. CFFs
3. GPDs
4. Gepard

Family tree of hadron structure functions



[Fig. by Markus Diehl]

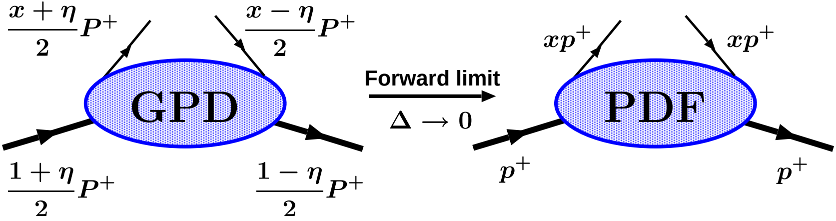
Definition of GPDs

- In QCD GPDs are defined as [Müller '92, et al. '94, Ji, Radyushkin '96]

$$F^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+ q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

$$\widetilde{F}^q(x,\eta,t) = \int \frac{dz^-}{2\pi}\, e^{ixP^+z^-} \langle P_2|\bar{q}(-z)\gamma^+\gamma_5 q(z)|P_1\rangle\Big|_{z^+=0,\,\mathbf{z}_\perp=\mathbf{0}}$$

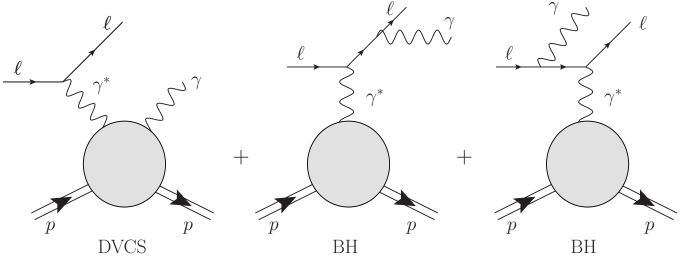
(and similarly for gluons F^g and $\widetilde{F}^g$).



$P = P_1 + P_2$; $\quad t = \Delta^2 = (P_2 - P_1)^2$; $\quad \eta = -\frac{\Delta^+}{P^+}$ (skewedness)

Deeply virtual Compton scattering

- Today GPDs are exctracted mostly from studies of deeply virtual Compton scattering (DVCS)
- measured via leptoproduction of a photon



- Interference with Bethe-Heitler process gives unique access to both real and imaginary part of DVCS amplitude.

DVCS cross section

$$d\sigma \propto |\mathcal{T}|^2 = |\mathcal{T}_{\rm BH}|^2 + |\mathcal{T}_{\rm DVCS}|^2 + \mathcal{I}\,.$$

$$\mathcal{I} \quad \propto \quad \frac{-e_\ell}{\mathcal{P}_1(\phi)\mathcal{P}_2(\phi)}\left\{ c_0^{\mathcal{I}} + \sum_{n=1}^{3}\left[c_n^{\mathcal{I}}\cos(n\phi) + s_n^{\mathcal{I}}\sin(n\phi)\right]\right\},$$

$$|\mathcal{T}_{\rm DVCS}|^2 \quad \propto \quad \left\{ c_0^{\rm DVCS} + \sum_{n=1}^{2}\left[c_n^{\rm DVCS}\cos(n\phi) + s_n^{\rm DVCS}\sin(n\phi)\right]\right\},$$

- Choosing polarizations (and charges) we focus on particular harmonics:

$$c^{\mathcal{I}}_{1,\rm unpol.} \propto \left[F_1\,\mathfrak{Re}\,\mathcal{H} - \frac{t}{4M_p^2}F_2\,\mathfrak{Re}\,\mathcal{E} + \frac{x_{\rm B}}{2-x_{\rm B}}(F_1+F_2)\,\mathfrak{Re}\,\widetilde{\mathcal{H}}\right]$$

[Belitsky, Müller et. al '01–'14]

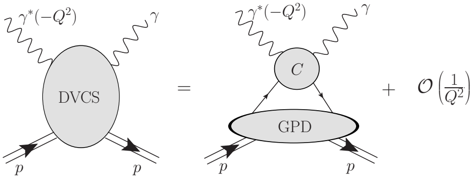
DVCS ⟶ CFFs ⟶ GPDs

- At leading order DVCS cross-section depends on four complex

Compton form factors (CFFs)

$$\mathcal{H}(\xi, t, Q^2), \quad \mathcal{E}(\xi, t, Q^2), \quad \widetilde{\mathcal{H}}(\xi, t, Q^2), \quad \widetilde{\mathcal{E}}(\xi, t, Q^2)$$

- [Collins et al. '98]

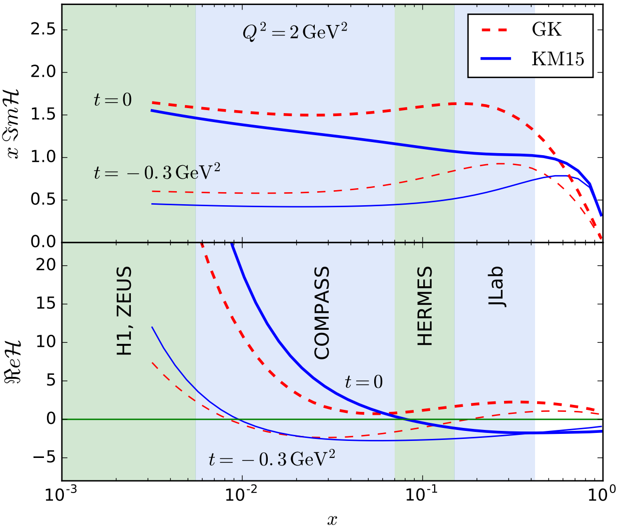


- CFFs are convolution:

$$^{a}\mathcal{H}(\xi, t, Q^2) = \int \mathrm{d}x \; C^a(x, \xi, \frac{Q^2}{Q_0^2}) \; H^a(x, \eta = \xi, t, Q_0^2) \qquad a = q, G$$

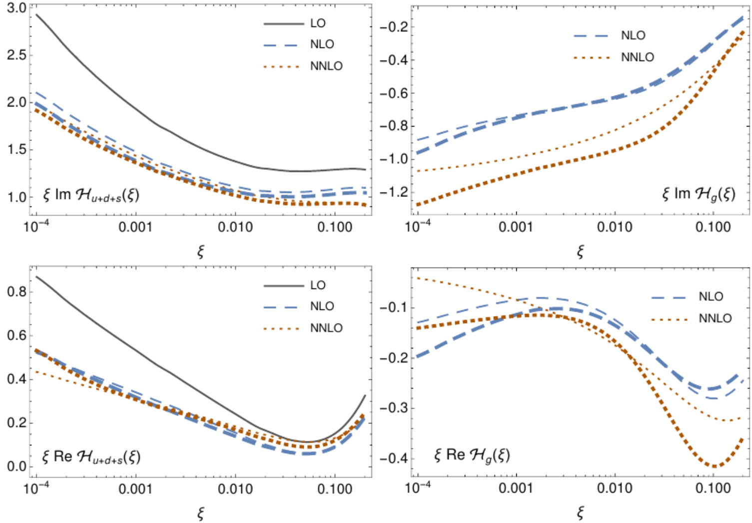
- $H^a(x, \eta, t, Q_0^2)$ — Generalized parton distribution (GPD)
  [Müller '92, et al. '94, Ji, Radyushkin '96]

Experimental coverage



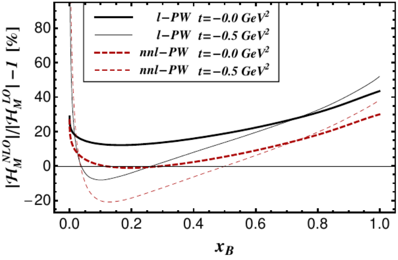
(N)NLO corrections - DVCS (new developments)

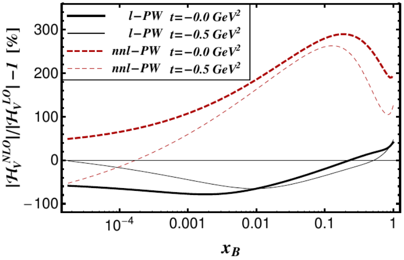
- [Braun, Ji, Schoenleber '22] (NNLO in MS-bar scheme)



NLO corrections - DVMP

- Deeply virtual meson production (DVMP) is complementary to DVCS (access to flavors!)
- NLO corrections: [Belitsky and Müller '01], [Ivanov, Szymanowski, Krasnikov '04], [Müller, Lautenschlager, Passek-K., Schäfer '13], [Duplančić, Müller, Passek-K. '16]
- nonsinglet: singlet:





[see talk by Kornelija tomorrow]

Recent developments and status

- New GPD-relevant exclusive processes are added to the mix, many of them calculated at NLO
  - timelike DVCS [Berger, Diehl, Pire, '01]
  - double DVCS [Guidal, Vanderhaeghen '02]
  - diphoton production [Grocholski et al. '22]
  - photon-meson production [Duplančić et al. '18]
- Global GPD extraction efforts should be improved
  - latest global fits don't even use proper full LO evolution!
- GPD evolution codes in disarray
  - [Vinnikov], [Freund]
  - new releases: Gepard [K.K. '22], APFEL++ in PARTONS [Bertone et al. '22]

Extracting CFFs

Modelling CFFs (1/2)

- LO relation:

$$\Im\mathfrak{m}\,\mathcal{H}(\xi, t) = \pi\left[\frac{4}{9}H^{u_{\rm val}}(\xi,\xi,t) + \frac{1}{9}H^{d_{\rm val}}(\xi,\xi,t)\right]$$

- enables direct modelling of CFFs

$$H(x,x,t) = n\,r\,2^{\alpha}\left(\frac{2x}{1+x}\right)^{-\alpha(t)}\left(\frac{1-x}{1+x}\right)^{b}\frac{1}{\left(1-\frac{1-x}{1+x}\frac{t}{M^2}\right)^{p}}\,.$$

Modelling CFFs (2/2)

- $\Re\mathfrak{e}\,\mathcal{H}$ determined by dispersion relations

$$\Re\mathfrak{e}\,\mathcal{H}(\xi, t) = \frac{1}{\pi}\mathrm{PV}\int_0^1 d\xi' \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'}\right) \Im\mathfrak{m}\,\mathcal{H}(\xi', t) - \frac{C}{\left(1 - \frac{t}{{M_C}^2}\right)^2}$$

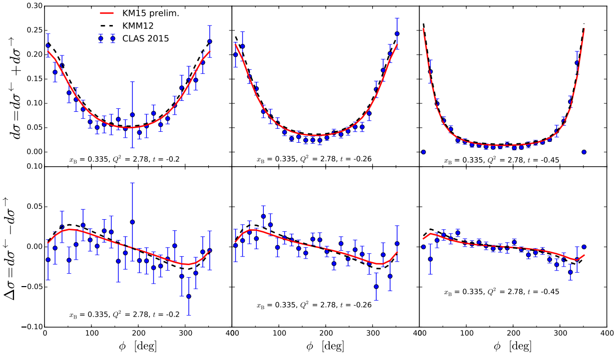
- Usually combined with GPD model for sea partons (see next section):
- [K.K., Müller '09] (Hybrid "KM model") (10-15 parameters)

Progression of fits over the years

Model	KM09a	KM09b	KM10	KM10a	KM10b	KMS11	KMM12	KM15
free params.	{3}+(3)+5	{3}+(3)+6	{3}+15	{3}+10	{3}+15	NNet	{3}+15	{3}+15
$\chi^2/\mathrm{d.o.f.}$	32.0/31	33.4/34	135.7/160	129.2/149	115.5/126	13.8/36	123.5/80	240./275
F_2	{85}	{85}	{85}	{85}	{85}		{85}	{85}
σ_{DVCS}	(45)	(45)	51	51	45		11	11
$d\sigma_{\mathrm{DVCS}}/dt$	(56)	(56)	56	56	56		24	24
$A_{LU}^{\sin\phi}$	12+12	12+12	12	16	12+12		4	13
$A_{LU,I}^{\sin\phi}$			18	18		18	6	6
$A_{C}^{\cos 0\phi}$							6	6
$A_{C}^{\cos\phi}$	12	12	18	18	12	18	6	6
$\Delta\sigma^{\sin\phi,w}$			12				12	63
$\sigma^{\cos 0\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}$			4				4	58
$\sigma^{\cos\phi,w}/\sigma^{\cos 0\phi,w}$		4			4			
$A_{UL}^{\sin\phi}$							10	17
$A_{LL}^{\cos 0\phi}$							4	14
$A_{LL}^{\cos\phi}$								10
$A_{UT,I}^{\sin(\phi-\phi_S)\cos\phi}$							4	4

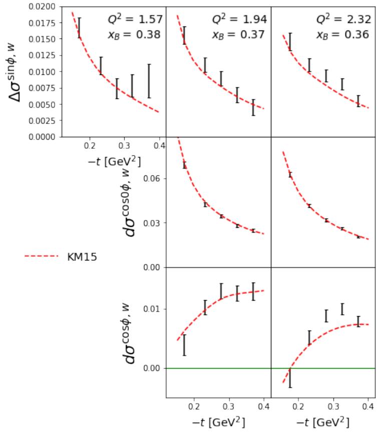
- [K.K., Müller, et al. '09–'15]

Example: JLab's CLAS cross-sections



-
- $\chi^2/\mathrm{npts} = 1032.0/1014$ for $d\sigma$ and $936.1/1012$ for $\Delta\sigma$

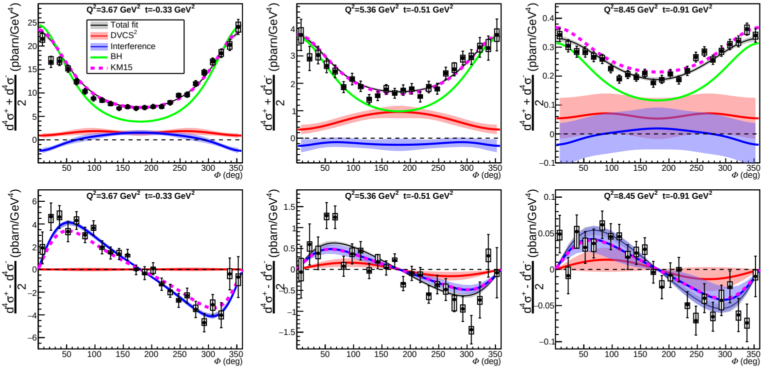
Example: JLab's Hall A cross-sections



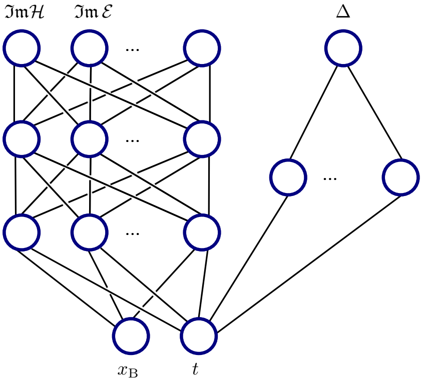
- KM15 global fit is fine. $\chi^2/n_{\mathrm{d.o.f.}} = 240./275$ ✓

Prediction of large-x JLab DVCS data

- [Hall A '22]



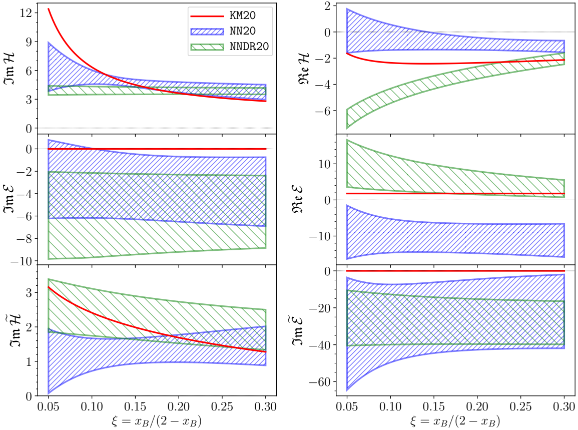
Unbiased model: Neural Networks



- Only imaginary part of CFFs and one subtraction constant $\Delta(t)$ are parametrized by neural nets
- Real parts are then fixed by dispersion relations

Neural nets: Extraction of 6 (out of 8) CFFs

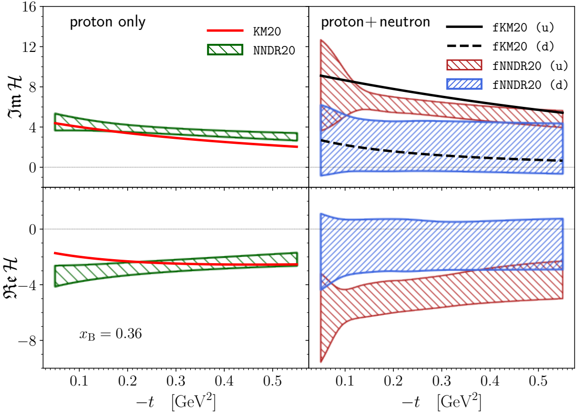
[M. Čuić, K.K., A. Schäfer, '20], from JLab data



- NNets: $\chi^2/n_{\text{pts}} = 1.6\text{-}1.7$ (harmonics only) / 1.3 (full data)

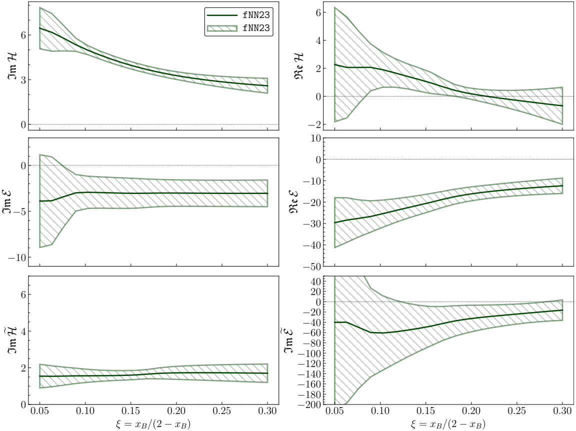
Separating flavored CFFs

- Using neutron DVCS data [Benali et al. '20], contributions of u and d quarks to CFF $\mathcal{H}$ can be cleanly separated



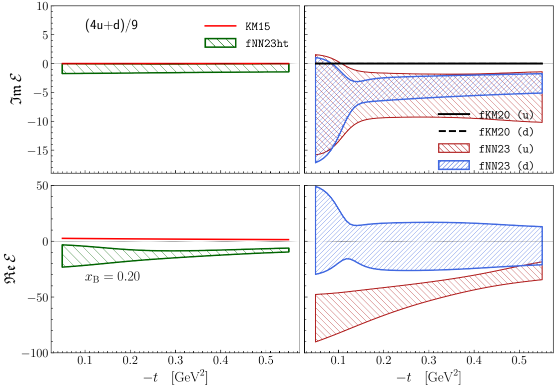
New (2023) analysis (1/2)

- Adding preliminary CLAS proton and neutron DVCS BSA measurements: fixes the sign of ReE



New (2023) analysis (2/2)

- Flavor separation for ReE is now possible:



Extraction of GPDs

Two representations

1. Momentum fraction space (x-space)

$$H(x,\eta,t) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \, F_{DD}(\beta,\alpha,t)\delta(x-\beta-\alpha\eta) + D\left(\frac{x}{\eta}, t\right) .$$

$$F_{DD}(\beta,\alpha,t) = \pi_N(\beta,\alpha) \underbrace{q(\beta,t)}_{\text{PDF + t dep.}}$$ – double distribution

2. Conformal moment space (j-space)

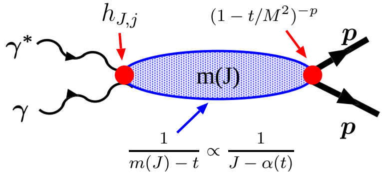
$$H(x,\eta,t) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj \frac{1}{\sin \pi j} p_j(x,\eta) H_j(\eta,t) ,$$

p_j – proportional to Gegenbauer polynomial

(relation between the two studied in [Müller, Polyakov, Semenov-Tian-Shansky '15])

SO(3) partial wave expansion

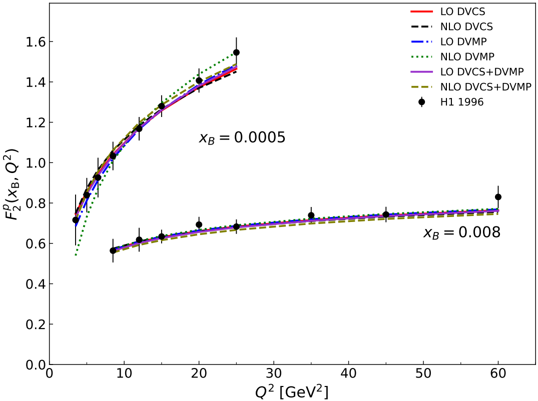
- To model ξ-dependence of GPD's $H_j(\xi, t)$ consider crossed t-channel process $\gamma^*\gamma \to p\bar{p}$ ($\frac{1}{\xi} \leftrightarrow \cos\theta$) and perform SO(3) partial wave expansion:



$$H_j(\xi, t) = \sum_{J=J_{\min}}^{j+1} h_{J,j} \frac{1}{J-\alpha(t)} \frac{1}{\left(1-\frac{t}{M^2}\right)^p} \xi^{j+1-J} d^J_{0,\nu}\left(\frac{1}{\xi}\right)$$

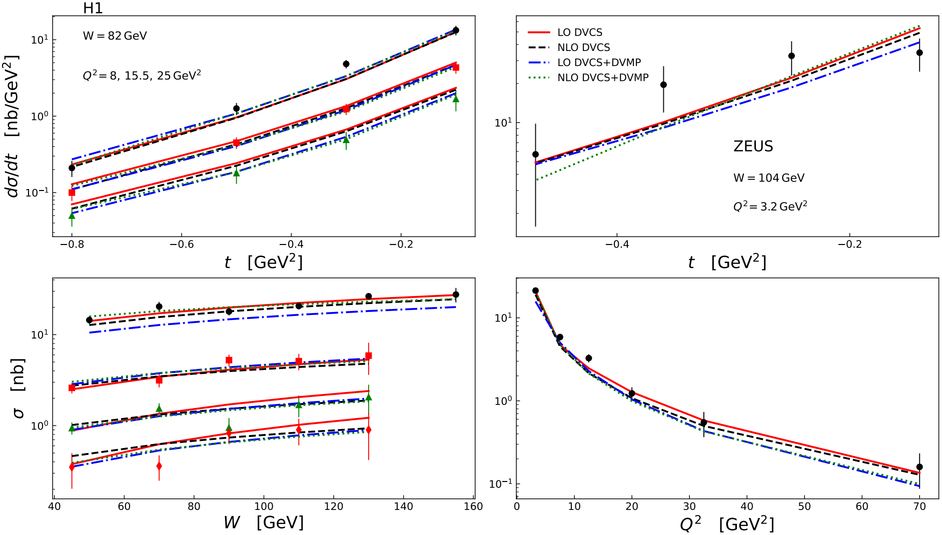
- $d^J_{0,\nu}$ — Wigner SO(3) functions (Legendre, Gegenbauer,...)
  $\nu = 0, \pm 1$ — depending on hadron helicities
- Similar to "dual" parametrization [Polyakov, Shuvaev '02]

DIS F2 data description

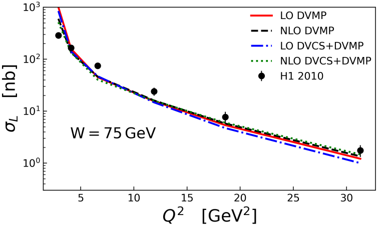


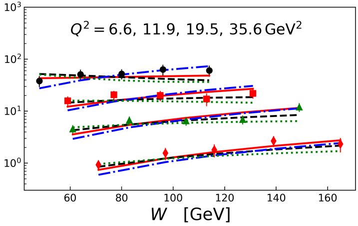
- May seem trivial, but not all popular GPD models describe DIS

DVCS data description



DVMP data description

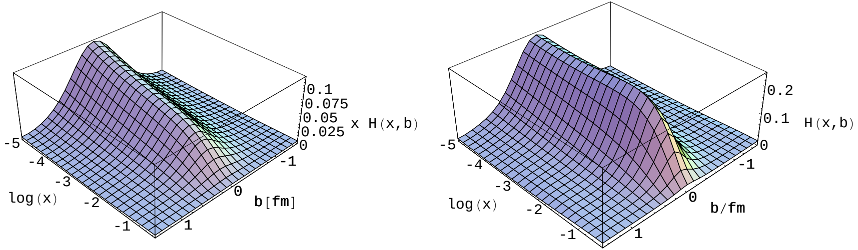




- Things we can learn from fits:
  - Effects of NLO corrections
  - Universality of GPD shape — separately from DVCS and DVMP

Tomography

- Resulting sea quark and gluon distributions $H(x, \vec{b}_\perp)$:



- (large-x part is still very model dependent)

Gepard - public code for GPD analysis

gepard

Search docs

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Software documentation

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Credits

https://gepard.phy.hr/datasets.html

» Tool for studying the 3D quark and gluon distributions in the nucleon   View page source

Tool for studying the 3D quark and gluon distributions in the nucleon

Gepard is software for analysis of three-dimensional distribution of quarks and gluons in hadrons, encoded in terms of the so-called Generalized Parton Distributions (GPDs).

This web site has manifold purpose:

- Documentation of the software
- Examples of the use of software
- Interface to various representations of results: numerical and graphical
- Interface to datasets used in analyses: numerical and graphical

Contents:

- Software documentation
  - Installation
  - Quickstart
  - Tutorial
  - Data points, sets and files

Gepard - publications

- Aiming for full reproducibility of results.

gepard

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Accompanying code runs with the latest version of Gepard package

Accompanying code runs only with old versions Gepard package

Credits

Accompanying code runs with the latest version of Gepard package

These papers have accompanying Jupyter notebooks, published on the github, which are easily runnable after installing the latest version of Gepard:

- K. Kumerički, D. Mueller, K. Passek-Kumerički and A. Schaefer, *Deeply virtual Compton scattering beyond next-to-leading order: the flavor singlet case*, Phys. Lett. B **648** (2007), 186-194, arXiv:hep-ph/0605237 [Code at github]
- K. Kumerički, D. Mueller, and K. Pasek-Kumerički, *Towards a fitting procedure for deeply virtual Compton scattering at next-to-leading order and beyond*, Nucl. Phys. B **794** (2008) 244-323, arXiv:hep-ph/0703179 [Code at github]
- K. Kumerički and D. Mueller, *Deeply virtual Compton scattering at small xB and the access to the GPD H*, Nucl. Phys. B **841** (2010) 1-58, arXiv:0904.0458 [Code at github]

Accompanying code runs only with old versions Gepard package

These papers have accompanying Jupyter notebooks, published on the github, but need old version of Gepard (available as *pyfortran* branch on the Gepard's github page), which can be tricky to compile and run

- M. Čuić, K. Kumerički, and A. Schäfer, *Separation of Quark Flavors using DVCS Data*, Phys. Rev. Lett. **125** (2020) 23, 232005, arXiv:2007.00029 [Code at github]
- K. Kumerički, *Measurability of pressure inside the proton*, Nature, 570 (2019) no. 7759, E1-E2,

Thank you!