2.1. Highlights of results of the first funding period

Our main achievements in the first funding period can be summarized as follows:

- We made significant contributions to impact studies on nucleon 3D imaging and nuclear modifications of parton distribution functions (PDFs) and fragmentation functions (FFs) for the EIC Yellow Report [1] and EIC detector-design proposals (projects **P4** and **JRP**).

  The EIC Yellow Report was an effort by the worldwide community to formulate requirements for the EIC accelerator and detector characteristics, using the present state-of-the-art in hadron physics theory. Our group led the impact studies for TMDs and contributed to the nuclear modification program. These studies are highly nontrivial because the expected accuracy of the EIC is much better than that of any present experiment, and also because the data will cover a previously unexplored kinematic range. This required creation of dedicated frameworks and updates of previous theoretical predictions. In particular, to obtain a reliable estimate of the evolution effects for single-spin asymmetries, we carried out a combined fit to the Drell-Yan and semi-inclusive DIS (SIDIS) data in the TMD factorization framework [4, 5], the first of its kind. Our results on the impact of the projected EIC measurements on the Sivers function were shown in Fig. 2.3 above. Our EIC projections for nuclear modifications of PDFs and FFs are shown in Fig. 2.4. These studies were performed in a feedback loop with experimental groups which supplied us with simulations for the expected EIC conditions. They are being continued in close contact with the detector-design groups.

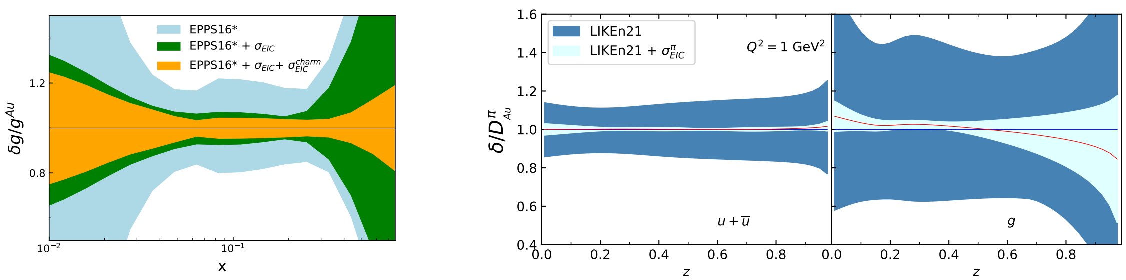


Figure 2.4.: Left: impact of EIC inclusive and charm production DIS pseudo-data on the relative uncertainty of the gluon nPDF in a gold nucleus; figure adapted from [1]. Center: improvement of the relative uncertainty of the $u + \bar{u} \to \pi^+$ FF in scattering off gold nuclei when including EIC SIDIS data at $\sqrt{s} \approx 29$ GeV. Right: same as center panel but for the gluon FF.

- The Collins-Soper (CS) kernel is a crucial element of the TMD factorization formalism. Until recently, it could only be parametrized as an unknown nonperturbative function governing TMD evolution. We were able to prove that the CS kernel is a universal (process-independent) function and developed new methods to determine it (projects **P3** and **P4**). In Ref. [6], a self-contained operator definition of the CS kernel has been presented. Using this definition one can relate the CS kernel to properties of the QCD vacuum and perform systematic first-principle studies. Based on this development, we have carried out several extractions of the CS kernel using experimental data [4], lattice QCD computations [7, 8], and event generators.

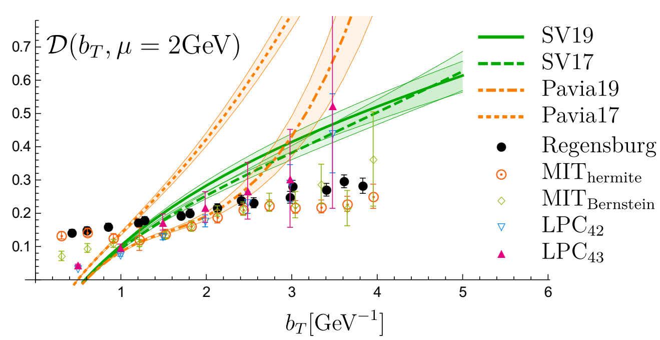


Figure 2.5.: Comparison of various extractions of the Collins-Soper kernel from experiment (dashed and continuous lines) and lattice simulation (points). The extraction labeled "SV19" has been carried out as part of project **P4** [4]. The lattice simulation labeled as "Regensburg" [8] has been obtained using a method also developed within **P4** [7].

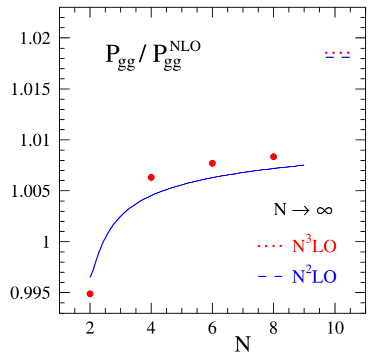
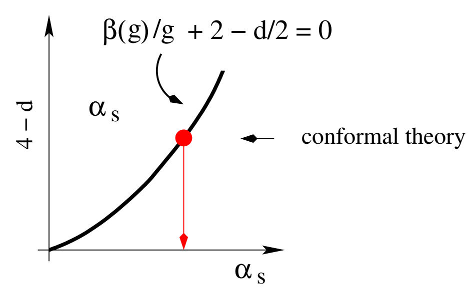


Figure 2.6.: The Mellin moments $N = 2, 4, 6, 8$ of the gluon splitting function at NNLO (lines) and N^{3}LO (points) at $\alpha_s = 0.2$ and $n_f = 4$ flavors, normalized to the NLO results. Also shown are the corresponding large-N limits (upper right corner). Figure taken from [9].

- We have computed the four lowest even-N Mellin moments of all four splitting functions $P_{\rm qq}, P_{\rm qg}, P_{\rm gq}, P_{\rm gg}$ for the evolution of the quark flavor-singlet distribution and the gluon distribution at the fourth order in the strong coupling constant α_s. Fig. 2.6 illustrates the apparent convergence of the perturbative expansion for the most complicated case of the $P_{\rm gg}$ splitting function, in a comparison of the new N^{3}LO results with the currently standard NNLO approximation. The computing time for the calculation of the $N = 8$ moment proves to be significant. For momentum fractions $x \gtrsim 0.1$, knowledge of the first four moments is sufficient: the perturbative series for the splitting functions, convoluted with flavor-singlet PDFs, is found to be well behaved with relative α_s-coefficients of order one and sub-percent effects on the scale derivatives of the respective PDFs at $\alpha_s \lesssim 0.2$.

- We have developed a new technique for the calculation of high-order and power corrections to deeply-virtual Compton scattering (DVCS) using the methods of conformal field theories [10] (projects **P1** and **P3**).



The starting point is that QCD in noninteger space-time dimensions $d = 4 - 2\epsilon$ enjoys conformal symmetry at a specially chosen value of the coupling constant (Wilson-Fisher critical point [11], see the sketch on the left side). For any given quantity, the result of the QCD calculation in "physical" four dimensions can be represented as the sum of the corresponding (conformal) result at the critical point, and addenda proportional to the QCD β-function that arise from moving to "physical" four dimensions (shown by red arrow). A salient feature of any conformal theory is that the two-point and three-point correlation functions of any gauge-invariant operator are known exactly up to normalization factors. We were able to show [10] that this classical result can be used to calculate matrices of anomalous dimensions in high orders of perturbation theory, and also "kinematic" corrections to off-forward hard processes to all orders in the power expansion by a relatively simple algebraic procedure. Applications to particular DVCS observables will be considered in the second funding period.

- We have performed several calculations of the matching coefficients relating lattice and continuum theory renormalization schemes (projects **P1** and **P3**). This includes three-loop matching for operators with up to two derivatives [12, 13], three-loop renormalization of quasi-PDFs [14, 15], and the QCD factorization of twist-three quasi-PDFs [16, 17]. The three-loop correction to the matching between the RI'-SMOM and $\overline{\rm MS}$ schemes [13] proves to be very large and has significant impact on the lattice determination of the second moment of the pion distribution amplitude [18].

- We have derived approximate results for the NNLO QCD corrections to the SIDIS cross section [19] (project **P2**). They have been obtained using the threshold resummation formalism, which we have fully developed to NNLL accuracy and recently even to N^{3}LL [20]. The approximate NNLO expressions include all five classes of terms in Mellin space that are logarithmically enhanced at threshold, or that are constant. In

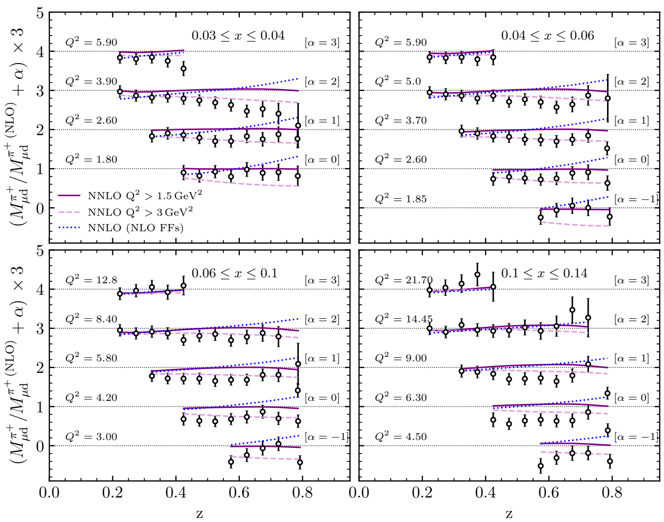


Figure 2.7.: Comparison of our NLO and NNLO fits to the COMPASS π^+ multiplicities [21] for some representative bins of x. Taken from Ref. [22].

terms of the customary SIDIS variables x and z they include all double distributions (that is, "plus" distributions and δ-functions) in the partonic variables.

The approximate NNLO results we have obtained are readily suited for an initial phenomenological NNLO analysis of SIDIS data in terms of FFs. We have carried out the first ever *global* analysis of FFs at NNLO accuracy [22] by performing a fit to the combined set of single-inclusive electron-positron annihilation and, now using our new NNLO corrections, SIDIS multiplicity data. We have found that the NNLO fit shows an improved overall quality compared to the NLO one, at least once an appropriate lower cut on Q^2 is applied. Fig. 2.7 shows a comparison of our NLO and NNLO fits to the COMPASS π^+ multiplicities [21] for some representative bins of x. The FFs we find at NNLO are overall close to the NLO ones, indicating good perturbative stability of the processes used for their extraction.

2.2. Bibliography

[1] R. Abdul Khalek et al. "Science Requirements and Detector Concepts for the Electron-Ion Collider: EIC Yellow Report" (Mar. 2021). arXiv: `2103.05419 [physics.ins-det]`.

[2] D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang. "Evidence for polarization of gluons in the proton." In: *Phys. Rev. Lett.* 113.1 (2014), p. 012001. DOI: `10.1103/PhysRevLett.113.012001`.

[3] J. Ashman et al. "A Measurement of the Spin Asymmetry and Determination of the Structure Function g(1) in Deep Inelastic Muon-Proton Scattering." In: *Phys. Lett. B* 206 (1988). Ed. by V. W. Hughes and C. Cavata, p. 364. DOI: `10.1016/0370-2693(88)91523-7`.

[4] I. Scimemi and A. Vladimirov. "Non-perturbative structure of semi-inclusive deep-inelastic and Drell-Yan scattering at small transverse momentum." In: *JHEP* 06 (2020), p. 137. DOI: `10.1007/JHEP06(2020)137`.

[5] M. Bury, A. Prokudin, and A. Vladimirov. "Extraction of the Sivers Function from SIDIS, Drell-Yan, and $W^\pm/Z$ Data at Next-to-Next-to-Next-to Leading Order." In: *Phys. Rev. Lett.* 126.11 (2021), p. 112002. DOI: `10.1103/PhysRevLett.126.112002`.

[6] A. A. Vladimirov. "Self-contained definition of the Collins-Soper kernel." In: *Phys. Rev. Lett.* 125.19 (2020), p. 192002. DOI: `10.1103/PhysRevLett.125.192002`.

[7] A. A. Vladimirov and A. Schäfer. "Transverse momentum dependent factorization for lattice observables." In: *Phys. Rev. D* 101.7 (2020), p. 074517. DOI: `10.1103/PhysRevD.101.074517`.

[8] M. Schlemmer, A. Vladimirov, C. Zimmermann, M. Engelhardt, and A. Schäfer. "Determination of the Collins-Soper Kernel from Lattice QCD." In: *JHEP* 08 (2021), p. 004. DOI: `10.1007/JHEP08(2021)004`.

[9] S. Moch, B. Ruijl, T. Ueda, J. A. M. Vermaseren, and A. Vogt. "Low moments of the four-loop splitting functions in QCD." In: *Phys. Lett. B* 825 (2022), p. 136853. DOI: `10.1016/j.physletb.2021.136853`.

[10] V. M. Braun, Y. Ji, and A. N. Manashov. "Two-photon processes in conformal QCD: Resummation of the descendants of leading-twist operators." In: *JHEP* 51 (2021), p. 51. DOI: `10.1007/JHEP03(2021)051`.

[11] K. G. Wilson and J. B. Kogut. "The Renormalization group and the epsilon expansion." In: *Phys. Rept.* 12 (1974), pp. 75–199. DOI: `10.1016/0370-1573(74)90023-4`.

[12] B. A. Kniehl and O. L. Veretin. "Bilinear quark operators in the RI/SMOM scheme at three loops." In: *Phys. Lett. B* 804 (2020), p. 135398. DOI: `10.1016/j.physletb.2020.135398`.

[13] B. A. Kniehl and O. L. Veretin. "Moments $n = 2$ and $n = 3$ of the Wilson twist-two operators at three loops in the RI′/SMOM scheme." In: *Nucl. Phys. B* 961 (2020), p. 115229. DOI: 10.1016/j.nuclphysb.2020.115229.

[14] V. M. Braun, K. G. Chetyrkin, and B. A. Kniehl. "Renormalization of parton quasi-distributions beyond the leading order: spacelike vs. timelike." In: *JHEP* 07 (2020), p. 161. DOI: 10.1007/JHEP07(2020)161.

[15] V. M. Braun, K. G. Chetyrkin, and B. A. Kniehl. "Operator product expansion of the non-local gluon condensate." In: *JHEP* 05 (2021), p. 231. DOI: 10.1007/JHEP05(2021)231.

[16] V. M. Braun, Y. Ji, and A. Vladimirov. "QCD factorization for twist-three axial-vector parton quasidistributions." In: *JHEP* 05 (2021), p. 086. DOI: 10.1007/JHEP05(2021)086.

[17] V. M. Braun, Y. Ji, and A. Vladimirov. "QCD factorization for chiral-odd parton quasi- and pseudodistributions." In: *JHEP* 10 (2021), p. 087. DOI: 10.1007/JHEP10(2021)087.

[18] G. S. Bali, V. M. Braun, S. Bürger, M. Göckeler, M. Gruber, F. Hutzler, P. Korcyl, A. Schäfer, A. Sternbeck, and P. Wein. "Light-cone distribution amplitudes of pseudoscalar mesons from lattice QCD." In: *JHEP* 08 (2019). [Addendum: JHEP 11, 037 (2020)], p. 065. DOI: 10.1007/JHEP08(2019)065.

[19] M. Abele, D. de Florian, and W. Vogelsang. "Approximate NNLO QCD corrections to semi-inclusive DIS." In: *Phys. Rev. D* 104.9 (2021), p. 094046. DOI: 10.1103/PhysRevD.104.094046.

[20] M. Abele, D. de Florian, and W. Vogelsang. "Threshold resummation at N^3LL accuracy and approximate N^3LO corrections to semi-inclusive DIS" (Mar. 2022). arXiv: 2203.07928 [hep-ph].

[21] C. Adolph et al. "Multiplicities of charged pions and charged hadrons from deep-inelastic scattering of muons off an isoscalar target." In: *Phys. Lett. B* 764 (2017), pp. 1–10. DOI: 10.1016/j.physletb.2016.09.042.

[22] I. Borsa, D. de Florian, R. Sassot, M. Stratmann, and W. Vogelsang. "Towards a Global QCD Analysis of Fragmentation Functions at Next-To-Next-To-Leading Order Accuracy" (Feb. 2022). arXiv: 2202.05060 [hep-ph].