Parton distribution functions from lattice QCD: facts and fancy

Sara Collins

Universität Regensburg

RQCD Collaboration: G. Bali, V. Braun, A. Schäfer, E. Scholz, W. Söldner, M. Göckeler, P. Wein, T. Wurm, D. Jenkins, S. Bürger, S. Weishäupl, L. Walter, M. Gruber, P. Korcyl, A. Sternbeck, . . .

FOR2926 workshop, Regensburg Feb. 16th 2023

Overview

Moments of PDFs

- ⋆ Lattice details
- ⋆ Results for lowest two moments of helicity, transversity, unpolarised PDFs
- ⋆ Phenomenological analysis of transversity with lattice constraints
- ⋆ Matrix elements of higher twist operators

Position space lattice methods

- ⋆ Quasi- and pseudo-PDFs
- ⋆ Challenges
- ⋆ Results for the isovector helicity, transversity, unpolarised PDFs.

Moments of PDFs

Mellin moment n, example of unpolarised PDF:

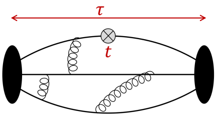
$$\langle x^{n-1}(\mu)\rangle_{q^{(-)^n}} = \int_0^1 \mathrm{d}x\, x^{n-1}\left[q(x,\mu) + (-)^n \bar{q}(x,\mu)\right]$$

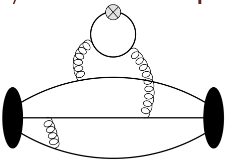
OPE connects the moments to matrix elements of local operators

$$\left\langle N(p)\left|\left[\mathcal{O}^n_{\mu_1\ldots\mu_n}\right]\right| N(p)\right\rangle = \langle N(p)\left|[\mathcal{O}^n]\right| N(p)\rangle\,((p_{\mu_1}\cdots p_{\mu_n}) - \text{traces})$$
$$\left\langle x^{n-1}\right\rangle_{q^{(-)^n}} \sim \langle N(p)\left|[\mathcal{O}^n]\right| N(p)\rangle$$
$$\mathcal{O}^n_{\mu_1\ldots\mu_n} = \bar{q}\gamma_{(\mu_1} i\overleftrightarrow{D}_{\mu_2}\ldots i\overleftrightarrow{D}_{\mu_n)} q$$

where $(\mu_1\cdots)$: symmetrized, traceless.
Different flavour combinations are possible, also gluonic operators. Second moment ($n=2$, $\langle x\rangle_{q^+}$) has one derivative (dimension 4 operator).
Note that there are no higher twist corrections but $\mathcal{O}(a)$ lattice corrections: continuum limit $a\to 0$.
Lattice scheme: if mixing with lower $n' = n-\ell$ is not prohibited by symmetry $\Rightarrow a^{-\ell}$ power divergencies! This (and statistical noise) complicates $n>3$.
Notation [PDFLattice,1711.07916]: $q^- := q - \bar{q}\,(= q_v)$, $q^+ := q + \bar{q}\,(= q_v + 2\bar{q})$.

Lattice details: computing $\langle N|\mathcal{O}|N\rangle$ in the isospin limit.





Isovector combinations only connected. Isoscalar also disconnected (computationally more expensive, additional stochastic noise).

Steps in the analysis:

- Fit the two and three-point correlation functions
- Renormalisation+ improvement: for $\vec{p} = \vec{p}\,' = 0$/some operators/actions $c_{\mathcal{O}} = 0$ or $b_{\mathcal{O}} = 0$
  $\mathcal{O}^{\overline{\mathrm{MS}}}(\mu) = Z_{\mathcal{O}}^{\overline{\mathrm{MS}},latt}(a\mu)\left[(1 + b_{\mathcal{O}}\, am_q)\mathcal{O}^{latt} + ac_{\mathcal{O}}\mathcal{O}_1^{latt}\right]$
  Non-perturbative matching: lattice → RI′-(S)MOM scheme,
  Perturbative: RI′-(S)MOM → $\overline{\mathrm{MS}}$ (typically 3-loop, 4-loop is desirable).

Repeat analysis on several ensembles to explore

- Finite volume effects: exponentially suppressed $\sim m_\pi^2 e^{-Lm_\pi}/(m_\pi L)^{3/2}$, $Lm_\pi > 4$.
- Discretisation effects: $\mathcal{O}(a)$ or $\mathcal{O}(a^2)$.
- Quark mass dependence: chiral pert. theory (ChPT) $m_\pi \to m_\pi^{phys}$.

Challenges

Lattice provides (very) precise results for (see [FLAG21,2111.09849])

- α_s, m_q, $q \in \{u/d, s, c, b\}$, $K \to \pi\ell\nu$, form factor at $q^2 = 0$, $f_+(0) = 0.9698(17)$, $f_K/f_\pi = 1.1932(21)$, ...

Difficulties in the baryon sector

- **Statistical noise**: signal vs noise decays with $e^{-(E-3m_\pi/2)\tau}$.
- **Excited state pollution**: significant since τ cannot be too large. Dense spectrum of multiparticle states at m_π^{phys}.
- Renormalisation: quark flavours mix under renormalization ($N_f = 2 + 1$):
  $\mathcal{O}_{u-d}(\mu) = Z_{\mathcal{O}}^{ns}(a\mu, \alpha_s)\mathcal{O}_{u-d}^{\text{latt}}(a)$ for all isovector currents.
  $\mathcal{O}_{u+d+s}(\mu) = Z_{\mathcal{O}}^{s}(a\mu, \alpha_s)\mathcal{O}_{u+d+s}^{\text{latt}}(a)$ for 1st moments of isosinglet.

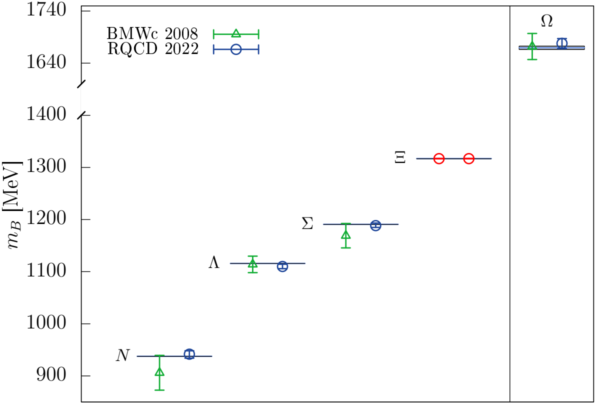
$$\text{Isosinglet } \langle x \rangle : \begin{pmatrix} J_{u+d+s}(\mu) \\ GG(\mu) \end{pmatrix} = \begin{pmatrix} Z_q^s(a\mu) & N_f Z_{qg}(a\mu) \\ Z_{gq}(a\mu) & Z_g(a\mu) \end{pmatrix} \begin{pmatrix} J_{u+d+s}^{\text{latt}}(a) \\ GG(a) \end{pmatrix}$$

- Quark mass dependence: not clear how well ChPT describes the quark mass dependence in the range $m_\pi \sim (m_\pi^{phys} - 300 \text{ MeV})$.

  Need to simulate at or close to m_π^{phys}.

(Stable) baryon spectrum $N_f = 2 + 1$

Example of progress.



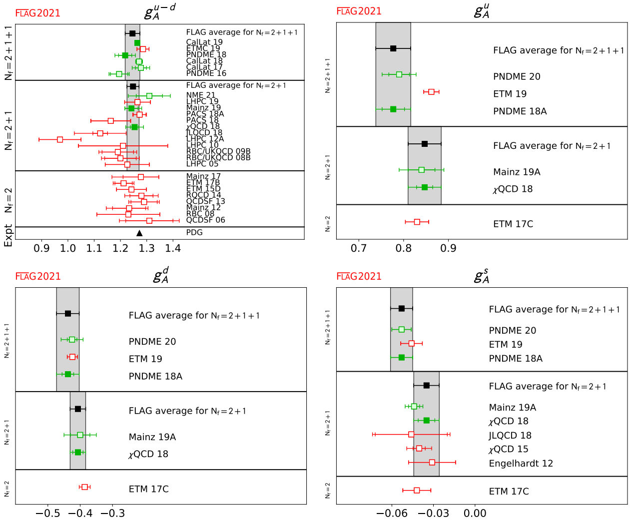
[BMWc,0906.3599]
[RQCD,2211.03744]: 42 ensembles, $m_\pi = 420 - 135$ MeV, six lattice spacings, $a = 0.1 - 0.04$ fm, $Lm_\pi \gtrsim 4$, ...

m_N with $< 1\%$ overall uncertainty.

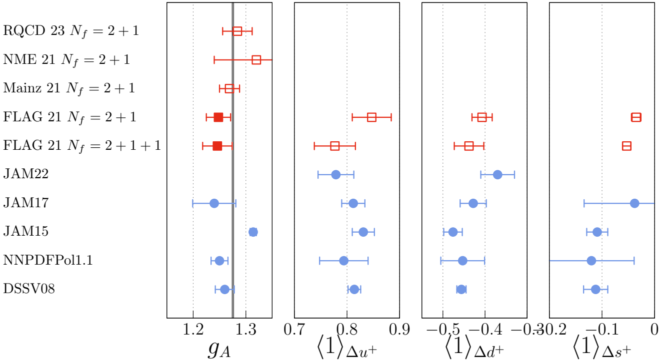
Lowest moments: helicity, $\langle 1 \rangle_{\Delta u^+ - \Delta d^+} = g_A$, $\langle 1 \rangle_{\Delta q^+} = g_A^q$

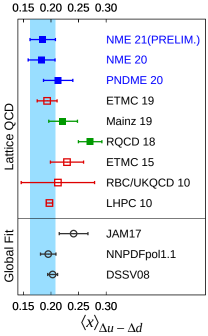
g_A is a benchmark quantity, sensitive to excited state contamination, finite volume, quark mass dependence, . . .

$\mu = 2$ GeV for g_A^q. [FLAG21,2111.09849]



Comparison with phenomenological fits, $\mu = 2$ GeV





RQCD 23 Weishäupl et al.

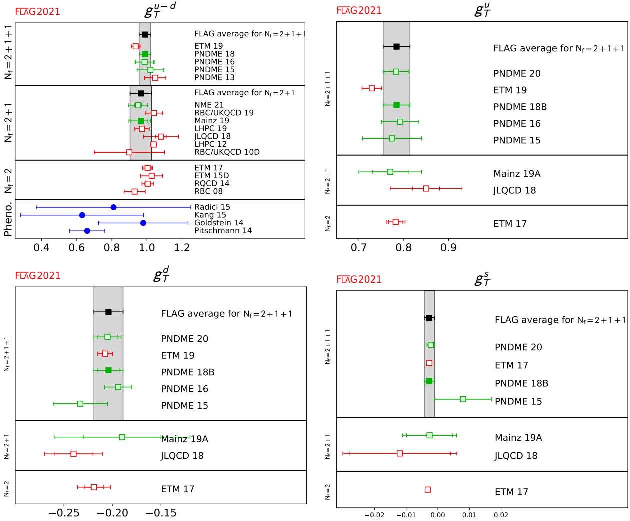
[NME21,2103.05599], [Mainz21,2103.05599] Left: [JAM22,2202.03372], [JAM17,1705.05889], [JAM15,1601.07782], [NNPDF,1406.5539], [DSSV08,0904.3821] Right: [NME,2201.00067]

Charm: $\langle 1 \rangle_{\Delta c^+} = -0.0026(18)$ [PNDME,2109.01191], $N_f = 2+1+1$

$\langle 1 \rangle_{\Delta c^+} = -0.0098(34)$ [ETMC,1909.00485], $N_f = 2+1+1$.

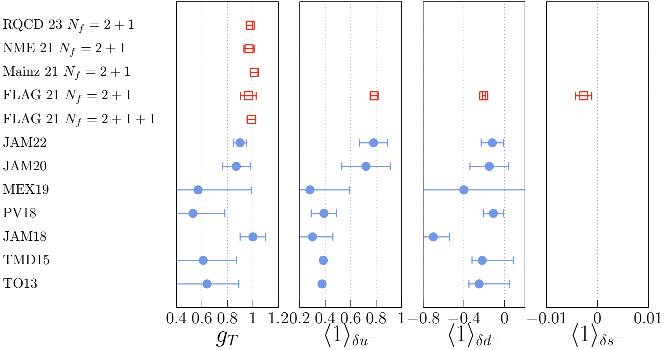
Transversity, $\langle 1\rangle_{\delta u^- - \delta d^-} = g_T$, $\langle 1\rangle_{\delta q^-} = g_T^q$, $\mu = 2$ GeV

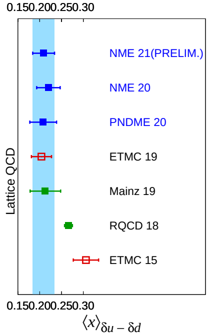
[FLAG21,2111.09849]



Comparison with phenomenological fits, $\mu = 2$ GeV

JAM18 and JAM22 impose lattice g_T as a constraint.



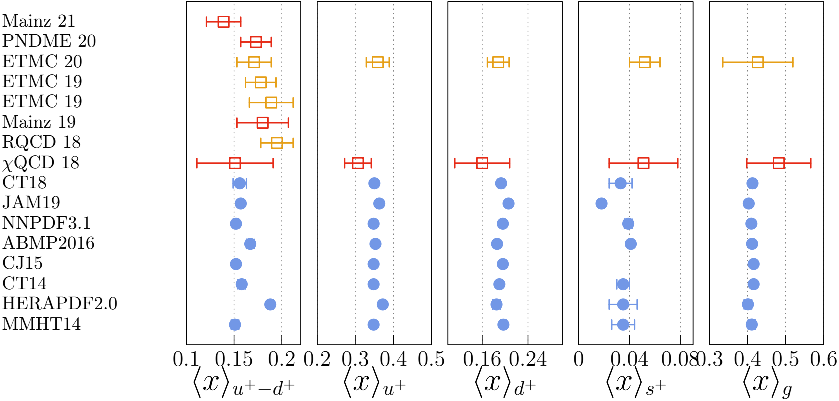


RQCD 23 Weishäupl et al.

[NME21,2103.05599], [Mainz21,2103.05599] Left: [JAM22,2205.00999], [JAM20,2002.08384], [MEX19,1912.03289], [PV18,1802.05212], [JAM18,1710.09858], [TM15,1505.05589], [TO13,1303.3822], Right: [NME,2201.00067]

Charm: $\langle 1 \rangle_{\delta c^-} = 0.0004(13)$ [PNDME,2109.01191], $N_f = 2+1+1$.
$\langle 1 \rangle_{\delta c^-} = -0.00024(16)$ [ETMC,1909.00485], $N_f = 2+1+1$.

Unpolarised second moment, $\mu = 2$ GeV



[MMHT14,1412.3989], [HERAPDF2.0,1506.06042], [CT14,1506.07443], [CJ15,1602.03154], [ABMP16,1701.05838], [NNPDF3.1,1706.00428], [JAM19,1905.03788], [CT18,1912.10053], [PNDME20,2005.13779], [ETMC20,2003.08486], [ETMC19,1908.10706], [Mainz19,1905.01291], [RQCD18,1812.08256], [χQCD18,1808.08677], [Mainz21,2110.10500]

Charm: $\langle x \rangle_{c^+} = 0.070(25)$ [PNDME,2109.01191], $N_f = 2+1+1$.

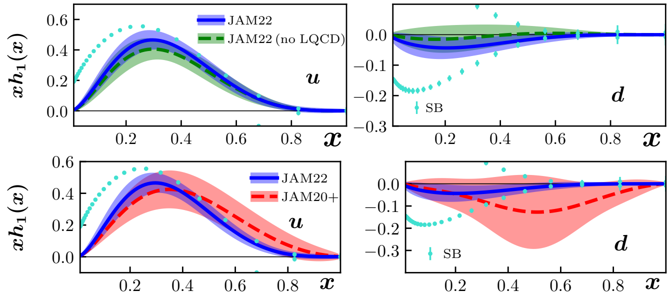
$\langle x \rangle_{c^+} = 0.019(09)$ [ETMC,1909.00485], $N_f = 2+1+1$.

Global analysis of transversity with lattice constraints

Global analysis of single transverse-spin asymmetries.

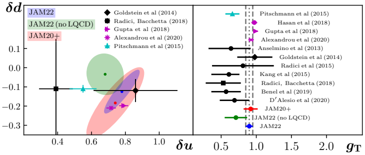
Transversity PDFs h_1^u and h_1^d with anti-quark functions set to zero.
Top, blue: [ETMC,1909.00485] g_T included as a data point.

[JAM22,2205.00999]



SB: (parton model)
[Soffer,hep-ph/9409254]
$|h_1^q(x)| \leq (q(x) + \Delta q(x))/2.$

Right: $\delta q = \langle 1 \rangle_{q^-}$.



Higher moments and twist: helicity

LO in OPE, third Mellin moments of the helicity structure functions g_1 and g_2

$$2\int_0^1 dx\, \mathbf{x^2 g_1(x, Q^2)} = \frac{1}{2}\sum_q Q_q^2\, E_{1,2}^{(q)}\left(\frac{\mu^2}{Q^2}, \alpha_s(\mu)\right) \mathbf{a_2^{(q)}(\mu)}\,,$$

$$2\int_0^1 dx\, \mathbf{x^2 g_2(x, Q^2)} = \frac{1}{3}\sum_q Q_q^2 \left[E_{2,2}^{(q)}\left(\frac{\mu^2}{Q^2}, \alpha_s(\mu)\right) \mathbf{d_2^{(q)}(\mu)} - E_{1,2}^{(q)}\left(\frac{\mu^2}{Q^2}, \alpha_s(\mu)\right) \mathbf{a_2^{(q)}(\mu}\right.$$

Wandzura-Wilczek relation: if $d_2^q \ll a_2^q$, g_2 can be obtained from g_1.

$a_2^{(q)}$, twist-2 contribution to g_1. $d_2^{(q)}$ twist-3 contribution to g_2 is not power suppressed in $1/Q$. Wilson coefficients $E_{1,n}^{(q)}$ are known to 2-loop, $E_{2,n}^{(q)}$ only known at tree-level.

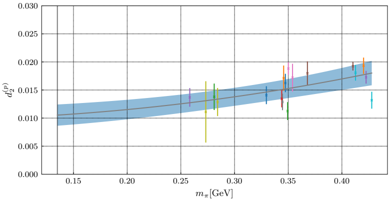
Both involve operators with two derivatives. Lattice: d_2 mixing with operators of lower dimension. Cancellation of $1/a$ contributions via renormalisation.

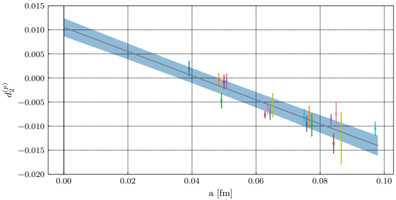
[RQCD,2111.08306] S. Bürger et al.: some approximations (no disconnected diagrams, . . .), however, unlikely to be the main uncertainty.

Higher moments and twist: helicity

Continuum, quark mass extrapolation.

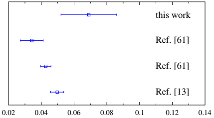
$d_2^{(p)} = \left(\frac{2}{3}\right)^2 d_2^{(u)} + \left(-\frac{1}{3}\right)^2 d_2^{(d)}, \quad d_2^{(n)} = \left(-\frac{1}{3}\right)^2 d_2^{(u)} + \left(\frac{2}{3}\right)^2 d_2^{(d)}$. Similarly for $a_2^{(p,n)}$.



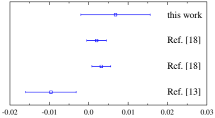


Comparison with phenomenology (some approximations also made) ($\mu = 2$ GeV)

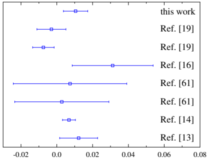
$a_2^{(p)}$



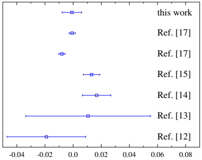
$a_2^{(n)}$



$d_2^{(p)}$



$d_2^{(n)}$

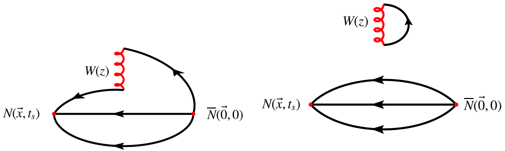


[Ref 12, E154,hep-ex/9705017],
[Ref 13, E143,hep-ex/9705017],
[Ref 14, E155,hep-ex/0204028],
[Ref 15, JLab Hall A,nucl-ex/0405006],
[Ref 16, HERMES,1112.5584],
[Ref 17, JLab Hall A,1404.4003],
[Ref 18, JLab Hall A,1603.03612],
[Ref 19, SANE,1805.08835],
[Ref 61, Osipenko,hep-ph/0503018].

Previous lattice studies:
[QCDSF,hep-lat/0506017],
[LHPC,1001.3620],
[LHPC,SESAM,hep-lat/0201021].

Position space lattice methods: quasi- and pseudo-PDFs

Evaluate 4-point function and extract matrix element with a non-local (equal time) current e.g. unpolarized case.



$$M^0(z, P_3) = \langle P|\bar{\psi}(z)\gamma^0 W(z)\psi(0)|P\rangle, \quad P = (P_0, 0, 0, P_3),\ z = (0, 0, 0, z_3)$$

Collinear factorisation used to extract information on the PDFs.

- Quasi-PDFs and large-momentum effective theory (LaMET) [Ji,1305.1539]:

$$\tilde{q}(y, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izyP_3} M^0(z, P_3),$$

$$\tilde{q}(y, P_3) = \int_{-1}^{1} \frac{dx}{|x|} C^{(\tilde{q})}\left(\frac{y}{x}, \frac{xP_3}{\mu}\right) q(x, \mu^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{y^2(1-y)P_3^2}\right)$$

Off light-cone $|y| > 1$ possible, y no longer Bjorken-x.
Higher twist effects at small and large y, range of y depends on P_3.

Position space lattice methods: quasi- and pseudo-PDFs

- Pseudo-PDFs [Radyushkin,1612.05170]: Ioffe-time $\boldsymbol{\nu = z \cdot P}$.

$$\mathcal{M}^0(P, z) = 2P_0\mathcal{M}_P(\nu, z^2)$$

Off the light-cone **Ioffe-time distribution (ITD)** $\mathcal{M}_P(\nu, z^2)$.
Related to a **Pseudo-PDF** $\mathcal{P}(x, z^2)$: $\mathcal{M}_P(\nu, z^2) = \int_{-1}^{1} dx\, e^{ix\nu}\mathcal{P}(x, z^2)$.
Pseudo-PDF can be related for small $|z|^2$ (fixed ν) via factorization to PDF $q(x)$. $|x| < 1$. However, the pseudo ITD is directly related to the PDF.

$$\mathcal{M}_P(\nu, z^2) = \int_{-1}^{1} dx\, C(x\nu, \mu^2 z^2) q(x, \mu^2) + O(z^2\Lambda^2)$$

$C(x\nu, \mu^2 z^2) = e^{i\nu x} + O(\alpha_s)$. z should be small, P_3 can also be small.
Wide range of ν required $\rightarrow$ large P_3 required.

Other approaches, e.g. lattice cross-sections [Ma and Qiu,1404.6860], hadronic tensor method [Liu et al.,hep-ph/9910306,1906.05312], current-current correlations [Braun and Müller,0709.1348], ...

Challenges

Achieving large momentum: with current methods (momentum smearing [RQCD,1602.05525]) reasonable signals up to $P_3 \lesssim 3$ GeV are achieved.

With higher P discretisation effects $O(aP)$, $O(a^2P^2)$, can be significant $\rightarrow$ need small a.

Signal to noise: signal of correlation functions deteriorates with large Euclidean separations (and higher P).

Excited state contamination: spectrum of states becomes even more dense for finite P.

Renormalisation: bare (extended) operator $\bar{\psi}(z)\gamma^0 W(z)\psi(0)$ has power divergences. Non-trivial to renormalise if z is not small. Alternative, current-current method.

Inverse problem/reaching long range correlations: discrete lattice data, over a finite range.

Quasi-PDFs: finite range of z for $\tilde{q}(y, P_3) = \mathrm{FT}_z[M^0(z, P_3)]$

Pseudo-PDFs: finite range of ν for the inverse transform of $M^0(\nu, z^2) \rightarrow q(x, \mu)$.

Affects large x (oscillations) and small x. Less problematic as P_3 increases. Either assumptions made or models used for the PDFs that are transformed and fitted directly to the (renormalised) lattice data.

Challenges

Perturbative matching: mostly 1-loop available (apart from current-current methods).

In addition, discretisation, finite volume, quark mass dependence effects.

Cross-checks:

- Comparison with phenomenological determinations of the PDFs. Most meaningful for lattice results at the physical point, after continuum extrapolation.
- Comparison with Mellin moments.
- Closure tests with artificial data (e.g. [Candido et al.][PoS(LATTICE2022)098]).

So far studies of isovector and flavour singlet unpolarised, helicity, transversity PDFs, gluon PDFs. Also GPDs, higher twist PDFs, TMDs, . . .

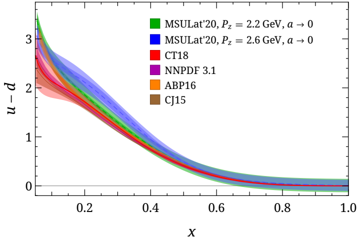
Snowmass 21 review of lattice PDF calculations [Constantinou et al.,2202.07193]

Quoted aim: 5% precision or better for isovector PDFs for $0.2 < x < 0.6$.

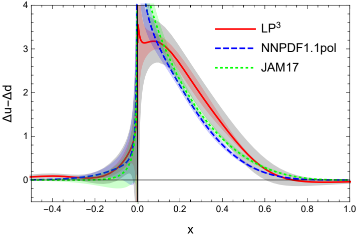
$10 - 20\%$ precision for flavour singlet quark and gluon PDFs.

Isovector results from the quasi-PDF approach

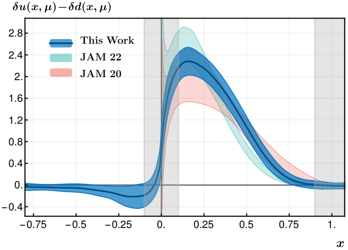
Unpolarised: [MSULat,2011.14971]



Helicity: [LP[3],1807.07431]



Transversity: [LPC,2208.08008]

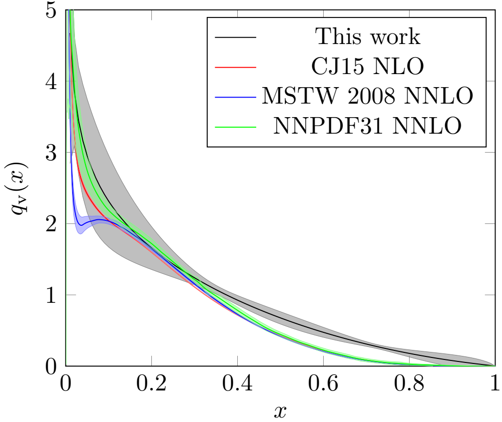


[MSULat,2011.14971], [LPC,2208.08008] results at m_π^{phys} after continuum extrapolation. Resp. $P_3^{max} \sim 2.6$ GeV and $P_3^{max} \sim 2.8$ GeV.

[LP[3],1807.07431] single m_π^{phys} ensemble result, $a = 0.09$ fm, $P_3^{max} \sim 3$ GeV.

Isovector unpolarized pseudo-PDF results

[Hadstruc,2004.01687] $a = 0.09$ fm, results extrapolated to m_π^{phys}.



Summary

- ⋆ Lower moments of unpolarised, helicity and transversity PDFs can be computed on the lattice (for the isovector flavour combination, individual quark flavours and gluon).
- ⋆ Calculations of first moments of helicity and transversity PDFs ($\langle 1\rangle_{\Delta q^+}$, $\langle 1\rangle_{\delta q^-}$) and isovector second moments are reasonably mature: main sources of systematic uncertainty investigated.
- ⋆ More results, in particular, for individual quark flavours will appear in the near future and precision will improve.
- ⋆ Provide complementary information to experiment where low and high x-regions are not well probed.
- ⋆ Global analyses using lattice moments as contraints can reduce uncertainties on transversity and helicity PDFs.
- ⋆ Flavour singlet and gluon second moments: renormalisation still needs investigation.
- ⋆ Twist two and three contributions to the second moment of helicity structure functions have been determined.

Summary

⋆ Position space methods face significant challenges.

⋆ Much progress has been made, however, new methods are required.

⋆ Resonable signals for larger momenta are needed $\rightarrow$ smaller a, with sufficiently large volume at m_π^{phys}. Significant computational resources will be required.